Simplifying the expressions in the next example requires a bit more work and/or caution.

Example 14: Simplifying Radical Expressions

a. Simplify the expression $\sqrt[4]{x^2}$.

b. Write $\sqrt[3]{2} \cdot \sqrt{3}$ as a single radical.

Solution

a. $\sqrt[4]{x^2} = \left(x^2\right)^{\frac{1}{4}}$
$= |x|^{\frac{1}{2}}$
$= \sqrt{|x|}$

We might be tempted to write $\sqrt[4]{x^2}$ as simply $\sqrt{x}$, but note that $\sqrt[4]{x^2}$ is defined for *all* real numbers x, while $\sqrt{x}$ is defined only for nonnegative real numbers. We can rectify this disparity by first making sure the radicand is nonnegative.

b. $\sqrt[3]{2} \cdot \sqrt{3} = 2^{\frac{1}{3}} \cdot 3^{\frac{1}{2}}$
$= 2^{\frac{2}{6}} \cdot 3^{\frac{3}{6}}$
$= \left(2^2\right)^{\frac{1}{6}}\left(3^3\right)^{\frac{1}{6}}$
$= 4^{\frac{1}{6}} \cdot 27^{\frac{1}{6}}$
$= 108^{\frac{1}{6}}$
$= \sqrt[6]{108}$

We can make use of the property $a^n b^n = (ab)^n$ if we can first make the exponents equal. We do so by finding the least common denominator of $\frac{1}{3}$ and $\frac{1}{2}$, writing both fractions with this common denominator, and then making use of the property $a^{nm} = \left(a^n\right)^m$.

1.2 EXERCISES

PRACTICE

Simplify each of the following expressions, writing your answer with only positive exponents. See Examples 1 and 2.

1. $(-2)^4$ **2.** -2^4 **3** $3^2 \cdot 3^2$

4. $\frac{7^4}{7^5}$ **5.** $\frac{x^5}{x^2}$ **6.** $n^2 \cdot n^5$

Use the properties of exponents to simplify each of the following expressions, writing your answer with only positive exponents. See Examples 1, 2, and 3.

7. $\frac{3t^{-2}}{t^3}$ **8.** $9^0 x^3 y^0$ **9.** $\frac{2n^3}{n^{-5}}$

10. $\dfrac{x^7y^{-3}z^{12}}{x^{-1}z^9}$ **11.** $\dfrac{s^5y^{-5}z^{-11}}{s^8y^{-7}}$ **12.** $\dfrac{\left(3yz^{-2}\right)^0}{3y^2z}$

13. $\left[9m^2-\left(2n^2\right)^3\right]^{-1}$ **14.** $\dfrac{(-3a)^{-2}\left(bc^{-2}\right)^{-3}}{a^5c^4}$ **15.** $\left[\left(5m^4n^{-2}\right)^{-1}\right]^{-2}$

16. $\left[\left(4a^2b^{-5}\right)^{-1}\right]^{-3}$ **17.** $\left[\left(3^{-1}x^{-1}y\right)\left(x^2y\right)^{-1}\right]^{-3}$ **18.** $\left[\dfrac{100^0\left(x^{-1}y^3\right)^{-1}}{x^2y}\right]^{-3}$

19. $\left(5z^6-\left(3x^3\right)^4\right)^{-1}$ **20.** $\left[\dfrac{y^6\left(xy^2\right)^{-3}}{3x^{-3}z}\right]^{-2}$

Convert each number from scientific notation to standard notation, or vice versa, as indicated. See Example 4.

21. $-912,000,000$; convert to scientific **22.** 0.00000021; convert to scientific

23. 3.2×10^7; convert to standard **24.** 1.934×10^{-4}; convert to standard

25. There are approximately 31,536,000 seconds in a calendar year. Express the number of seconds in scientific notation.

26. Together, the 46 human chromosomes are estimated to contain some 3.0×10^9 base pairs of DNA. Express the number of pairs of DNA in standard notation.

27. A white blood cell is approximately 3.937×10^{-4} inches in diameter. Express this diameter in standard notation.

28. The probability of winning the lottery with one dollar is approximately 0.0000002605. Express this probability in scientific notation.

Evaluate each expression using the properties of exponents. Use a calculator only to check your final answer. See Example 5.

29. $\dfrac{\left(2\times10^3\right)\left(7\times10^{-2}\right)}{\left(5\times10^4\right)}$ **30.** $\dfrac{\left(8\times10^{-3}\right)\left(3\times10^{-2}\right)}{\left(2\times10^5\right)}$

31. $\left(2\times10^{-13}\right)\left(5.5\times10^{10}\right)\left(-1\times10^3\right)$ **32.** $\left(6\times10^{21}\right)\left(5\times10^{-19}\right)\left(5\times10^4\right)$

33. $\dfrac{4\times10^{-6}}{\left(5\times10^4\right)\left(8\times10^{-3}\right)}$ **34.** $\dfrac{\left(4.6\times10^{12}\right)\left(9\times10^3\right)}{\left(1.5\times10^8\right)\left(2.3\times10^{-5}\right)}$

Apply the definition of integer exponents to demonstrate the following properties.

35. $a^n\cdot a^m=a^{n+m}$ **36.** $\left(a^n\right)^m=a^{nm}$ **37.** $(ab)^n=a^nb^n$

Evaluate the following radical expressions. See Example 7.

38. $-\sqrt{9}$

39. $\sqrt[3]{-27}$

40. $\sqrt{-25}$

41. $\sqrt[3]{-\frac{27}{125}}$

42. $\sqrt{\frac{25}{121}}$

43. $-\sqrt[3]{-8}$

44. $\sqrt[4]{\sqrt{16}-\sqrt[3]{-27}+\sqrt{81}}$

45. $\sqrt{\frac{\sqrt[3]{-64}}{-\sqrt{144}-\sqrt{169}}}$

Simplify the following radical expressions. See Example 9.

46. $\sqrt[3]{-8x^6y^9}$

47. $\sqrt{2x^6y}$

48. $\sqrt[7]{x^{14}y^{49}z^{21}}$

49. $\sqrt{\frac{x^2}{4x^4y^6}}$

50. $\sqrt[3]{\frac{a^3b^{12}}{27c^6}}$

51. $\sqrt[4]{\frac{x^{12}y^8}{16}}$

52. $\sqrt[5]{\frac{y^{30}z^{25}}{32x^{35}}}$

53. $\sqrt[5]{32x^7y^{10}}$

Simplify the following radicals by rationalizing the denominators. See Example 10.

54. $\sqrt[3]{\frac{4x^2}{3y^4}}$

55. $\frac{-\sqrt{3a^3}}{\sqrt{6a}}$

56. $\frac{10}{\sqrt{7}-\sqrt{2}}$

57. $\frac{3}{\sqrt{6}-\sqrt{3}}$

58. $\frac{\sqrt{x}}{\sqrt{x}-\sqrt{2}}$

59. $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

Rationalize the numerators of the following expressions. See Example 11.

60. $\frac{3+\sqrt{y}}{6}$

61. $\frac{\sqrt{13}+\sqrt{t}}{13-t}$

62. $\frac{2\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

Combine the radical expressions, if possible. See Example 12.

63. $\sqrt[3]{-16x^4}+5x\sqrt[3]{2x}$

64. $\sqrt{27xy^2}-4\sqrt{3xy^2}$

65. $\sqrt{7x}-\sqrt[3]{7x}$

66. $-x^2\sqrt[3]{54x}+3\sqrt[3]{2x^7}$

67. $\sqrt[5]{32x^{13}}+3x\sqrt[5]{x^8}$

68. $\sqrt[3]{-16z^4}+6z\sqrt[3]{2z}$

Simplify the following expressions, writing your answer using the same notation as the original expression. See Example 13.

69. $\sqrt[3]{\sqrt[4]{x^{36}}}$

70. $32^{-\frac{3}{5}}$

71. $(3x^2-4)^{\frac{1}{3}}(3x^2-4)^{\frac{5}{3}}$

72. $81^{\frac{3}{4}}$

73. $(-8)^{\frac{2}{3}}$

74. $625^{-\frac{3}{4}}$

75. $\sqrt[3]{\sqrt[5]{y^{25}}}$

76. $\frac{(a-b)^{-\frac{2}{3}}}{(a-b)^{-2}}$

Convert the following expressions from radical notation to exponential notation, or vice versa. Simplify each expression in the process, if possible.

77. $256^{-\frac{3}{4}}$ **78.** $\sqrt[12]{x^3}$ **79.** $\sqrt[6]{\frac{2}{72}}$ **80.** $\left(36n^4\right)^{\frac{5}{6}}$

Simplify the following expressions. See Example 14.

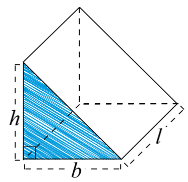
81. $\sqrt{5}\cdot\sqrt[4]{5}$ **82.** $\sqrt[3]{x^7}\cdot\sqrt[9]{x^6}$ **83.** $\sqrt[5]{y^{16}}\cdot\sqrt[25]{y^{20}}$ **84.** $\sqrt[4]{7}\cdot\sqrt[16]{7}$

Apply the definition of rational exponents to demonstrate the following properties.

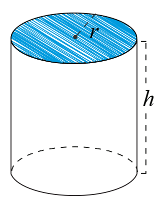
85. $\sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$ **86.** $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ **87.** $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}$

APPLICATIONS

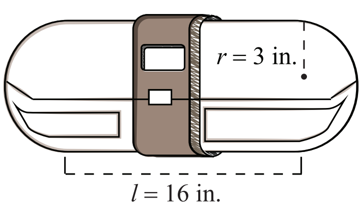
88. The prism shown below is a right triangular cylinder, where the base is a right triangle. Find the volume of the prism in terms of b, h, and l.



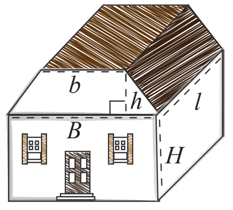
89. Determine the volume of the right circular cylinder shown, in terms of r and h.



90. Matt wants to let people in the future know what life is like today, so he goes shopping for a time capsule. Capacity, along with price and quality, is an important consideration for him. One time capsule he looks at is a right circular cylinder with a hemisphere on each end. Find the volume of the time capsule, given that the length l of the cylinder is 16 inches and the radius r is 3 inches.



91. Bill and Dee are buying a new house. The house is a right cylinder based on a trapezoid atop a rectangular prism. The bases of the trapezoid are $B = 10$ m and $b = 8$ m, and the length of the house is $l = 15$ m. The height of the house up to the bottom of the roof is $H = 3$ m, and the height of the roof is $h = 1$ m. Find the volume of the house.

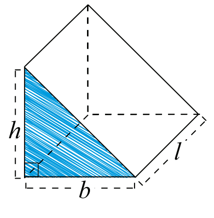


92. Determine the expression for the volume of water contained in an above-ground circular swimming pool that has a diameter of 18 feet, assuming the water has a uniform depth of *d* feet.

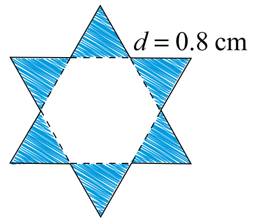
93. The floor of a rectangular bedroom measures *N* feet wide and *M* feet long. The height of the walls is 7 feet. Find an expression for the number of square feet of wallpaper needed to cover all the walls. (Ignore the presence of doors and windows.)

94. The interior surface of the birdbath in Example 6c needs to be painted with a waterproof (and nontoxic) coating. Determine the expression for the interior surface area.

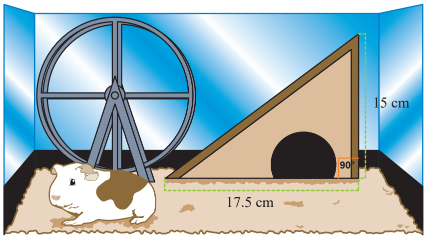
95. The prism shown below is a right triangular cylinder, where the base is a right triangle. Find the surface area of the prism in terms of *b*, *h*, and *l*.



96. A jeweler decides to construct a pendant for a necklace by simply attaching equilateral triangles to each edge of a regular hexagon. The edge length of one of the points of the resulting star is $d = 0.8$ cm. Find the formula for the area of the star in terms of *d* and then evaluate for $d = 0.8$ cm (rounding to three decimal places). Use the fact that the area of an equilateral triangle of side length *d* is $A = \frac{d^2\sqrt{3}}{4}$.



97. Ilyana has made a home for her pet guinea pig (Ralph) in the shape of a right triangular cylinder. Before she can put the new home in Ralph's cage, she must paint it with a nontoxic outer coat. If the front of the home has a base of 17.5 cm and a height of 15 cm and the length of the home is 25 cm, what is the surface area of Ralph's home, rounded to the nearest square centimeter? The small bottle of nontoxic coating will cover up to 1500 cm^2. Will the small bottle contain enough nontoxic coating to cover Ralph's home?



98. Einstein's Theory of Special Relativity tells us that $E = mc^2$, where E is energy (in joules, J), m is mass (in kilograms, kg), and c is the speed of light (in meters per second, m/s). This equation may also be written as $\sqrt{\frac{E}{m}} = c$. Assume you know $E = 418{,}400$ J and $m = 4.655 \times 10^{-12}$ kg. Use this information to estimate the speed of light.

WRITING & THINKING

99. Give a few examples of instances in which it would be more useful to use scientific notation rather than standard.

100. In February of 2006, the US national debt was approximately 8.2 trillion dollars. How is saying 8.2 trillion similar to scientific notation? How is it different?

101. In your own words, explain why $a^0 = 1$.

102. Explain, in your own words, why the square root of a negative number is not a real number.

103. Explain, in your own words, why exponents and roots are evaluated at the same time in the order of operations.