b.
$$-6x = 30$$
$$-6x\left(-\frac{1}{6}\right) = 30\left(-\frac{1}{6}\right)$$
$$x = -5$$

Using multiplicative cancellation, we multiply both sides by $-\frac{1}{6}$, then simplify.

Alternatively, dividing both sides by -6 yields the same result. Thus, the equation $-6x = 30$ is equivalent to equation $x = -5$. We can see how cancellation properties can help us *solve* equations for variables.

c. Multiplying both sides of the equation $x^2 - x = 2$ by 0 leads to the equation $0 = 0$ a true statement. However, these two equations are not equivalent!

While replacing x in the first equation by -1 or 2 leads to a true statement, any other value for x leads to a false statement. By contrast, the equation $0 = 0$ is true for all values of x, as there is no x in the equation to replace with a number. This example illustrates why we must multiply both sides of an equation by a nonzero quantity to apply multiplicative cancellation.

d. The equation $(x - y)(x + y) = 0$ means that $x - y = 0$ or $x + y = 0$, by the Zero-Factor Property. Remember that the only way for a product of two (or more) factors to be 0 is for *at least* one of the factors to be 0 itself. For instance, in this example it might be that *both* $x - y = 0$ and $x + y = 0$. If $x = 0$ and $y = 0$, this is indeed the case.

1.1 EXERCISES

PRACTICE

Which elements of the following sets are **a.** natural numbers, **b.** whole numbers, **c.** integers, **d.** rational numbers, **e.** irrational numbers, **f.** real numbers? See Example 1.

1. $\left\{19,\ -4.3,\ -\sqrt{3},\ \frac{0}{15},\ 2^5,\ -33\right\}$

2. $\left\{5\sqrt{7},\ 4\pi,\ \sqrt{16},\ 3.\overline{3},\ -1,\ \frac{22}{7},\ |-8|\right\}$

Plot the real numbers in the following sets on a number line. Choose the unit length appropriately for each set.

3. $\{-4.5, -1, 2.5\}$ **4.** $\{-24, 2, 15\}$ **5.** $\{5.1, 5.2, 5.8\}$ **6.** $\left\{0, \frac{1}{2}, \frac{5}{6}\right\}$

Select all of the symbols from the set $\{<, \leq, >, \geq\}$ that can be placed in the blank to make each statement true.

7. $12 \_\_\_ 14$ **8.** $-102 \_\_\_ 9$ **9.** $3 \_\_\_ 3$

10. $-50 \_\_\_ -45$ **11.** $-3.4 \_\_\_ -3.5$ **12.** $-\frac{1}{4} \_\_\_ -\frac{1}{3}$

Write each statement as an inequality, using the appropriate inequality symbol.

13. "$2a+b$ is strictly greater than c"

14. "2 is less than or equal to x"

15. "$2c$ is no more than $3d$"

16. "$6+x$ is greater than or equal to $4x$"

Describe each of the following sets using set-builder notation. There may be more than one correct way to do this. See Example 4.

17. $\{5,6,7,\ldots,105\}$

18. $\{2,3,5,7,11,13,17,\ldots\}$

19. $\{1,2,4,8,16,32,\ldots\}$

20. $\{-6,-3,0,3,6,9\}$

21. $\left\{\ldots,\frac{1}{3},\frac{1}{5},\frac{1}{7},\frac{1}{9},\ldots\right\}$

22. $\{0,1,2,3,4,5,\ldots\}$

Write each set as an interval using interval notation. See Example 5.

23. $\{x\,|\,-3\le x<19\}$

24. $\{x\,|\,x<4\}$

25. $x<15$

26. $-9\le x\le 6$

27. The positive real numbers

28. The nonnegative real numbers

Graph the following intervals.

29. $[5,14)$

30. $[-9,-1]$

31. $(0,2)$

32. $(-3,18]$

33. $(-\infty,7]$

34. $(25,\infty)$

Simplify the following set expressions. See Example 6.

35. $[-7,7)\cup(2,5)$

36. $(-5,2]\cup(2,4]$

37. $(-5,2]\cap(2,4]$

38. $[3,5]\cap[2,4]$

39. $(-\infty,4]\cup(0,\infty)$

40. $(-\infty,\infty)\cap[-\pi,21)$

Simplify the following set expressions. See Example 7.

41. $\mathbb{Q}\cap\mathbb{Z}$

42. $\mathbb{N}\cup\mathbb{R}$

43. $\mathbb{N}\cup\mathbb{Z}\cap\mathbb{Q}$

44. $(-4.8,-3.5)\cap\mathbb{Z}$

Evaluate the absolute value expressions. See Examples 8 and 9.

45. $-|-11|$

46. $|3-7|$

47. $\left|\sqrt{3}-\sqrt{5}\right|$

48. $\left|-\sqrt{2}\right|$

49. $\frac{|-x|}{|x|}\,(x\neq 0)$

50. $-|4-9|$

51. $-|-4-|-11||$

52. $-\left|-\sqrt{|-9|}-|-9|\right|$

Find the distance on the real number line between each pair of numbers given. See Example 8.

53. $a = 8, b = 3$

54. $a = 5, b = 5$

55. $a = 4, b = -2$

56. $a = -12, b = -1$

Identify the components of the algebraic expressions, as indicated. See Example 10.

57. Identify the terms in the expression $3x^2y^3 - 2\sqrt{x+y} + 7z$.

58. Identify the factors in the term $-2\sqrt{x+y}$.

59. Identify the coefficients in the expression $x^2 + 8.5x - 14y^3$.

60. Identify the factors in the term $8.5x$.

61. Identify the terms in the expression $\dfrac{-5x}{2yz} - 8x^5y^3 + 6.9z$.

62. Identify the coefficients in the expression $\dfrac{-5x}{2yz} - 8x^5y^3 + 6.9z$.

Evaluate the following algebraic expressions for the given values of the variables. See Example 11.

63. $3x^3 + 5x - 2$ for $x = -3$

64. $\sqrt{2x} + \dfrac{3x}{4}$ for $x = 8$

65. $-3\pi y + 8x + y^3$ for $x = 2$ and $y = -2$

66. $y\sqrt{x^3 - 2} + \sqrt{x - 2y} - 3y$ for $x = 3$ and $y = -\dfrac{1}{2}$

67. $|x - 9y| - (8z - 8)$ for $x = -3$, $y = 1$, and $z = 5$

68. $\dfrac{x^2y^3}{8z} - \dfrac{|2xy|}{8z}$ for $x = 2$, $y = -1$, and $z = 3$

Identify the property that justifies each of the following statements. If one of the cancellation properties is being used to transform an equation, identify the quantity that is being added to both sides or the quantity by which both sides are being multiplied. See Examples 12 and 14.

69. $(x - y)(z^2) = (z^2)(x - y)$

70. $3 - 7 = -7 + 3$

71. $4(y - 3) = 4y - 12$

72. $-3(4x^6z) = (-3)(4)(x^6z) = -12x^6z$

73. $4 + (-3 + x) = (4 - 3) + x = 1 + x$

74. $(x + y)\left(\dfrac{1}{x + y}\right) = 1$

75. $25x^3 = 10y \Leftrightarrow 5x^3 = 2y$

76. $-14y = 7 \Leftrightarrow y = -\dfrac{1}{2}$

77. $14 - x = 2x \Leftrightarrow 14 = 3x$

78. $(a + b)(x) = 0 \Rightarrow a + b = 0$ or $x = 0$

79. $\frac{x}{6}+\frac{y}{3}-2=0 \Leftrightarrow x+2y-12=0$

80. $x^2z=0 \Rightarrow x^2=0$ or $z=0$

81. $5+3x-y=2x-y \Leftrightarrow 5+x=0$

82. $(x-3)(x+2)=0 \Rightarrow x-3=0$ or $x+2=0$

APPLICATIONS

83. Jess, Stan, Nina, and Michele are in a marathon. Twenty-five minutes after beginning, Jess has run 3.4 miles, Stan has run 4 miles, Nina has run 2.25 miles, and Michele has walked 1.6 miles. Using 0 as the beginning point, plot each competitor's location on a real number line using an appropriate interval.

84. Freddie, Sarah, Elizabeth, JR, and Aubrey are trying to line up by height for a photo shoot. JR is the tallest and Elizabeth is the shortest. Freddie is taller than Sarah, and Sarah is taller than Aubrey. Express their lineup using appropriate inequality symbols.

85. Sue boards an eastbound train in Center Station at the same time Joy boards a westbound train in Center Station. After riding the Straight Line for 20 minutes, Sue's train has traveled 13 miles east, while Joy's train (also on the Straight Line) has traveled 7 miles west. Find the distance between the two trains at this time. (Assume the Straight Line is true to its name and that the tracks lie literally along a straight line.)

86. The admission prices at the local zoo are as follows.

Admission Prices

Children under 2	free
Children under 12	\$3
Adults	\$7
Seniors (65 and up)	\$5

Express the age range for each of these prices in set-builder notation and interval notation.

87. A particular fudge recipe calls for at least 3 but no more than 4 cups of sugar and at least $\frac{1}{2}$ but no more than $\frac{2}{3}$ of a cup of walnuts. Express the amount of sugar and nuts needed in both set-builder and interval notation.

88. At the beginning of the month, your checking account contains \$128. For your birthday, your mother deposits \$50 and your grandmother deposits \$25. After you write three checks for \$17, \$23, and \$62, you make a deposit of \$41. At the end of the month, your bank removes half of the balance to put in your savings account and then charges you a \$5 fee for doing so. How much do you have remaining in your checking account?

89. A particular liquid boils at 268 °F. Given the formula $C=\frac{5}{9}(F-32)$ for converting temperatures from Celsius (C) to Fahrenheit (F), find the boiling point of this liquid in the Celsius scale. Round your answer to two decimal places.

90. Stephen received \$75 as a gift from his aunt. With this money, he decided to start saving to buy the newest gaming console, which costs \$398 after tax. After working two weeks at his part-time job, he got one check for \$123 and a second check for \$98. How much more does Stephen need to save to buy his gaming console?

91. Body mass index, abbreviated BMI, is one way doctors determine an adult's weight status. A BMI below 18.5 is considered underweight, the range 18.5–24.9 is normal, the range 25.0–29.9 is overweight, and a BMI above 30.0 indicates obesity. The formula used to determine BMI is $\text{BMI} = 703\left(\dfrac{\text{weight in pounds}}{(\text{height in inches})^2}\right)$.

Derek weighs 180 pounds and is 73 inches tall. Use this formula to determine Derek's BMI and weight status. Round your answer to one decimal place.

92. The Du Bois Method provides a formula used to estimate your body's surface area in meters squared: $\text{BSA} = 0.007184h^{0.725}w^{0.425}$ where h is height in centimeters and w is weight in kilograms. Assume Juan is 193 cm tall and weighs 88 kg. Use the Du Bois Method to estimate his body's surface area in square meters. Round your answer to two decimal places.

93. Samantha drops a tennis ball from the top of the mathematics building. If it takes the ball 3.42 seconds to hit the ground, use the formula $\text{distance} = \frac{1}{2}(\text{acceleration})(\text{time})^2$ to find the height of the building, which is equivalent to the distance the ball falls. Use the value of $32\ \text{ft/s}^2$ for the acceleration of a falling object. Round your answer to the nearest foot.

WRITING & THINKING

94. Choose a number. Multiply it by 3 and then add 4. Now multiply by 2 and subtract 8. Finally divide by 6. What do you notice about your final answer? Explain why you got this as a result.

95. After taking a poll in her town, Sally began grouping the citizens into various sets. One set contained all the citizens with brown hair and another set contained all the citizens with blue eyes. What do you know about the citizens who would be listed in the union of these two sets? What do you know about the citizens who would be listed in the intersection of these two sets?

96. Can a natural number be irrational? Explain.

97. Are all whole numbers also integers? Are all integers also whole numbers? Explain your answers.

98. In your own words, define absolute value.

99. Write a short paragraph explaining the similarities and differences between $>$ and $\geq$.

100. In your own words, explain the difference between a union and an intersection of two sets.

TECHNOLOGY

Select all of the symbols from the set $\{<, \leq, >, \geq\}$ that can be placed in the blank to make each statement true. Use a graphing utility to check your answers.

101. $-2.9 \_\_\_\_ -3.1$ **102.** $2.1 \_\_\_\_ -5.5$ **103.** $100 \_\_\_\_ -4$

104. $0.001 \_\_\_\_ -99.8$ **105.** $\frac{1}{3} \_\_\_\_ \frac{1}{4}$ **106.** $-\frac{1}{5} \_\_\_\_ -\frac{3}{4}$

Use a graphing utility to evaluate the following algebraic expressions.

107. $\sqrt{x^4y - z} + \dfrac{x - y^3}{z^2}$ for $x = -3$, $y = 2$, and $z = -2$

108. $\dfrac{\left(x - pq^2\right)^3}{2q^3}$ for $x = -5$, $p = 2$, and $q = -3$

109. $\dfrac{\left|x^2 - y^3\right| - 4x}{3y^5}$ for $x = 2$ and $y = 3$

110. $\sqrt{p^3q - q^3} - \left|p + q^2\right|$ for $p = -5$ and $q = 2$