CHAPTER 1 REVIEW EXERCISES

Section 1.1

Which elements of the following set are **a.** natural numbers, **b.** whole numbers, **c.** integers, **d.** rational numbers, **e.** irrational numbers, **f.** real numbers?

1. $\left\{\frac{3}{7},\ -\sqrt{4},\ 2^3,\ 5.3,\ |-2.1|,\ \sqrt{17},\ 0\right\}$

Describe the following set using set-builder notation. There may be more than one correct way to do this.

2. $\left\{\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10},\ldots\right\}$

Write each set as an interval using interval notation.

3. $4 \le x < 17$

4. $\{x \mid -8 \le x \le -1\}$

Evaluate the absolute value expressions.

5. $-|-4-3|$

6. $-|11-2|$

7. $|\sqrt{9}-7|$

8. $|\sqrt{5}-\sqrt{11}|$

9. $-\frac{|x|}{|-x|}$

10. Liz, Monica, Peter, James, and Melissa are comparing their ages. Liz is older than Peter and Melissa is the youngest. James is the oldest and Peter is older than Monica. Order them from youngest to oldest.

Identify the components of the algebraic expressions, as indicated.

11. Identify the terms in the expression $\frac{x^2}{2y}+12.1x-\sqrt{y+5}$.

12. Identify the coefficients in the expression $\frac{x^2}{2y}+12.1x-\sqrt{y+5}$.

Evaluate the following algebraic expressions for the given values of the variables.

13. $7y^2-\frac{1}{3}\pi xy+8x^3$ for $x=-2$ and $y=2$

14. $x^2z^3+5\sqrt{3x-2y}$ for $x=2, y=1,$ and $z=-1$

15. $|-3x+x^2y|-\frac{xy}{2}$ for $x=-3$ and $y=4$

16. $3\sqrt{\frac{xy}{3}}-2y^2$ for $x=2$ and $y=6$

Identify the property that justifies each of the following statements. If one of the cancellation properties is being used to transform an equation, identify the quantity that is being added to both sides or the quantity by which both sides are being multiplied.

17. $-4+x=x-4$

18. $12a^2=8b \Leftrightarrow 3a^2=2b$

19. $(x-3)(z-2)=0 \Rightarrow x-3=0$ or $z-2=0$

Simplify the following set expressions.

20. $(-4,8)\cup[5,13]$

21. $(-4,8)\cap[5,13]$

Section 1.2

Use the properties of exponents to simplify each of the following expressions, writing your answer with only positive exponents.

22. $\left(2^3a^{-2}b^4\right)^{-1}c^{-3}$

23. $\dfrac{-4t^0\left(s^2t^{-2}\right)^{-3}}{2^3st^{-3}}$

24. $\left[\left(3y^{-2}z\right)^{-1}\right]^{-3}$

25. $\dfrac{3^2x^{-4}\left(y^2z\right)^{-2}}{\left(2z^{-3}\right)^{-1}y^{-6}}$

Convert each number from scientific notation to standard notation, or vice versa, as indicated.

26. -3.005×10^{-4}; convert to standard

27. 69,520,000; convert to scientific

Evaluate each expression using the properties of exponents. Use a calculator only to check your final answer.

28. $\left(3.46\times10^8\right)\left(1.2\times10^4\right)$

29. $\dfrac{2.4\times10^{-12}}{(1.2)\times10^{-4}}$

30. Sam is making a piñata in the shape of a sphere and needs to know how much candy to buy to fill it. If the radius of the piñata is 10 inches, what is the volume of the piñata?

Evaluate the following radical expressions.

31. $\sqrt{3^2+4^2}$

32. $\dfrac{\sqrt[3]{\sqrt{15}}}{\sqrt{\sqrt[3]{5}}}$

Simplify the following radical expressions. Rationalize all denominators and use only positive exponents.

33. $\sqrt{25x^{20}}$

34. $\sqrt{16x^2}$

35. $\sqrt[3]{-64x^{-9}y^3}$

36. $\dfrac{\sqrt{3a^3}}{\sqrt{12a}}$

37. $\sqrt[3]{\dfrac{8x^2}{3y^{-4}}}$

38. $\sqrt[4]{\dfrac{a^9b^{-4}}{81}}$

39. $\dfrac{4}{\sqrt{2}-\sqrt{6}}$

40. $\dfrac{3}{\sqrt{x}+\sqrt{2}}$

Simplify the following expressions.

41. $\sqrt{18x^3y}-\sqrt[3]{16x^4y}$

42. $\left(2\sqrt{3}-5\sqrt{2}\right)^2$

Convert the following expressions from radical notation to exponential notation, or vice versa. Simplify each expression in the process, if possible.

43. $\sqrt{x^{-5}} \cdot \sqrt[4]{x^3}$

44. $\left(49x^4\right)^{\frac{1}{2}}\left(16x^{12}\right)^{\frac{3}{4}}$

Section 1.3

Add or subtract the polynomials, as indicated.

45. $\left(-4m^2 - 5m^3 + 4\right) + \left(m^4 + 7m^2 - 2\right)$

46. $\left(2xy + 3x\right) - \left(8x^2y - 6xy + 3x - y\right)$

Multiply the polynomials, as indicated.

47. $\left(x^2 + y\right)\left(3x - 4y^3\right)$

48. $\left(a + 5b\right)\left(5a - 7ab + 2b\right)$

Factor each of the following polynomials.

49. $x^2 - x - 12$

50. $2x^2 + x - 15$

51. $6a^2 - 7a - 5$

52. $4a^2 - 9b^4$

53. $36x^6 - y^2$

54. $nx + 3mx - 2ny - 6my$

55. $2x^2 + 6x - 5xy - 15y$

56. $8x^3y^2 + 4x^3y - 12xy^2$

Factor the following algebraic expressions.

57. $(3x - 2y)^{\frac{4}{3}} - (3x - 2y)^{\frac{2}{3}}$

58. $8x^{-2} + 5x^{-1}$

Section 1.4

Simplify the following rational expressions, indicating which real values of the variable must be excluded.

59. $\dfrac{x^3 + 6x^2 + 9x}{x^3 - 9x}$

60. $\dfrac{x^2 - 9}{x^3 - 27}$

Perform the indicated operations on the rational expressions and simplify your answer.

61. $\dfrac{1}{x} - \dfrac{3}{x+2} - \dfrac{6}{x^2 + 2x}$

62. $\dfrac{a^3 - 8}{a^2 - 4} \div \dfrac{a^3 + 2a^2 + 4a}{a^3 + 2a^2} \cdot \dfrac{1}{a^2 + a}$

Simplify the complex rational expressions.

63. $\dfrac{\dfrac{1}{2a} - \dfrac{1}{2b}}{\dfrac{2}{a} + \dfrac{2}{b}}$

64. $\dfrac{\dfrac{x}{3} - \dfrac{3}{x}}{-\dfrac{3}{x} + 1}$

65. $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{x^{-1} - y^{-1}}$

Section 1.5

Evaluate the following square root expressions.

66. $-\sqrt{-8x}$

67. $i^3\sqrt{-9}$

Simplify the following expressions.

68. $(7-2i)+(9i-5)$

69. $(5-3i)-(-12i)$

70. $(3-i)(6i^2-4)$

71. $\dfrac{17}{4-i}$

72. $\dfrac{2i}{3-i}$

73. $\dfrac{3+4i}{3-4i}$

74. $(\sqrt{-3})(\sqrt{-16})$

75. $(8-\sqrt{-2})^2$

76. $\dfrac{2i\sqrt{-27}}{\sqrt{-16}}$

Section 1.6

Solve the following linear equations.

77. $2y-(1-y)=y+2(y-1)$

78. $\dfrac{x}{2}-\dfrac{1}{3}=x-\dfrac{1}{3}-\dfrac{x}{2}$

79. $-0.2x-0.5=-0.4x+0.75$

80. $-2(x-5)+1=3+(7x-2)$

Solve the following absolute value equations.

81. $|2x-7|=1$

82. $|2y-5|-1=|3-y|$

83. $|7z+5|+3=8$

84. $|w-5|=|3w+1|$

Solve the following absolute value equations geometrically and algebraically.

85. $|-2x+1|=7$

86. $|x+4|-|x-1|=0$

Solve each of the following equations for the indicated variable.

87. Area of a trapezoid: $A=\dfrac{1}{2}h(b+c)$; solve for c

88. Volume of a rectangular pyramid: $V=\dfrac{1}{3}lwh$; solve for l

89. Temperature conversions: $F=\dfrac{9}{5}C+32$; solve for C

90. Two trains leave the station at the same time in opposite directions. One travels at an average rate of 90 miles per hour, and the other at an average rate of 95 miles per hour. How far apart are the two trains after an hour and twenty minutes? Round your answer to one decimal place.

91. Two firefighters, Jake and Rose, each have \$5000 to invest. Jake invests his money in a money market account with an annual return of 3.25%, while Rose invests hers in a CD paying 4.95% annually. How much more money does Rose have than Jake after 1 year?

Section 1.7

Solve the following linear inequalities. Describe each solution set using interval notation and by graphing.

92. $-8x+3\geq -9x+10$

93. $4(2x-5)<-3(-3x+8)$

94. $\frac{-2(x-1)}{3}\leq \frac{-2x}{4}$

95. $3.1(2x-1)>7.2-4.1x$

96. $-5<3m+1<13$

97. $-14<-2(3+y)\leq 8$

98. $2<\frac{x+1}{4}\leq 7$

99. $-5|3+t|>-10$

100. $3+|2x-1|<1$

101. $-2|x-1|+|3x-3|\geq 7$

102. $6+\frac{x}{5}\leq \frac{4}{5}$ or $5+2x\geq x-2$

103. $\frac{8x-5}{9}\leq 3$ or $2(3x-16)\geq 4(x-3)$

104. $2.9x+1.8<3(1.3x+6)$ and $7x<5x+34$

Section 1.8

Solve the following quadratic equations.

105. $5x^2-13x-6=0$

106. $x^2=7$

107. $2(x-2)^2=-18$

108. $15x^2+3x+2=-8x$

109. $x^2-8x+14=0$

110. $3x^2-x+3=-7x$

111. $x^2=6x-16$

112. $-2x-7=-4x^2$

113. $2x^2+3x-10=10$

114. $x^2-7x-2=-12$

115. $1.7z^2-3.8z-2=0$

116. $2x^2+7x=x^2+2x-6$

Solve the following quadratic-like equations.

117. $(x^2+2)^2-7(x^2+2)+12=0$

118. $y^{\frac{2}{3}}+y^{\frac{1}{3}}-6=0$

119. $(t+2)^2-2(t+2)=24$

120. $x^4-13x^2+36=0$

Solve the following equations by factoring.

121. $x^3-4x^2-2x+8=0$

122. $2x^3+2x=5x^2$

123. $x^3-x^2+4x-4=0$

124. $x^4+7x^2-18=0$

125. $x^{\frac{7}{2}}-3x^{\frac{5}{2}}-4x^{\frac{3}{2}}=0$

126. $x^{\frac{7}{3}}+7x^{\frac{4}{3}}-8x^{\frac{1}{3}}=0$

127. $(x-2)^{\frac{3}{4}}+2(x-2)^{\frac{7}{4}}=0$

128. $(x-1)^{-\frac{1}{2}}+4(x-1)^{\frac{1}{2}}=0$

Use the connection between solutions of polynomial equations and polynomial factoring to answer the following questions.

129. Find b and c so the equation $x^3 + bx^2 + cx = 0$ has solutions of −2, 0, and 4.

130. Given that the equation $x^2 - 6x + m - 1 = 0$ has only one root, find m.

131. If the sum of the roots of the equation $x^2 + mx - 6 = 0$ is 5, then what is m?

Section 1.9

Solve the following rational equations.

132. $\frac{1}{x+2} + \frac{1}{x-3} - \frac{x}{x-3} = 0$

133. $\frac{1}{x-2} - \frac{x}{x+2} = \frac{2}{x^2-4}$

134. $\frac{y}{y-1} + \frac{1}{y-4} = \frac{y^2}{y^2-5y+4}$

135. $\frac{2}{x+1} - \frac{x}{x-3} = \frac{3x-21}{x^2-2x-3}$

136. Jim cleans a house in 6 hours. John cleans the same house in 8 hours. How long does it take for them to clean the house together?

Solve the following equations.

137. $\sqrt{-4-x} - 4 = x$

138. $\sqrt{5x-1} = 4 + \sqrt{x+3}$

139. $\sqrt{2x^2+8x+1} - x - 3 = 0$

140. $\sqrt{10x^2-14x+16} + 1 = 3x$

141. $x + 2 = \left(-x^2 + 11x + 19\right)^{\frac{1}{2}}$

142. $\left(2x^2 + 14x\right)^{\frac{1}{4}} = \left(-x^2 - 8\right)^{\frac{1}{4}}$

143. $(2x-5)^{\frac{1}{6}} = (x-2)^{\frac{1}{6}}$

144. $\left(x^2 + x - 16\right)^{\frac{1}{3}} = 2(x-1)^{\frac{1}{3}}$

145. The formula for the volume of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$. Solve the equation for r.

CHAPTER 2 REVIEW EXERCISES

Section 2.1

Plot the following sets of points in the Cartesian plane.

1. $\{(7,3),(-2,4),(3,0),(-1,-6)\}$

2. $\{(4,-4),(-6,3),(-3,-1),(-4,2)\}$

3. $\{(2,1),(-4,5),(3,-7),(2,3)\}$

Identify the quadrant in which each point lies, if possible. If a point lies on an axis, specify which part (positive or negative) of which axis (*x* or *y*).

4. $(0,0)$

5. $(1,0)$

6. $(3,-2)$

For each of the following equations, determine the value of the missing entries in the accompanying table of ordered pairs. Then plot the ordered pairs and sketch your guess of the complete graph of the equation.

7. $3x-2y=6$

x	y
?	0
0	?
-1	?
?	-2
-2	?

8. $3x=y^2-4$

x	y
0	?
?	0
?	$-\sqrt{7}$
-1	?
?	3

Determine **a.** the distance between the following pairs of points, and **b.** the midpoint of the line segment joining each pair of points.

9. $(2,-6)$ and $(3,-7)$

10. $(-4,-3)$ and $(4,-9)$

11. $(-3,6)$ and $(-7,0)$

12. $(5,-1)$ and $(-4,3)$

13. Given $A(-4,2)$, $B(x,y)$, and $C(1,-1)$, find $x+y$ if C is the midpoint of the line segment $\overline{AB}$.

Find the perimeter of the triangle whose vertices are the specified points in the plane.

14. $(-3,2)$, $(-3,0)$, and $(-6,-3)$

15. $(8,-3)$, $(2,-3)$, and $(2,5)$

16. Use the distance formula to prove that the triangle with vertices at the points $(-2,2),(0,3)$, and $(4,-5)$ is a right triangle and determine the area of the triangle.

Section 2.2

Find the standard form of the equation for each circle described below.

17. Radius 4; center $\left(\sqrt{5}, -\sqrt{2}\right)$

18. Endpoints of a diameter are $(1,-3)$ and $(-5,3)$.

19. Center at $(2,-1)$; passes through $(4,3)$

20. Endpoints of a diameter are $(1,2)$ and $(-5,8)$.

21. What is the radius and center of the circle $(x+3)^2+(y-1)^2=8$?

22. Given that point $(a, 4)$ is on the circle $x^2+y^2=25$, find a.

Sketch a graph of the circle defined by the given equation. Then state the radius and center of the circle.

23. $(x+5)^2+(y-2)^2=16$

24. $x^2+(y-3)^2=10$

25. $(x-1)^2+(y+4)^2=9$

26. $x^2+y^2+6x-10y=-5$

Section 2.3

Determine if the following equations are linear.

27. $3x+y(4-2x)=8$

28. $y-3(y-x)=8x$

29. $9x^2-(3x+1)^2=y-3$

30. $8x-3y=4(x-1)+y$

31. $2x(3y-1)=7$

32. $3x^2+2=(x+2)^2-1$

Find the *x*- and *y*-intercepts of the given equations, if possible, and then sketch their graphs.

33. $4y-12=8x$

34. $3(2y+1)=5y-4x+3$

35. $2x+y-2=2(3+x)$

36. $3y-4x=-2(3x-y)$

37. $2x+3y=18$

38. $4x+y=12+y$

Section 2.4

Determine the slope of the line passing through the specified points.

39. $(-2, 5)$ and $(-3, -7)$

40. $(3, 6)$ and $(7, -10)$

41. $(3, 5)$ and $(3, -7)$

Use the slope-intercept form to graph the equations.

42. $6x-3y=9$ **43.** $2y+5x+9=0$ **44.** $15y-5x=0$

Find the equation, in standard form, of the line passing through the given point with the given slope.

45. point $(4,-1)$; slope of 1 **46.** point $(-2,3)$; slope of $\frac{3}{2}$

Find the equation, in slope-intercept form, of the line with the given *y*-intercept and slope.

47. *y*-intercept $(0,-2)$; slope of $\frac{5}{9}$ **48.** *y*-intercept $(0,9)$; slope of $-\frac{7}{3}$

Find the equation, in standard form, of the line passing through the specified points.

49. $(5,7)$ and $(3,-2)$ **50.** $\left(\frac{3}{2},1\right)$ and $\left(-3,\frac{5}{2}\right)$

51. A sales person receives a monthly salary of \$2800 plus a commission of 8% of sales. Write a linear equation for the sales person's monthly wage *W*, in terms of monthly sales, *s*.

Section 2.5

Determine if the two lines are perpendicular, parallel, or neither.

52. $x-4y=3$ and $4x-y=2$

53. $3x+y=2$ and $x-3y=25$

54. $\frac{3x-y}{3}=x+2$ and $\frac{y}{3}+x=9$

Find the equation, in slope-intercept form, for the line parallel to the given line and passing through the indicated point.

55. $y-3x=10$; $(-2,4)$ **56.** $3(y+1)=\frac{x-3}{2}$; $(-6,3)$

57. $y=2x+1$; $(1,-1)$ **58.** $3y-2=-5(2x-1)$; $(2,-5)$

Find the equation, in slope-intercept form, for the line perpendicular to the given line and passing through the indicated point.

59. $y=\frac{3}{4}x-1$; $(6,-2)$ **60.** $2(y-3)=\frac{2x+3}{3}$; $(-5,-4)$

61. $y=8$; $(7,1)$ **62.** $5x+7y-2=10$; $\left(\frac{2}{7},-1\right)$

Each set of four ordered pairs defines the vertices, in counterclockwise order, of a quadrilateral. Determine if the quadrilateral is a rectangle.

63. $\{(-2,1),(-1,-1),(3,1),(2,3)\}$ **64.** $\{(-2,2),(-3,-1),(2,-3),(2,1)\}$

Section 2.6

Solve the following linear inequalities by graphing their solution sets.

65. $x - 2y < 4$ **66.** $y < 3x + 2$ **67.** $\frac{4x + y}{3} \geq 2$

Graph the solution sets that satisfy the following inequalities.

68. $7x - 2y \geq 8$ and $y < 5$ **69.** $x - 4y \geq 6$ or $y > -2$

70. $y - x > 0$ and $x < 2$

Graph the solution sets that satisfy the following linear absolute value inequalities.

71. $|2x + 5| < 3$ **72.** $|2x - 1| < 5$

73. $|x - y| < 3$ **74.** $-5 + |x - 3| > -1$

75. $|2x + 1| < 3$ or $|y + 3| \geq 4$ **76.** $|x| > 4$ and $\left|\frac{2y - 1}{3}\right| < 3$

77. A candle store makes a $3 profit for every novelty candle sold and a $4 profit for every accompanying candle holder sold. Write a linear inequality describing the number of each type of item that needs to be sold in order to make a total profit of at least $1500.

CHAPTER 3 REVIEW EXERCISES

Section 3.1

For each of the following relations, determine the domain and range and determine whether the relation is a function.

1. $R = \{(-2,9),(-3,-3),(-2,2),(-2,-9)\}$

2. $f = \{(-3,0),(-1,4),(0,3),(3,3),(4,-1)\}$

3. $R = \{(x,2) \mid x \in \mathbb{R}\}$

4. $S = \{(x,4x) \mid x \in \mathbb{Z}\}$

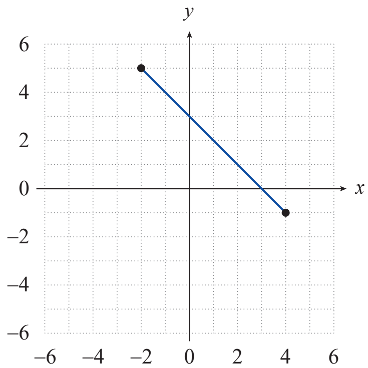
5. $3x - 4y = 17$

6. $x = y^2 - 6$

7. $x = \sqrt{y-4}$

8. $y = -5$

9.



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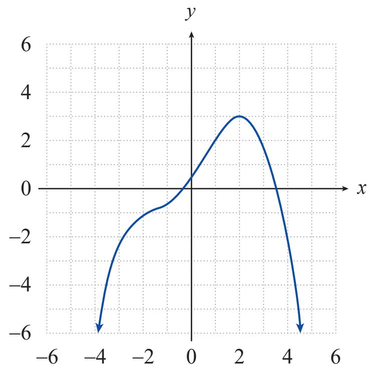
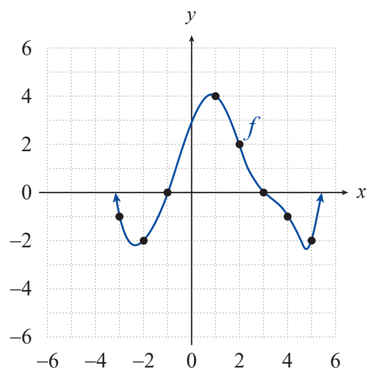


Rewrite each of the following relations as a function of *x*. Then evaluate the function at $x = -2$.

11. $\dfrac{y+4}{\sqrt{x+11}} - 3y = 3(1-y)$

12. $x^2 - 4x + 3y = x + 2y$

Use the graph below of the function *f* to answer each of the following questions.



13. What is the value of $f(1)$?

14. What is the value of $f(3)$?

15. For what integer value(s) of *x* is $f(x) = 0$?

16. For what integer values(s) of *x* is $f(x) = -2$?

Given $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x^2}$, evaluate the following.

17. $f(x+h)$

18. $\dfrac{f(x+h)-f(x)}{x+h}$

19. $g(x+h)$

20. $\dfrac{g(x+h)-g(x)}{h}$

Identify the domain, the codomain, and the range of each of the following functions.

21. $g:\mathbb{N}\to\mathbb{R}$ by $g(x)=\dfrac{3x}{4}$

22. $h:\mathbb{R}\to\mathbb{R}$ by $h(x)=5x+1$

Determine the implied domain of each of the following functions.

23. $f(x)=\sqrt{|-x-3|}$

24. $f(x)=\dfrac{x}{1-x}$

Section 3.2

Graph the following linear functions.

25. $f(x)=7x-2$

26. $g(x)=\dfrac{2x-6}{3}$

27. $p(x)=-2x-3$

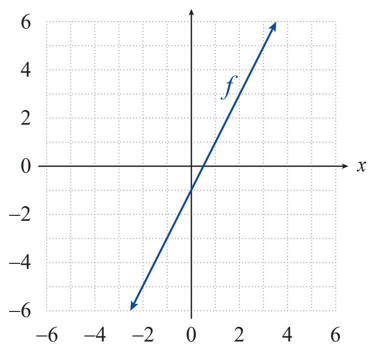
28. $h(x)=-5+\dfrac{x}{4}$

29. $k(x)=2(1-2x)+x$

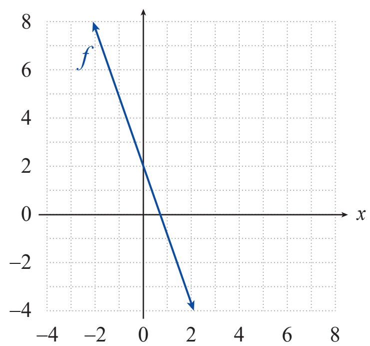
30. $f(x)=\dfrac{4x+3}{2}-2x$

Find a formula for the linear function depicted in each of the following graphs.

31.



32.



For the given points, **a.** use linear regression to find and graph the line of best fit along with the points, and **b.** find the Pearson correlation coefficient *r*.

33. $(1,2),(2,2),(3,4),(4,6),(5,5)$

Section 3.3

Graph the following quadratic functions, accurately locating the vertices and *x*-intercepts (if any).

34. $f(x)=(x-1)^2-1$

35. $g(x)=-(x+3)^2-2$

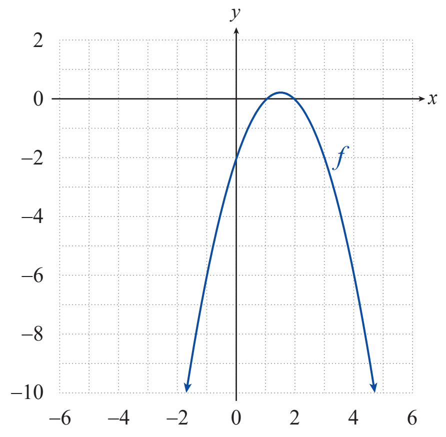
36. $p(x)=x^2-2$

37. $k(x)=-x^2+4x$

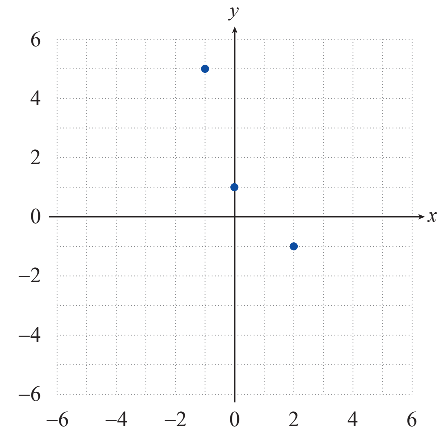
38. $h(x)=x^2+2x-3$

39. $f(x)=-x^2+5$

40. For the parabolic graph, **a.** find a formula for the corresponding quadratic function, and **b.** use the formula to determine the coordinates of the parabola's vertex.



41. Given the points graphed in the following figure, **a.** find the quadratic function that best fits the points, and **b.** use your result to determine the coordinates of the vertex of the best-fitting parabola.



42. The total revenue for McDaniel's Storage Plus is given as the function

$$R(x) = -0.4x^2 + 100x - 5250,$$

where x is the number of storage units rented. What number of units rented produces the maximum revenue?

Section 3.4

Sketch the graphs of the following functions. Pay particular attention to intercepts, if any, and locate these accurately.

43. $f(x) = -4|x|$

44. $g(x) = 3\sqrt{x}$

45. $r(x) = \dfrac{1}{x^2}$

46. $p(x) = -2x^4$

47. $q(x) = -\dfrac{1}{x^3}$

48. $k(x) = \dfrac{\sqrt[3]{x}}{2}$

49. $f(x) = 4x^3$

50. $f(x) = -\dfrac{2}{x^2}$

51. $f(x) = \left[\!\left[\dfrac{2x}{3}\right]\!\right]$

52. $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \dfrac{1}{x} & \text{if } x \ge 1 \end{cases}$

53. $g(x) = \begin{cases} (x+1)^2 - 1 & \text{if } x \le 0 \\ \sqrt[3]{x} & \text{if } x > 0 \end{cases}$

54. $h(x) = \begin{cases} -|x| & \text{if } x < 3 \\ (x-4)^2 + 1 & \text{if } x \ge 3 \end{cases}$

55. $f(x) = \begin{cases} x^2 & \text{if } x \le -2 \\ \dfrac{1}{x^2} & \text{if } x > -2 \end{cases}$

56. $q(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ x^4 & \text{if } x \ge 1 \end{cases}$

57. $g(x) = \begin{cases} 2|x| & \text{if } x < 2 \\ \sqrt{x} & \text{if } x \ge 2 \end{cases}$

Section 3.5

Find the mathematical model for each of the following verbal statements.

58. The quantity V varies directly as the product of r squared and h.

59. The value of y varies directly as the cube of a and inversely as the square root of b.

Solve the following variation problems.

60. Suppose that y varies directly as the square of x and that $y = 567$ when $x = 9$. What is y when $x = 4$?

61. Suppose that y is inversely proportional to the square root of x and that $y = 45$ when $x = 64$. What is y when $x = 25$?

62. A video store manager observes that the number of videos rented seems to vary inversely as the price of a rental. If the store's customers rent 1050 videos per month when the price per rental is \$3.49, how many videos per month does he expect to rent if he lowers the price to \$2.99?

Section 3.6

63. Determine the approximate distance between Earth, which has a mass of approximately 6.4×10^{24} kg, and an object that has a mass of 6.42×10^{22} kg, if the gravitational force between them equals approximately 4.95×10^{21} N. Remember, $F=\frac{km_1m_2}{d^2}$ and the universal gravitational constant equals $6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

64. **a.** Find a model for the volume of a cylindrical can in terms of the can's radius r, given that the surface area of the rectangle used to make the cylindrical portion of the can is constrained to be $100\ \text{in}^2$.

b. Find the height of such a can if the volume is to be $150\ \text{in}^3$.

65. Robert is planning to build a fenced rectangular garden by the side of a road, with fencing that costs \$8/ft for the length along the road and fencing that costs \$5/ft for the other three sides. He wants the garden to have an area of $1200\ \text{ft}^2$.

a. Find a model for the total cost of the fencing.

b. Estimate the size of the garden that will minimize the total cost of the fencing and estimate that minimum cost.

66. Carlotta throws a baseball straight up as hard as she can. It reaches a maximum height of 30 meters, and the table below shows its height in quarter-second intervals from that point on.

a. Graph the heights (either by hand or with a graphing utility) and estimate the time the ball hits the ground.

b. Find the linear function of best fit that models the height of the ball, and graph the function along with the given heights. By the linear model, what is the extrapolated time when the ball hits the ground? What is the calculated linear-model height of the ball at time $t=0$?

c. Find the quadratic function of best fit that models the height of the ball, and graph the function along with the given heights. By the quadratic model, what is the extrapolated time when the ball hits the ground? What is the calculated quadratic-model height of the ball at time $t=0$?

Time t (in seconds)	Height (in meters)
0	30
0.25	29.4
0.5	27.6
0.75	24.5
1.0	20.2
1.25	14.7
1.5	8.0

CHAPTER 4 REVIEW EXERCISES

Section 4.1

Sketch the graphs of the following functions by first identifying the more basic functions that have been shifted, reflected, stretched, or compressed. Then determine the domain and range of each function.

1. $f(x) = (x-1)^3 + 2$

2. $G(x) = 4|x+3|$

3. $m(x) = \dfrac{1}{(x+2)^2}$

4. $g(x) = -\sqrt[3]{x} + 4$

5. $r(x) = \dfrac{1}{x-2} - 3$

6. $f(x) = \sqrt{x-1} + 3$

7. $g(x) = \sqrt{\dfrac{x}{2}} + 1$

8. $f(x) = -\sqrt{-4x}$

Write a formula for each of the functions described below.

9. Use the function $g(x) = x^2$. Move the function 1 unit to the right and 2 units down.

10. Use the function $g(x) = |x|$. Move the functions 3 units to the right and reflect across the x-axis.

11. Use the function $g(x) = \sqrt{x}$. Reflect the function across the x-axis and move it 4 units up.

Section 4.2

Determine if each of the following relations is a function. If so, determine whether it is even, odd, or neither. Also determine if it has *y*-axis symmetry, *x*-axis symmetry, origin symmetry, or none of the above.

12. $y = |2x-1|$

13. $y = \dfrac{1}{x^2} + 1$

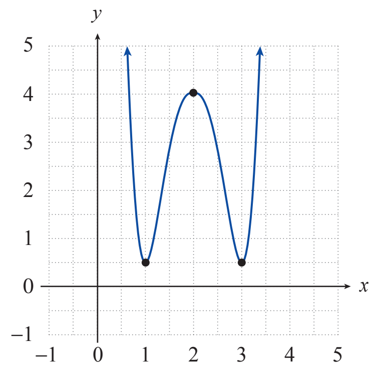
14. $x = -5|y|$

For each of the following functions, find the open intervals of monotonicity where the function is increasing, decreasing, or constant.

15. $f(x) = (x-2)^4 - 6$

16. $R(x) = \begin{cases} (x+2)^2 & \text{if } x < -1 \\ -x & \text{if } x \geq -1 \end{cases}$

17. Given the following graph of a function determine, **a.** the locations and types of the local extrema, and **b.** the values of the local extrema.



For each given function and interval, determine the average rate of change of the function over the interval.

18. $f(x) = x^2; \quad [3,4]$

19. $f(x) = \frac{1}{x}; \quad [1,3]$

20. $f(x) = \sqrt{x}; \quad [1,4]$

21. $f(x) = x^2 - x^3; \quad [-1,2]$

Section 4.3

In each of the following exercises, use the information given to determine **a.** $(f+g)(2)$, **b.** $(f-g)(2)$, **c.** $(fg)(2)$, and **d.** $\left(\frac{f}{g}\right)(2)$.

22. $f(x) = -x^2 + x$ and $g(x) = \frac{1}{x}$

23. $f(2) = 4$ and $g(2) = -1$

24. $f(x) = \sqrt{2x}$ and $g(x) = x + 3$

25. $f = \{(0,4),(2,8)\}$ and $g = \{(-2,2),(0,3),(2,-10)\}$

In each of the following exercises, find **a.** the formula and domain for $f+g$ and **b.** the formula and domain for $\frac{f}{g}$.

26. $f(x)=x^2$ and $g(x)=\sqrt{x}$

27. $f(x)=\frac{1}{x-2}$ and $g(x)=\sqrt[3]{x}$

28. $f(x)=3x$ and $g(x)=(x-1)^2$

29. $f(x)=x^2-4$ and $g(x)=\sqrt[3]{x}-1$

In each of the following exercises, use the information given to determine $(f\circ g)(3)$.

30. $f(x)=-x+1$ and $g(x)=-x-1$

31. $f(x)=\frac{x^{-1}}{18}-3$ and $g(x)=\frac{x-4}{x^3}$

32. $f(-3)=4$ and $g(3)=-3$

33. $f(x)=\frac{x}{3}$ and $g(x)=-\sqrt{x+1}$

In each of the following exercises, find **a.** the formula and domain for $f\circ g$ and **b.** the formula and domain for $g\circ f$.

34. $f(x)=4x-1$ and $g(x)=x^3+2$

35. $f(x)=\frac{1}{\sqrt{x-4}}$ and $g(x)=x+2$

36. $f(x)=2x^2+1$ and $g(x)=x-4$

37. $f(x)=3x$ and $g(x)=\sqrt{x-3}$

Write each of the following functions as a composition of two functions. Answers may vary.

38. $f(x)=\frac{3}{3x^2+1}$

39. $f(x)=\frac{\sqrt{x+2}}{x^2+4x+4}$

In each of the following exercises, use the information given to find $g(x)$.

40. $f(x)=6x-1$ and $(f\circ g)(x)=x+3$

41. $f(x)=\sqrt{x}+3$ and $(g\circ f)(x)=\frac{2}{\sqrt{x}+3}+1$

Section 4.4

Graph the inverse of each of the following relations, and state its domain and range.

42. $R=\{(3,4),(-1,5),(0,2),(-6,-1)\}$

43. $y=3x+1$

44. $y=\frac{\sqrt{x}}{2}$

Find a formula for the inverse of each of the following functions.

45. $r(x)=\dfrac{2}{7x-1}$

46. $g(x)=\dfrac{4x-3}{x}$

47. $f(x)=x^{\frac{1}{5}}-6$

48. $p(x)=2\sqrt{x-1}+3$

49. $f(x)=\dfrac{2x-3}{x+1}$

50. $f(x)=\sqrt[3]{x+2}-1$

51. $f(x)=8x+3$

52. $f(x)=(x-1)^2-3,\ x\geq 1$

Verify that $f\left(f^{-1}(x)\right)=x$ and that $f^{-1}\left(f(x)\right)=x$.

53. $f(x)=\dfrac{6x-7}{2-x}$ and $f^{-1}(x)=\dfrac{2x+7}{6+x}$

CHAPTER 5 REVIEW EXERCISES

Section 5.1

Verify that the given values of x solve the corresponding polynomial equations.

1. $4x^3 - 5x^2 = -3x + 18;\ x = 2$

2. $x^2 - 6x = -13;\ x = 3 + 2i$

3. $x^3 + x = 6x^2 - 164;\ x = 5 - 4i$

4. $x^3 + (1+4i)x = (7-2i)x^2 - 2i + 36;\ x = -2i$

Solve the following polynomial equations by factoring and/or using the quadratic formula, making sure to identify all the solutions.

5. $x^4 - 7x^2 + 10 = 0$

6. $x^5 - x^3 - 2x = 0$

7. $x^4 + 4 = 4x^2$

8. $6x^2 + 8x = -x^3$

9. $x^4 + x^3 = x^2$

10. $x^2 + 4x + 7 = 0$

For each of the following polynomial functions, determine the behavior of its graph as $x \to \pm\infty$ and identify the x- and y-intercepts. Use this information to then sketch the graph of each polynomial.

11. $f(x) = (x+2)(x-1)(x-3)$

12. $f(x) = (x-2)^2(x+1)^2$

13. $g(x) = x^2 - 5x + 4$

14. $h(x) = -x^3 - 7x^2 - 10x$

Solve the following polynomial inequalities.

15. $2x^2 + 15 \le 11x$

16. $(x-3)^2(x+1)^2 > 0$

17. $(x-4)(x+2)(x^2-1) \le 0$

18. $x^3 - 2x^2 - 8x \ge 0$

19. $x^2(x-2)(1-x) < 0$

20. $-3x^2 + 7x - 2 > 0$

21. A manufacturer has determined that the revenue from the sale of x video games is given by $r(x) = -x^2 + 12x$. The cost of producing x video games is $C(x) = 120 - 22x$. Given that profit is revenue minus cost, what value(s) for x will give the company a nonnegative profit?

Section 5.2

Use polynomial long division to rewrite each of the following fractions in the form $q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

22. $\dfrac{8x^4 - 6x^3 + 2x^2 + 3x + 4}{2x^2 - 1}$

23. $\dfrac{11x^2 + 2x - 5}{x - 3}$

24. $\dfrac{x^4 - 3x^2 + x - 8}{x^2 + 3x + 2}$

25. $\dfrac{2x^5 - 4x^3 - x^2 + x - 2}{x^2 - x}$

26. $\dfrac{2x^3 + ix^2 - 12x - 4 + i}{2x + i}$

Use synthetic division to determine if the given value for c is a zero of the corresponding polynomial. If not, determine $p(c)$.

27. $p(x) = 6x^5 - 23x^4 - 95x^3 + 70x^2 + 204x - 72;\ \ c = 1$

28. $p(x) = 48x^4 + 10x^3 - 51x^2 - 10x + 3;\ \ c = \frac{1}{6}$

29. $p(x) = 18x^5 - 87x^4 + 110x^3 - 28x^2 - 16x + 3;\ \ c = \frac{2}{3}$

Use synthetic division to rewrite each of the following fractions in the form $q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

30. $\dfrac{x^4 - 2x^3 - x^2 + x - 21}{x - 3}$

31. $\dfrac{-x^4 - x^3 - x^2 + 2x + 69}{x + 3}$

32. $\dfrac{x^5 + 2x^4 + 3x^3 + 6x^2 - 5x + 13}{x + 2}$

33. $\dfrac{-x^4 + 8x^3 - 6x^2 - 4x + 2}{x - 1}$

34. $\dfrac{x^4 + (4 - 2i)x^3 - (1 + 8i)x^2 + (3 + 2i)x - 6i}{x - 2i}$

Construct a polynomial function with the stated properties.

35. Second-degree, zeros of –2 and 6, and goes to ∞ as $x \to \infty$.

36. Fourth-degree and a single x-intercept of –4 and y-intercept $(0,128)$.

37. Third-degree, zeros of ± 2 and 3 and passing through the point $(4,24)$.

Section 5.3

List all of the potential rational zeros of the following polynomials. Then use polynomial division and the quadratic formula, if necessary, to identify the actual zeros.

38. $f(x) = x^4 + 3x^3 - 3x^2 - 11x - 6$

39. $g(x) = 2x^3 - 11x^2 + 18x - 9$

40. $h(x) = 2x^3 + 2x^2 - 9x + 9$

41. $p(x) = x^4 + 8x^3 + 22x^2 + 24x + 9$

Using the Rational Zero Theorem or your answers to the preceding problems, solve the following polynomial equations.

42. $2x^4 - 6x^2 = -6x^3 + 22x + 12$

43. $2x^3 - 9x^2 + 18x = 9 + 2x^2$

44. $2x^3 + 9 = 9x - 2x^2$

45. $x^4 - x^5 = -x^5 - 8x^3 - 22x^2 - 24x - 9$

Use Descartes' Rule of Signs to determine the possible numbers of positive and negative real zeros of each of the following polynomials.

46. $f(x) = 2x^4 - 3x^3 - x^2 + 3x + 10$

47. $g(x) = x^6 - 4x^5 - 2x^4 + x^3 - 6x^2 - 11x + 6$

Use synthetic division to identify integer upper and lower bounds of the real zeros of the following polynomials.

48. $f(x) = 2x^3 - 11x^2 + 3x + 36$

49. $g(x) = 4x^3 - 16x^2 - 79x - 35$

Using your answers to the preceding problems, polynomial division, and the quadratic formula, if necessary, find all of the zeros of the following polynomials.

50. $f(x) = 2x^3 - 11x^2 + 3x + 36$

51. $g(x) = 4x^3 - 16x^2 - 79x - 35$

Use the Intermediate Value Theorem to show that each of the following polynomials has a real zero between the indicated values.

52. $f(x) = 2x^4 - 6x^3 + x - 5;\ -2$ and 0

53. $f(x) = -x^3 + 3x^2 + x - 3;\ 2$ and 4

Find all of the real zeros of the following functions.

54. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

55. $g(x) = x^3 - 4x^2 + 9x - 36$

56. $f(x) = x^3 + 6x^2 + 11x + 6$

57. $f(x) = x^3 - 7x^2 + 13x - 3$

Solve the following equations.

58. $x^4 - 2x^3 + 10x^2 = 9(2x - 1)$

59. $2x^3 = 7x^2 - 4x - 4$

60. $-8 = 3x^3 + 4x^2 + 6x$

Section 5.4

Throughout these exercises, a graphing utility may be helpful in identifying zeros and in checking your graphs.

Sketch the graph of each factored polynomial.

61. $f(x) = (x+4)^2(x-1)$

62. $g(x) = x(x-3)(x+4)^3$

Factor each of the following polynomials completely, and then sketch the graph of each one.

63. $f(x) = x^3 - 3x^2 + x - 3$

64. $f(x) = x^5 - x^4 - 2x^3 - x^2 + x + 2$

Solve each polynomial equation.

65. $3x^5 + x^4 + 5x^3 = x^2 + 28x + 20$

66. $8x^5 + 12x^4 - 18x^3 - 35x^2 = 18x + 3$

67. $x^5 + 3x^4 + 3x^3 + 9x^2 = 4(x + 3)$

Factor each of the following polynomials completely, making use of the given zero.

68. $f(x)=14x^4-109x^3+296x^2-321x+70;\ 2+i$ is a zero

69. $f(x)=x^4-5x^3+19x^2-125x-150;\ -5i$ is a zero

70. $f(x)=2x^4+3x^3-7x^2+8x+6;\ 1+i$ is a zero

71. $f(x)=4x^3+10x^2-x+15;\ -3$ is a zero

Construct polynomial functions with the stated properties.

72. Fourth-degree, only real coefficients, $\frac{1}{2}$ and $1+2i$ are two of the zeros, y-intercept is -30, leading coefficient is 2.

73. Fifth-degree, only real coefficients, -1 is a zero of multiplicity 3, $\sqrt{6}$ is a zero, y-intercept is -6, leading coefficient is 1.

74. Fifth-degree, only real coefficients, 1 is a zero of multiplicity 3, $\sqrt{3}$ is a zero, y-intercept is 3, leading coefficient is 1.

Section 5.5

Find equations for the vertical asymptotes, if any, for each of the following rational functions.

75. $f(x)=\frac{4}{2x-5}$

76. $f(x)=\frac{x^2-3x+2}{x-1}$

77. $f(x)=\frac{x^2-1}{x-x^2}$

78. $f(x)=\frac{x^2-x-6}{x^2-6x+9}$

Find equations for the horizontal or oblique asymptotes, if any, for each of the following rational functions.

79. $f(x)=\frac{2x^3+5x^2-1}{x^2-2x}$

80. $f(x)=\frac{x^2-x+8}{3x^2-7}$

81. $f(x)=\frac{x^2-9}{x+3}$

82. $f(x)=\frac{x^2+2x-3}{(x+1)^3}$

Sketch the graphs of the following rational functions.

83. $f(x)=\frac{2x}{x+1}$

84. $f(x)=\frac{4x^2}{x^2+3x}$

85. $f(x)=\frac{x^2+2}{x+2}$

86. $f(x)=\frac{x+1}{x^2-4}$

Solve the following rational inequalities.

87. $\frac{7}{x+3}\geq\frac{2x}{x+3}$

88. $\frac{x}{x^2-5x+6}\leq\frac{3}{x^2-5x+6}$

89. $\frac{x-4}{x+3}<\frac{x}{x-2}$

90. $\frac{x-2}{x+3}<2$

CHAPTER 6 REVIEW EXERCISES

Section 6.1

Sketch the graphs of the following functions. State their domain and range.

1. $f(x) = \left(\frac{1}{2}\right)^{x-1} + 3$

2. $r(x) = 2^{-x+4} - 2$

3. $h(x) = 3^x$

4. $f(x) = 1 - 2^{-x}$

5. $p(x) = \left(\frac{1}{4}\right)^x$

6. $s(x) = (0.2)^{x-2}$

7. $g(x) = 4 - 2^x$

8. $m(x) = \frac{1}{2^x} - 3$

9. $f(x) = \frac{1}{2^{4-x}}$

10. $r(x) = \left(\frac{9}{2}\right)^{3-x}$

Solve the following exponential equations.

11. $3^x = 243$

12. $2^{-x} = 16$

13. $0.5^x = 0.25$

14. $3^{3x-5} = 81$

15. $\left(\frac{2}{5}\right)^{-4x} = \left(\frac{25}{4}\right)^{x-1}$

16. $10{,}000^x = 10^{-2x-12}$

17. $9^{x-1} = 27^{-x+2}$

18. $\left(\frac{1}{3}\right)^{x-1} = 81^{\frac{1}{2}}$

19. $5^{3x-6} = 1$

Section 6.2

20. Melissa has recently inherited \$15,000 that she wants to deposit into a savings account for 10 years. She has determined that her two best bets are an account that compounds annually at a rate of 3.95% and an account that compounds continuously at an annual rate of 3.85%. Which account would pay Melissa more interest?

21. Bill has come upon a 37-gram sample of iodine-131. He isolates the sample and waits for 2 weeks. After this time period, only 11 grams of iodine-131 remain. What is the half-life of this isotope?

22. Katherine is working in a lab testing bacteria populations. Starting out with a population of 870 bacteria, she notices that the population doubles every 22 minutes. Find **a.** the equation for the population P in terms of time t in minutes, and **b.** the time it would take for the population to reach 7500 bacteria.

23. The number of fruit flies in an experimental population after t hours is given by $Q(t) = 20e^{0.03t},\ t \geq 0.$

a. How large is the population of fruit flies after 72 hours?
b. Find the initial number of fruit flies in the population.

24. In 1986, a nuclear reactor accident occurred in Chernobyl in what was then the Soviet Union. The explosion spread radioactive chemicals over hundreds of square miles and the government evacuated the city and surrounding areas. To see why the city is now uninhabited, consider the model $P = 10e^{-0.00002845t}$.

This model represents the amount of plutonium that remains (from an initial amount of 10 pounds) after t years. Sketch the graph of this function over the interval from $t = 0$ to $t = 100{,}000$. How much of the 10 pounds will remain after 100,000 years?

Section 6.3

Write the following equations in logarithmic terms.

25. $3^x = 8$ **26.** $(3a)^{\sqrt{2}} = 10$ **27.** $4^{3a} = 4096$

Write the following logarithmic equations as exponential equations.

28. $\log_4 64 = x$ **29.** $\log_3\left(\frac{1}{27}\right) = -3$ **30.** $\log_8(2A) = 3$

Sketch the graphs of the following functions. State their domain and range.

31. $f(x) = -\log_3(-x)$ **32.** $f(x) = \log_{\frac{1}{2}} x$ **33.** $m(x) = \log_{\frac{1}{2}}(x+2)$

Evaluate the following logarithmic expressions without the use of a calculator.

34. $\log_{27} 9^{\log 1000}$ **35.** $\log_{\frac{1}{3}} 9$ **36.** $\log_4\left(\frac{1}{64}\right)$

37. $\log_{\frac{1}{2}} 8$ **38.** $\log_{\sqrt{3}}\left(\frac{1}{3}\right)$ **39.** $\ln\left(\sqrt[3]{e^2}\right)$

Use the elementary properties of logarithms to solve the following equations.

40. $\log_6 6^{\log_5 x} = 3$ **41.** $\log_9 x^{\frac{1}{2}} = \frac{3}{4}$

42. $\log_x\left(\log_{\frac{1}{2}}\left(\frac{1}{16}\right)\right) = 2$ **43.** $\log_4(2x-1) = 2$

Solve the following logarithmic equations, using a calculator if necessary to evaluate the logarithms. Express your answer either as a fraction or a decimal rounded to two decimal places.

44. $\ln(4x) = 3.2$ **45.** $\ln(x-7) = 5$ **46.** $\log_7(4x-3) = 4$

Section 6.4

Use the properties of logarithms to expand the following expressions as much as possible. Simplify any numerical expressions that can be evaluated without a calculator.

47. $\log\sqrt{\frac{x^3}{4\pi^5}}$

48. $\ln\left(\frac{\sqrt{a^5}\,mn^2}{e^5}\right)$

49. $\log_3\left(27a^3\right)$

50. $\ln\left(\ln\left(e^{2ex}\right)\right)$

Use the properties of logarithms to condense the following expressions as much as possible, writing each answer as a single term with a coefficient of 1.

51. $\frac{1}{3}\left(\log_2\left(a^5\right)-\log_2\left(bc^3\right)\right)$

52. $\ln 4-\ln\left(x^2\right)-7\ln y$

53. $\log_2\left(x^2-9\right)-\log_2\left(x+3\right)$

54. $2\log a+3\log b-\frac{1}{2}\log c-\log d$

55. $\log_3\left(x-2\right)+\log_3 x-\log_3\left(x^2+4\right)$

Use the properties of logarithms to write each of the following as a single term that does not contain a logarithm.

56. $6^{3\log_6 x}$

57. $5^{\log_5 x-2\log_5 y}$

Evaluate the following logarithmic expressions.

58. $\log_3 17$

59. $\log_{1.4} 8$

60. $4\log_{\frac{1}{2}} 3$

Without using a calculator, evaluate the following expressions.

61. $\ln\left(\frac{1}{e^2}\right)+\ln e^2$

62. $\log_4\left(64^2\right)$

63. On the Richter scale, the magnitude R on a earthquake of intensity I is given by $R=\log\frac{I}{I_0}$, where $I_0=1$ is the minimum intensity used for comparison. Find the intensity per unit of area for the following values of R.

a. $R=8.4$

b. $R=6.85$

c. $R=9.1$

Section 6.5

Solve the following exponential and logarithmic equations. When appropriate, write the answer as both an exact expression and as a decimal approximation. Round your answer to two decimal places if necessary.

64. $e^{8-5x}=16$

65. $10^{\frac{6}{x}}=321$

66. $7^{\frac{x}{3}-4}=19$

67. $e^{4x}=5^{3x+1}$

68. $24=3e^{x+2}$

69. $3^{2x-1}=2^{2-x}$

70. $\ln(x+1)+\ln(x-1)=\ln(x+5)$

71. $\log_2(x+3)+\log_2(x+4)=\log_2(3x+8)$

72. $\log_5(8x-3)=3$

73. $\log_7(4x)-\log_7 6=2$

74. $\ln(5x+8)=\ln(40-3x)$

Using the properties of logarithmic functions, simplify the following functions as much as possible. Write each function as a single term with a coefficient of 1, if possible.

75. $f(x)=0.75\ln x^4$

76. $f(x)=6\log\sqrt{2x}$

77. $f(x)=4\log x^3-\log x^2$

78. $f(x)=0.5\ln(9x^6)$

79. $f(x)=2\log 7^{\log_9 3}$

80. $f(x)=2\ln 3^{\log_4 8}$

81. Rick puts \$6500 in a high interest money market account at 4.36% annual interest compounded monthly. Assuming he makes no deposits or withdrawals, how long will it take for his investment to grow to \$7000?

82. Sodium-24 has a half-life of approximately 15 hours. How long would it take for 350 grams of sodium-24 to decay to 12 grams?

CHAPTER 7 REVIEW EXERCISES

Section 7.1

Convert each of the following angle measures as directed.

1. Express $\frac{\pi}{45}$ in degrees.

2. Express $\frac{6\pi}{5}$ in degrees.

3. Express $-\frac{7\pi}{4}$ in degrees.

4. Express $-\frac{3\pi}{10}$ in degrees.

5. Express 42° in radians.

6. Express 60° in radians.

7. Express –79° in radians.

8. Express –25° in radians.

Sketch the indicated angles.

9. 300°

10. $-\frac{9\pi}{4}$

Find the length of the arc subtended by the given central angle θ on a circle of radius r. Round your answers to two decimal places.

11. $r = 5$ ft; $\theta = 180°$

12. $r = 8$ km; $\theta = \frac{3\pi}{4}$

Find the indicated arc length in each of the following problems. Round your answers to two decimal places.

13. Given a circle of radius 16.3 meters, find the length of the arc subtended by a central angle of $\frac{3\pi}{5}$.

14. Given a circle of radius 8 inches, find the length of the arc subtended by a central angle of 72° (**Hint:** Convert to radians first).

15. Find the distance between Vancouver, British Columbia and San Francisco, California which have the same longitude. The latitude of Vancouver is 49.25° N and the latitude of San Francisco is 37.77° N. Assume the Earth's radius is 6470 km.

Find the radian measure of the central angle θ given the radius r and the length s of the arc subtended by θ. Leave your answers in fraction form.

16. $r = 18$ ft; $s = 52$ ft

17. $r = 6.4$ dm; $s = 19.2$ dm

Solve the following problems. Round your answers to two decimal places.

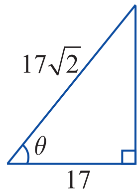
18. A gear in a machine has a radius of 14 cm and is rotating at a constant speed of 600 rpm. Find **a.** the angular speed of a tooth of the gear in radians per minute and **b.** the linear speed of the tooth in meters per second.

19. Find the area of the sector of a circle of radius 18 ft with a central angle of $\frac{2\pi}{3}$.

Section 7.2

Use the information contained in each figure to determine the values of the six trigonometric functions of θ. Rationalize all denominators in your answers.

20.



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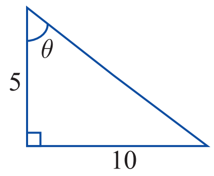

Use a calculator to evaluate each of the following expressions. Round your answers to four decimal places.

22. $\sin 82°$

23. $\cot 14°$

24. $\csc\left(\frac{5\pi}{12}\right)$

25. $\cos\left(\frac{3\pi}{7}\right)$

Convert each expression from degrees, minutes, seconds (DMS) notation to decimal notation. Round your answers to four decimal places.

26. $36°56'14''$

27. $15°12'73''$

Determine whether the following statements are true or false. Use a calculator when necessary.

28. If $\tan\theta = 1.6$, then $\cot\theta = 0.625$.

29. If $\csc\theta = 3.4$, then $\sin\theta = 1.7$.

Use an appropriate trigonometric function and a calculator if necessary to solve each of the following problems. Round your answer to two decimal places.

30. A wheelchair ramp touches the ground 15 feet away from the top of the steps. If the ramp makes an angle of 30° relative to the ground, how long is the ramp?

31. A building is 83 feet tall and a cable is stretched from the top of the building to the ground. If the angle between the cable and the ground is 40°, how long is the cable?

Section 7.3

Determine the values of the six trigonometric functions of each angle θ, using a calculator and rounding your answers to four decimal places if necessary.

32. $\theta = 90°$

33. $\theta = -460°$

34. $\theta = \frac{\pi}{4}$

35. $\theta = \frac{7\pi}{3}$

Determine the reference angle associated with the given angle.

36. $\theta = 86°$

37. $\theta = -143°$

38. $\theta = \frac{3\pi}{2}$

39. $\theta = \frac{11\pi}{4}$

Determine the quadrant in which the terminal side of the angle θ is located.

40. $\csc\theta > 0$ and $\tan\theta > 0$

41. $\sec\theta < 0$ and $\cot\theta > 0$

In Exercises 42 and 43, **a.** rewrite the expression in terms of the given angle's reference angle, and then **b.** evaluate the result, using a calculator and rounding your answer to four decimal places if necessary.

42. $\sin 290°$

43. $\tan\left(\frac{4\pi}{3}\right)$

Express each of the following in terms of the appropriate cofunction, and verify the equivalence of the two expressions, using a calculator and rounding your answer to four decimal places if necessary.

44. $\csc 193°$

45. $\sin(-42°)$

46. $\cot\left(\frac{3\pi}{4}\right)$

47. $\cos\left(\frac{5\pi}{4}\right)$

Use the given information about each angle to evaluate the expression, if possible. If no angle with the stated properties exists, determine the reason.

48. Given that $\sin\theta = \frac{\sqrt{2}}{2}$ and $\tan\theta$ is negative, determine θ and $\tan\theta$.

49. Given that $\csc\theta = \frac{13}{12}$ and θ lies in the first quadrant, determine $\sec\theta$.

Section 7.4

Determine the amplitude, period, and phase shift of each of the following functions.

50. $f(x) = 3\cos(4x)$

51. $h(x) = 10 + 6\cos x$

52. $g(x) = 6 - \frac{1}{2}\sin(3\theta - \pi)$

53. $f(x) = -3 + 9\sin(2\theta + 2\pi)$

Sketch the graph of each of the following functions.

54. $f(x) = \frac{1}{2}\cos(3\pi x)$

55. $g(x) = 4\sin(2x - 5)$

56. $f(\theta) = 5\cos\left(\theta - \frac{\pi}{3}\right)$

57. $h(x) = 2 + \sin(x - \pi)$

58. $g(x) = 1 - \frac{1}{2}\sin\left(\frac{1}{2}x + \frac{\pi}{4}\right)$

59. $f(t) = \frac{1}{2}e^{-t}\cos(t + 2\pi) - 1$

Section 7.5

Sketch the graph of each of the following functions.

60. $f(x) = 1 - \tan\left(x - \frac{\pi}{2}\right)$

61. $f(x) = \cot\left(\frac{\pi x}{4} + \frac{\pi}{4}\right)$

62. $f(x) = \frac{1}{2}\sec(2\pi x)$

63. $f(x) = -3\csc\left(\frac{x}{2} + \frac{\pi}{2}\right) + 1$

Section 7.6

Evaluate each of the following expressions without the use of a calculator.

64. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

65. $\cos^{-1} 0$

66. $\arctan(-1)$

67. $\csc^{-1}\left(\frac{2\sqrt{3}}{3}\right)$

Evaluate each of the following expressions, if possible, using a calculator and rounding your answer to four decimal places if necessary.

68. $\sin^{-1} 2$

69. $\tan^{-1}(0.5)$

Evaluate each of the following expressions, if possible.

70. $\arccos(\sin \pi)$

71. $\cos\left(\cos^{-1}(0.9)\right)$

72. $\tan\left(\tan^{-1}(0.75)\right)$

73. $\sin^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right)$

Find the value of each of the following expressions without using a calculator.

74. $\sin\left(\arctan\left(\frac{\sqrt{3}}{3}\right)\right)$

75. $\cot\left(\sec^{-1} 2\right)$

76. $\cos\left(\arcsin\left(\frac{-\sqrt{2}}{2}\right)\right)$

77. $\sec\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

Using a calculator, find the value of θ in radians. Remember to make sure your calculator is in the correct mode.

78. $\theta = \cos^{-1}(0.3492581)$

79. $\theta = \tan^{-1}(-4.18249588)$

Using a calculator, find the value of θ in degrees. Remember to make sure your calculator is in the correct mode.

80. $\theta = \arcsin(-0.66666667)$

81. $\theta = \arccos(0.56894372)$

Express the following function as a purely algebraic function.

82. $\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)\right)$

CHAPTER 8 REVIEW EXERCISES

Section 8.1

Use trigonometric identities to simplify the expressions. There may be more than one correct answer.

1. $\cot x \sec x$

2. $\left(\csc^2 x - 1\right)\cos^2\left(\frac{\pi}{2} - x\right)$

3. $\frac{\tan(-y)}{\cot(\pi + y)}$

4. $\frac{\tan^2 \alpha}{\csc^2\left(\frac{\pi}{2} - \alpha\right)} + \frac{\cos \alpha}{\sec(\alpha + 2\pi)}$

5. $\sin^2 \theta \sec^2 \theta - \csc^2\left(\frac{\pi}{2} - \theta\right) + \sin(-\theta)$

6. $\sin\left(\frac{\pi}{2} - x\right)\cos(x + 2\pi) + \sin(-x)\sec\left(\frac{\pi}{2} - x\right)$

Verify the following trigonometric identities.

7. $(\tan x + \sec x)(\sec x - \tan x) = 1$

8. $\cos^2 x \tan^2 x = 1 - \frac{1}{\sec^2 x}$

9. $\frac{\cos\left(\frac{\pi}{2} - t\right)}{\tan(-t)} = -\cos t$

10. $5 + \tan^2 y = 4 + \sec^2 y$

11. $\tan(\theta + \pi) = -\frac{\sec(\theta + 2\pi)}{\csc(-\theta)}$

12. $-\tan\left(\frac{\pi}{2} - x\right)\tan(-x) - \tan^2 x \sin^2\left(\frac{\pi}{2} - x\right) = \cos^2 x$

Use the suggested substitution to rewrite the given expression as a trigonometric expression. Assume $0 \le \theta \le \frac{\pi}{2}$.

13. $\sqrt{16 + x^2}$, $\tan\theta = \frac{x}{4}$

14. $\sqrt{64 - 16x^2}$, $2\sin\theta = x$

15. $\sqrt{25x^2 - 100}$, $\csc\theta = \frac{x}{2}$

16. $\sqrt{9x^2 + 36}$, $x = 2\tan\theta$

Section 8.2

Use the sum and difference identities to determine the exact value of each of the following expressions.

17. $\cos\left(\frac{\pi}{2} + \frac{5\pi}{3}\right)$

18. $\cos 255°$

19. $\sin(-15°)$

20. $\sin\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)$

21. $\tan\left(\pi - \frac{2\pi}{3}\right)$

22. $\tan 105°$

23. Suppose that $\sin\alpha = \frac{8}{17}$ and $\sin\beta = \frac{3}{5}$ and the terminal sides of both α and β are in quadrant II. Find $\tan(\alpha - \beta)$.

24. Suppose that $\sin\alpha = \frac{5}{13}$ and $\cos\beta = \frac{4}{5}$, the terminal side of α is in quadrant II, and the terminal side of β is in quadrant IV. Find $\sin(\alpha - \beta)$.

Use the sum and difference identities to rewrite each of the following expressions as a trigonometric function of one angle, and then evaluate the result.

25. $\sin 175°\cos 35° + \cos 175°\sin 35°$

26. $\dfrac{\tan\left(\frac{9\pi}{8}\right) - \tan\left(\frac{3\pi}{8}\right)}{1 + \tan\left(\frac{9\pi}{8}\right)\tan\left(\frac{3\pi}{8}\right)}$

Use the sum and difference identities to verify the following identities.

27. $\sin x = \cos\left(\frac{\pi}{2} - x\right)$

28. $\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2\sin\alpha\sin\beta$

Express each of the following as an algebraic function of *x*.

29. $\cos\left(\sin^{-1} x + \tan^{-1} x\right)$

30. $\cos\left(\cos^{-1}(2x) + \tan^{-1}(2x)\right)$

Express each of the following functions in terms of a single sine function.

31. $f(x) = \sqrt{2}\sin x - \sqrt{2}\cos x$

32. $h(\alpha) = \sqrt{3}\sin(4\alpha) - \cos(4\alpha)$

Section 8.3

Use the given information to determine $\cos(2x)$, $\sin(2x)$, and $\tan(2x)$ if possible.

33. $\tan x = \frac{4}{3}$ and $\sin x$ is positive

34. $\sin x = \frac{-1}{\sqrt{10}}$ and $\tan x$ is positive

Verify the following trigonometric identities.

35. $\cos(3x) = 4\cos^3 x - 3\cos x$

36. $\dfrac{\sin(4x)}{4} = \sin x\cos x - 2\sin^3 x\cos x$

Use a power-reducing identity to rewrite the given expression as directed.

37. Rewrite $\sin^3 x\cos^2 x$ in terms containing only the first powers of sine and cosine.

38. Rewrite $\tan^2 x\sin^3 x$ in terms containing only the first powers of sine and cosine.

Determine the exact value of each of the following expressions.

39. $\tan\left(\frac{5\pi}{12}\right)$

40. $\cos(157.5°)$

41. $\tan 15°$

42. $\sin\left(-\frac{5\pi}{8}\right)$

Use the product-to-sum identities to rewrite the given expression as a sum or difference.

43. $\cos(x+y)\sin(x-y)$

44. $\cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)$

45. $\sin 165°\cos 15°$

46. $\sin(4x)\sin(3x)$

Use the sum-to-product identities to rewrite the given expression as a product.

47. $\sin(5\alpha)-\sin(3\alpha)$

48. $\cos 225°+\cos 15°$

49. $\cos\left(\frac{3\pi}{4}\right)-\cos\left(\frac{2\pi}{3}\right)$

50. $\sin\left(\frac{5\pi}{6}\right)+\sin\left(\frac{\pi}{3}\right)$

Section 8.4

Use the trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equations.

51. $8\cos^2 x+1=7$

52. $2\sin^2 x=\sin x$

53. $-\sin^2 x+4\cos x+1=0$

54. $\tan^3 x=\tan x$

55. $-2\sin^2 x=-\cos x-1$

56. $\sin x+\cos x\cot x=-2$

Use trigonometric identities, algebraic methods, and inverse trigonometric functions, as necessary, to solve the following trigonometric equations on the interval $[0,2\pi)$.

57. $3\tan^2 x+9=10$

58. $\sin^2 x=3-2\sin x$

Determine if the value given is a solution to the trigonometric equation. If the value of *x* is not a solution, give all solutions to the equation.

59. $4\sin^2 x=3;\quad x=\frac{5\pi}{3}+2n\pi$

60. $\frac{1}{2}\csc x+1=2;\quad x=\frac{3\pi}{4}+2n\pi$

61. $\tan(2x)\cos x=-\frac{\sqrt{3}}{2};\quad x=\frac{\pi}{6}+2n\pi$

62. $\sin x+\cos(2x)=1;\quad x=\frac{5\pi}{6}+2n\pi$

Solve the following equations on the interval $[0°,360°)$. Give exact answers when appropriate; otherwise, round your answers to one decimal place.

63. $\cos^2 x\sin x=\sin x$

64. $2\cos^2 x+7\cos x=4$

CHAPTER 9 REVIEW EXERCISES

Section 9.1

Create a triangle, if possible, using the given information.

1. $A=30°, B=45°, b=4$

2. $a=15, c=13, C=57°$

3. $A=74°20', C=37°, c=23$

4. $b=8, c=13, C=78°$

5. Find the area of a triangle for which $a=3$, $b=7$, and $C=75°$.

Section 9.2

Create a triangle, if possible, using the given information and the Law of Cosines.

6. $A=62°, b=8, c=10$

7. $B=94°\ 7', a=6, c=14$

8. $a=9, b=2.5, c=7.3$

9. $a=10.8, b=13.4, c=6$

10. The base of a 25 ft ladder is positioned 7 ft away from an office building situated on a slight hill, and the ladder and ground form a 62° angle. At what angle and at what height does the ladder touch the building?

11. Find the area of a triangle for which $a=5$, $b=11$, and $c=9$.

Section 9.3

Convert the point from polar to Cartesian coordinates.

12. $\left(-3.45, \frac{\pi}{3}\right)$

13. $\left(7, \frac{7\pi}{6}\right)$

Convert the point from Cartesian to polar coordinates.

14. $\left(-\sqrt{3}, -1\right)$

15. $(10, 12)$

Rewrite the rectangular equation in polar form.

16. $x^2+y^2=16a^2$

17. $x^2+y^2=9ax$

Rewrite the polar equation in rectangular form.

18. $r=2\cos\theta$

19. $r=\frac{16}{4\cos\theta+4\sin\theta}$

Sketch a graph of the given polar equation.

20. $r=4\sin(3\theta)$

21. $r^2=25\cos(2\theta)$

Section 9.4

Sketch the graphs of the following parametric equations by eliminating the parameter.

22. $x=\frac{1}{36t}$ and $y=t^2$

23. $x=t+5$ and $y=|t-2|$

24. $x=\frac{3}{4t-2}$ and $y=2t-2$

25. $x=4\sin\theta$ and $y=\cos\theta+1$

Construct parametric equations describing the graphs of the following equations.

26. $y^2=x^2+4$

27. $6x=2-y$

Construct parametric equations for the line or conic with the given attributes.

28. Line: passing through $(14,4)$ and $(-3,-8)$

29. Circle: center $(1,1)$, radius 1

Section 9.5

Graph and determine the magnitudes of the following complex numbers.

30. $4+5i$

31. $-3+3i$

Sketch z_1, z_2, z_1+z_2, and z_1z_2 on the same complex plane.

32. $z_1=-2-3i, z_2=6+3i$

33. $z_1=4+2i, z_2=-5+i$

Graph the regions of the complex plane defined by the following.

34. $\{z \mid 2\le|z|\le3\}$

35. $\{z=a+bi \mid a>2, b>3\}$

Write each of the following complex numbers in trigonometric form.

36. $2\sqrt{3}-3i$

37. $1+4i$

Write each of the following complex numbers in standard form.

38. $4\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)$

39. $3(\cos60°+i\sin60°)$

Perform the following operations and show the answer in both trigonometric form and standard form.

40. $\left[\sqrt{3}\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)\right]\left[4\sqrt{3}\left(\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right)\right]$

41. $\frac{5(\cos240°+i\sin240°)}{(\cos120°+i\sin120°)}$

42. $\frac{-\sqrt{3}+i}{1-i\sqrt{3}}$

43. $\left(12e^{35°i}\right)\left(2e^{280°i}\right)$

Use De Moivre's Theorem to calculate the following.

44. $\left(1+\sqrt{3}i\right)^6$

45. $\left[3\left(\cos 240° + i\sin 240°\right)\right]^{11}$

Find the indicated roots of the following and graphically represent each set in the complex plane.

46. The square roots of $-144i$.

47. The cube roots of $125\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)$.

Solve the following equations.

48. $z^4 - 1 + i = 0$

49. $z^3 + 4\sqrt{2} - 4i\sqrt{2} = 0$

Section 9.6

Find the component form and the magnitude of a vector **v** defined by the given points. Assume the first point given is the initial point and the second point given is the terminal point.

50. $(-1, 0), (4, -5)$

51. $(6, 5), (-4, -1)$

For each of the following, calculate and graph **a.** 2**u** + **v**, **b.** –**u** + 3**v**, and **c.** –2**v**.

52. $\mathbf{u} = \langle 1,3\rangle$, $\mathbf{v} = \langle -5,2\rangle$

53. $\mathbf{u} = \langle 1,-1\rangle$, $\mathbf{v} = \langle 4,-3\rangle$

For each of the following graphs, determine the component forms of –**u**, 2**u** – **v**, and **u** + **v** and find the magnitudes of **u** and **v**.

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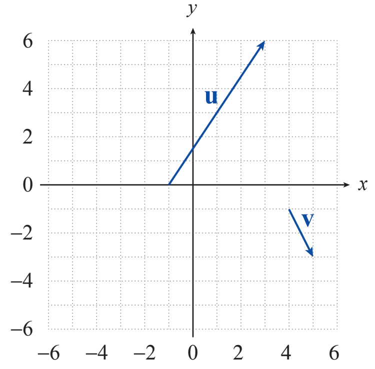

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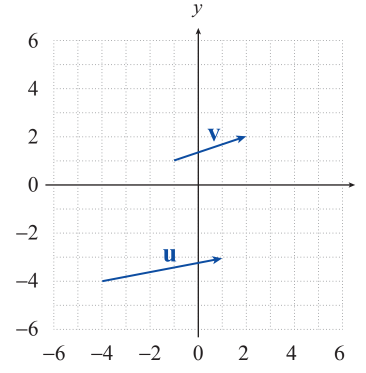


Given the vector **u**, find **a.** a unit vector pointing in the same direction as **u**, and **b.** the linear combination of **i** and **j** that is equivalent to **u**.

56. $\mathbf{u} = \langle -4,5\rangle$

57. $\mathbf{u} = \langle 6,3\rangle$

Find the magnitude and direction angle of the vector **v**.

58. $\mathbf{v} = 4\left(\cos 135°\mathbf{i} + \sin 135°\mathbf{j}\right)$

59. $\mathbf{v} = 5\mathbf{i} - \mathbf{j}$

Find the component form of **v** given its magnitude and the angle it makes with the positive *x*-axis.

60. $\|\mathbf{v}\| = 2\sqrt{2}, \theta = 60°$

61. $\|\mathbf{v}\| = 6$, $\mathbf{v}$ in the direction of $3\mathbf{i} - 4\mathbf{j}$

62. A golf ball is driven into the air at a speed of 90 miles per hour and an angle of 45° from the horizontal. Express this velocity in vector form.

63. A sailboat is traveling at a speed of 55 miles per hour with a bearing of N 24° W, when it encounters a front with winds blowing at 20 miles per hour with a bearing of S 10° W. What is the resultant true velocity of the sailboat?

Section 9.7

Find the indicated quantity given $\mathbf{u} = \langle 1, -4 \rangle$ and $\mathbf{v} = \langle 2, 5 \rangle$.

64. $3\mathbf{u} \cdot \mathbf{v}$

65. $(\mathbf{u} \cdot \mathbf{v})3\mathbf{v}$

Find the magnitude of **u** using the dot product.

66. $\mathbf{u} = \langle -2, -3 \rangle$

67. $\mathbf{u} = -\mathbf{i} - 3\mathbf{j}$

Find the angle between the given vectors.

68. $\mathbf{u} = \langle 5, -5 \rangle, \mathbf{v} = \langle 1, 4 \rangle$

69. $\mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j}, \mathbf{v} = \cos\left(\frac{2\pi}{3}\right)\mathbf{i} + \sin\left(\frac{2\pi}{3}\right)\mathbf{j}$

Find **u** · **v** where θ is the angle between **u** and **v**.

70. $\|\mathbf{u}\| = 16, \|\mathbf{v}\| = 2, \theta = 60°$

71. $\|\mathbf{u}\| = 8, \|\mathbf{v}\| = 9, \theta = \frac{2\pi}{3}$

Find the projection of **u** onto **v**, and then write **u** as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$.

72. $\mathbf{u} = \langle 2, 3 \rangle, \mathbf{v} = \langle -1, 5 \rangle$

73. $\mathbf{u} = \langle 4, -1 \rangle, \mathbf{v} = \langle 2, 2 \rangle$

Find the work done in a particle moving from *J* to *K* if the magnitude and direction of the force are given by **F**.

74. $J = (2, 4), K = (3, 6), \mathbf{F} = \langle 1, 3 \rangle$

75. $J = (-5, 3), K = (0, 4), \mathbf{F} = \langle 5, 6 \rangle$

76. A truck with a gross weight of 33,000 pounds is parked on a 6° slope. What force is required to prevent the truck from rolling down the hill?

77. The world's strongest man pulls a log 160 feet, and the tension in the cable connecting the man and log is 2650 pounds. What is the work being done if the cable is being held 10° from the horizontal?

Section 9.8

Evaluate each of the following expressions. Round your answers to two decimal places if necessary.

78. $\cosh(\ln 2)$

79. $\sinh(\ln 4)$

80. $\operatorname{sech}(-2)$

81. $\coth 1$

82. Verify the identity $\cosh x + \cosh y = 2\cosh\left(\frac{x+y}{2}\right)\cosh\left(\frac{x-y}{2}\right)$.

CHAPTER 10 REVIEW EXERCISES

Section 10.1

Find the center, foci, and vertices of the ellipse that each equation describes.

1. $(x-3)^2+4(y+1)^2=16$

2. $9x^2+4y^2+18x-16y+9=0$

Sketch the graphs of the following ellipses and determine the coordinates of the foci.

3. $\frac{(x+1)^2}{16}+\frac{(y-2)^2}{9}=1$

4. $x^2+9y^2-6x+18y=-9$

5. $3x^2+y^2=27$

6. $25x^2+4y^2-200x+300=0$

In each of the following exercises, an ellipse is described by either a picture or by the properties it possesses. Find the equation, in standard form, for each ellipse.

7. Center at $(-1,4)$, major axis is vertical and of length 8, foci $\sqrt{7}$ units from the center.

8. Foci at $(1,2)$ and $(7,2)$, $e=\frac{1}{2}$.

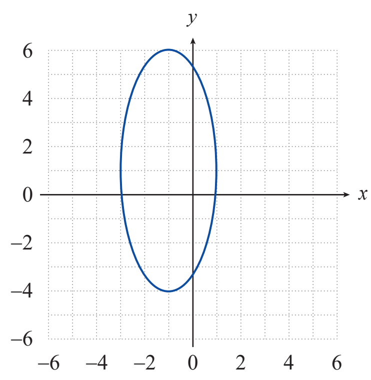
9. Vertices at $\left(\frac{7}{2},-1\right)$ and $\left(\frac{1}{2},-1\right)$, $e=0$.

10. Vertices at $(1,-8)$ and $(1,2)$, minor axis of length 6.

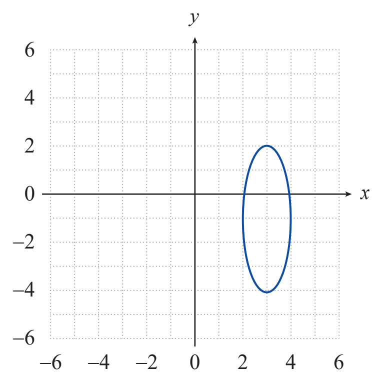
11. Foci at $(0,0)$ and $(4,0)$, major axis of length 8.

12. Center at $(0,4)$, $a=2c$, and vertices at $(-4,4)$ and $(4,4)$.

13.



14.



For exercises 15 and 16, use the fact that the area A of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $A=\pi\cdot a\cdot b$ and $a+b=30$.

15. Write the area of the ellipse as a function of a.

16. Find the equation of an ellipse with an area of 200π square inches.

Section 10.2

Graph the following parabolas and determine the focus and directrix of each.

17. $(y+1)^2 = -12(x+3)$

18. $y^2 - 8y + 2x + 14 = 0$

19. $y^2 + 2y = 4x - 1$

20. $x + \frac{1}{4}y^2 = 0$

21. $2y + 4x^2 = 8$

22. $y^2 - 4y + 2x + 24 = 0$

Find the equation, in standard form, for the parabola with the given properties or with the given graph.

23. Vertex at $(-2,3)$, directrix is the line $y = 2$.

24. Vertex at $(5,-3)$, focus at $(5,1)$.

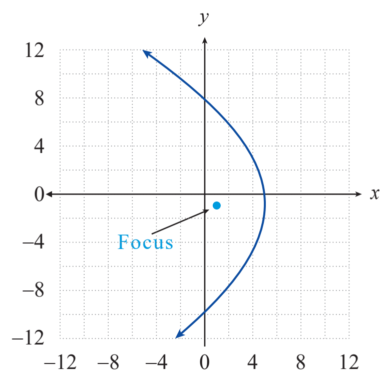
25. Focus at $(3,-1)$, directrix is the line $x = 2$.

26. Focus at $(1,-2)$, directrix is the x-axis.

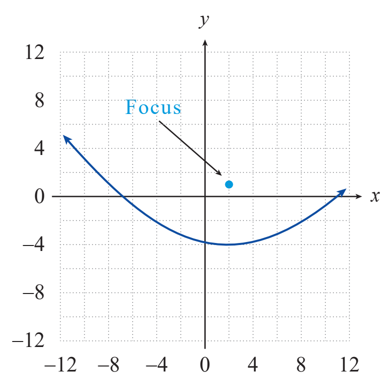
27. Vertex at $(2,-1)$, directrix is the line $x = -2$.

28. Symmetric with respect to the x-axis, focus at $(-3,0)$, and $p = 4$.

29.



30.



31. A motorcycle headlight is made by placing a strong light bulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 5y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?

Section 10.3

Sketch the graphs of the following hyperbolas, using asymptotes as guides. Determine the coordinates of the foci in each case.

32. $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$

33. $9x^2 - 4y^2 + 54x - 8y + 41 = 0$

34. $x^2 - y^2 = 1$

35. $\frac{y^2}{25} - \frac{x^2}{144} = 1$

Find the center, foci, and vertices of the hyperbola that each equation describes.

36. $(x+1)^2-4(y-2)^2=36$

37. $x^2-9y^2+36y-72=0$

38. $y^2-4x^2-2y-32x=67$

39. $\frac{(y-3)^2}{4}-\frac{(x-3)^2}{49}=1$

Find the equation, in standard form, for the hyperbola with the given properties or with the given graph.

40. Vertices at $(4,-1)$ and $(-2,-1)$ and foci at $(5,-1)$ and $(-3,-1)$.

41. Asymptotes of $y=\pm\frac{5}{2}(x+1)-2$ and vertices at $(-3,-2)$ and $(1,-2)$.

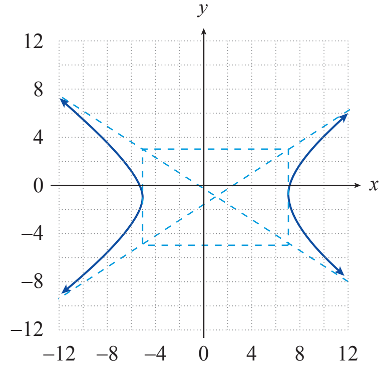
42. Foci at $(-1,-2)$ and $(-1,8)$ and asymptotes of $y=\pm\left(\frac{3}{4}x+\frac{3}{4}\right)+3$.

43. Asymptotes of $y=\pm(3x-6)+2$ and vertices at $(2,-1)$ and $(2,5)$.

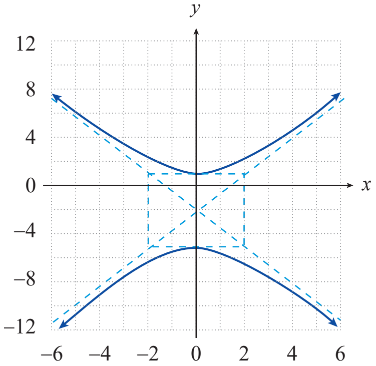
44. Vertices at $(\pm3,0)$ and foci at $(\pm5,0)$.

45. Foci at $\left(-1,7\pm\sqrt{13}\right)$ and asymptotes of $y=\pm\frac{2}{3}(x+1)+7$.

46.



47.



Section 10.4

Find the $x'y'$-coordinates of each point for the given rotation angle θ.

48. $(-8,7),\quad \theta=\frac{\pi}{4}$

49. $(22,86),\quad \theta=\frac{\pi}{3}$

50. $(4.6,-8.9),\quad \theta=53°$

51. $\left(2\sqrt{3},6\sqrt{3}\right),\quad \theta=30°$

Use the discriminant to classify each of the following conic sections. Then determine the angle θ that will allow you to convert the equation and eliminate the *xy*-term. Finally, sketch the graph of the conic section.

52. $xy-6=0$

53. $10x^2+2\sqrt{3}xy+12y^2-100y=0$

54. $10\sqrt{3}x^2+42xy-4\sqrt{3}y^2=187\sqrt{3}$

55. $x^2+2xy+y^2+x-y=0$

Section 10.5

Identify each conic section and find the equation for its directrix.

56. $r = \dfrac{5}{4 - 8\sin\theta}$

57. $r = \dfrac{7}{4 + 4\sin\theta}$

58. $r = \dfrac{4}{6 - 3\cos\theta}$

59. $r = \dfrac{7}{5 + 2\cos\theta}$

Construct a polar equation for each conic section with the focus at the origin and the given eccentricity and directrix.

	Conic	Eccentricity	Directrix
60.	Hyperbola	$e = 4$	$y = 3$
61.	Ellipse	$e = \dfrac{1}{4}$	$x = 16$
62.	Parabola	$e = 1$	$y = -7$
63.	Hyperbola	$e = 9$	$x = \dfrac{1}{3}$

CHAPTER 11 REVIEW EXERCISES

Section 11.1

Use any convenient method to solve the following systems of equations. If a system is dependent, express the solution set in terms of one or more of the variables, as appropriate.

1. $\begin{cases} 3x - y + z = 2 \\ -x + y - 2z = -4 \\ -6x + 2y - 2z = -7 \end{cases}$

2. $\begin{cases} 2x - y = 13 \\ 5x - 2y - z = 25 \\ 7x - 6z = -2 \end{cases}$

3. $\begin{cases} x + y - z = 1 \\ 3x - 4y - 5z = -1 \\ 6x - 3y + z = 20 \end{cases}$

4. $\begin{cases} 6x - 5y = 17 \\ -4x + 9y = -17 \end{cases}$

5. $\begin{cases} 3x - y = 2 \\ -6x + 2y = 5 \end{cases}$

6. $\begin{cases} 3x - 2y = -10 \\ x + 2y = 2 \end{cases}$

7. $\begin{cases} \frac{x}{3} + y - 1 = 0 \\ x + 3y = 3 \end{cases}$

8. $\begin{cases} \frac{x}{3} - \frac{y+1}{2} = 1 \\ \frac{x}{2} - \frac{y}{4} = \frac{3}{4} \end{cases}$

9. $\begin{cases} x + y = 5 \\ 2x - y = 4 \\ 5x + y = 17 \end{cases}$

10. $\begin{cases} 2x + 3y - 4z = -7 \\ x - y + 4z = 6 \\ x + y + z = 2 \end{cases}$

11. $\begin{cases} 3x - 2y + z = 10 \\ x + y + z = 30 \\ 2x - y - z = -6 \end{cases}$

12. $\begin{cases} 3x - 2y - 2z = -8 \\ x - y - z = -5 \\ x + y + z = -3 \end{cases}$

13. Find the equation of a parabola $y = ax^2 + bx + c$, passing through the points $(1,0)$, $(-4,9)$, and $(-1,2)$.

Section 11.2

14. Let $A = \begin{bmatrix} 2 & -8 & 9 \\ 7 & 3 & 0 \\ 11 & 6 & 1 \end{bmatrix}$. Determine the following, if possible:

a. The order of A **b.** The value of a_{12} **c.** The value of a_{21}

15. Let $B = \begin{bmatrix} 13 & 8 & 20 & 5 \end{bmatrix}$. Determine the following, if possible:

a. The order of B **b.** The value of b_{12} **c.** The value of b_{31}

Construct the augmented matrix that corresponds to each of the following systems of equations. (Answers may appear in slightly different, but equivalent, forms.)

16. $\begin{cases} 2x+(y-z)=3 \\ 2(y-x)+y-2=z \\ 3x-\dfrac{3-z}{2}=4y \end{cases}$

17. $\begin{cases} z-4x=5y \\ 14z+7(x+3y)=21 \\ 8x-y=-2(x-3z) \end{cases}$

Construct the system of equations that corresponds to each of the following matrices.

18. $\left[\begin{array}{cc|c} 8 & -2 & 2 \\ -1 & 5 & 3 \end{array}\right]$

19. $\left[\begin{array}{ccc|c} 8 & 0 & 7 & 5 \\ 0 & -3 & 4 & 16 \\ 16 & -2 & 1 & 2 \end{array}\right]$

20. $\left[\begin{array}{ccc|c} 3 & -7 & 6 & 9 \\ -11 & 0 & 3 & -14 \\ 0 & 0 & 8 & 2 \end{array}\right]$

Fill in the blanks by performing the indicated elementary row operations.

21. $\left[\begin{array}{cc|c} 3 & 1 & -2 \\ 1 & 2 & 3 \end{array}\right] \xrightarrow{-3R_2+R_1} \underline{?}$

22. $\left[\begin{array}{cc|c} 2 & 3 & 5 \\ -4 & -1 & 2 \end{array}\right] \xrightarrow{2R_1+R_2} \underline{?}$

23. $\left[\begin{array}{cc|c} 1 & -4 & -4 \\ 3 & -1 & 3 \end{array}\right] \xrightarrow{-2R_1+R_2} \underline{?}$

24. $\left[\begin{array}{ccc|c} -1 & 0 & 2 & -6 \\ 1 & -3 & 4 & 1 \\ -2 & -1 & -3 & 0 \end{array}\right] \xrightarrow[-R_1+R_3]{2R_2} \underline{?}$

Use Gaussian elimination and back-substitution to solve the following systems of equations.

25. $\begin{cases} 3x-y=7 \\ x-4y=6 \end{cases}$

26. $\begin{cases} \dfrac{x}{5}-\dfrac{y}{3}=2 \\ -6x+5y=20 \end{cases}$

Use Gauss-Jordan elimination to solve the following systems of equations.

27. $\begin{cases} 5x-4y=35 \\ 25x-18y=165 \end{cases}$

28. $\begin{cases} x-3y-4z=-5 \\ -x+7y+8z=17 \\ 2x-10y-12z=-10 \end{cases}$

Section 11.3

Evaluate the following determinants.

29. $\begin{vmatrix} x^3 & -x^2 \\ x^2 & x \end{vmatrix}$

30. $\begin{vmatrix} -1 & 3 & 1 \\ 1 & -4 & 0 \\ 0 & 2 & 3 \end{vmatrix}$

31. $\begin{vmatrix} -2 & -1 & -3 & 0 \\ 3 & 3 & 1 & 5 \\ 4 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{vmatrix}$

32. $\begin{vmatrix} x^4 & x & x & 2x \\ 0 & x & x^3 & x \\ 0 & 0 & x & x \\ 0 & 0 & 0 & x^2 \end{vmatrix}$

Use the matrix $A = \begin{bmatrix} 0 & -3 & 1 \\ 2 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}$ to evaluate the minor and cofactor of the following elements.

33. a_{12}

34. a_{31}

Use Cramer's Rule to solve each system of equations.

35. $\begin{cases} x + 6y = 2 \\ 3x - y = -13 \end{cases}$

36. $\begin{cases} x + 2y - 3z = -3 \\ -5x - y + 4z = -5 \\ 3x + y + z = 6 \end{cases}$

37. $\begin{cases} -4x + 2y = 3 \\ 2x - y = 4 \end{cases}$

38. $\begin{cases} x - 2y = 0 \\ x + y + z = 6 \\ 3x - y - 4z = 10 \end{cases}$

Section 11.4

Given $A = \begin{bmatrix} 2 & -8 & 3 \\ -1 & 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 1 & -6 \\ 8 & -3 & -7 \end{bmatrix}$, and $D = \begin{bmatrix} 0 & 4 \\ -3 & 11 \\ 7 & 1 \end{bmatrix}$, determine the following, if possible.

39. BA

40. B^2

41. $CD + C$

42. BD

43. $3A + C$

44. $AD + B$

Determine values of the variables that will make the following equations true, if possible.

45. $\begin{bmatrix} w & 5x \\ 2y & z \end{bmatrix} - 3\begin{bmatrix} w & x \\ 2 & -z \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ y-3 & -16 \end{bmatrix}$

46. $\begin{bmatrix} 4x & 2y^2 & z \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \\ -2 \end{bmatrix}$

47. $2\begin{bmatrix} x \\ -3y \end{bmatrix} - \begin{bmatrix} y \\ 2x \end{bmatrix} = \begin{bmatrix} 7 \\ 14 \end{bmatrix}$

48. $\begin{bmatrix} 3x \\ 5y \end{bmatrix} - \begin{bmatrix} y \\ -2x \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

Evaluate the following matrix products, if possible.

49. $\begin{bmatrix} 7 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 6 \\ 2 & 1 \\ -3 & -3 \end{bmatrix}$

50. $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}\begin{bmatrix} -3 & 2 & 3 \end{bmatrix}$

Section 11.5

Write each of the following systems of equations as a single matrix equation.

51. $\begin{cases} x_1 - x_2 + 2x_3 = -4 \\ 2x_1 - 3x_2 - x_3 = 1 \\ -3x_1 + 6x_3 = 5 \end{cases}$

52. $\begin{cases} 3x - y + z = 4 \\ 2x - 5z = 1 \\ 4x + 3y - 6 = 0 \end{cases}$

Find the inverse of each of the following matrices, if possible.

53. $\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}$

54. $\begin{bmatrix} 2 & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$

55. $\begin{bmatrix} 4 & 12 \\ 3 & 9 \end{bmatrix}$

56. $\begin{bmatrix} -1 & 2 & 3 \\ 1 & -1 & 4 \\ 2 & 0 & -2 \end{bmatrix}$

For each pair of matrices, determine if either matrix is the inverse of the other.

57. $\begin{bmatrix} 3 & 12 \\ 2 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ \frac{2}{3} & 3 \end{bmatrix}$

58. $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}, \begin{bmatrix} -2 & -1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

59. $\begin{bmatrix} -2 & 4 & -3 \\ 0 & 6 & -3 \\ 0 & 8 & -3 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{4}{3} & 1 \end{bmatrix}$

60. $\begin{bmatrix} 5 & -3 & 7 \\ 6 & 0 & 2 \\ -9 & 1 & 0 \end{bmatrix}, \begin{bmatrix} -9 & 2 & 1 \\ 7 & 0 & -3 \\ 1 & 8 & 2 \end{bmatrix}$

Solve the following systems by the inverse matrix method, if possible. If the inverse matrix method doesn't apply, use any other method to determine if the system is inconsistent or dependent.

61. $\begin{cases} 5x + 9y = 2 \\ -2x - 3y = -1 \end{cases}$

62. $\begin{cases} 2y + 3z = 3 \\ -2x = 0 \\ 8x + 4y + 5z = -1 \end{cases}$

Solve the following set of systems by the inverse matrix method.

63. $\begin{cases} 2x - z = 3 \\ x + 4y + 2z = -1 \\ x + y = 5 \end{cases}$ $\begin{cases} 2x - z = 0 \\ x + 4y + 2z = 2 \\ x + y = 1 \end{cases}$ $\begin{cases} 2x - z = -1 \\ x + 4y + 2z = 1 \\ x + y = 2 \end{cases}$

Section 11.6

Write the form of the partial fraction decomposition of each of the following rational functions. In each case, assume the degree of the numerator is less than the degree of the denominator.

64. $f(x) = \dfrac{p(x)}{9x^4 - 6x^3 + x^2}$

65. $f(x) = \dfrac{p(x)}{x^2 + 3x - 4}$

Find the partial fraction decomposition of each of the following rational functions.

66. $f(x) = \dfrac{2x}{(x-1)(x+1)}$

67. $f(x) = \dfrac{x-4}{(2x-5)^2}$

68. $f(x) = \dfrac{2x^2 + x + 8}{(x^2+4)^2}$

Section 11.7

Graph the solution set of each of the following systems of inequalities.

69. $\begin{cases} 7x - 2y \geq 8 \\ y < 5 \end{cases}$

70. $\begin{cases} y - x > 0 \\ x < 2 \end{cases}$

Construct the constraints and graph the feasible regions for the following situations.

71. Each bag of nuts contains peanuts and cashews. The total number of nuts in the bag cannot exceed 60. There must be at least 20 peanuts and 10 cashews per bag. There can be no more than 40 peanuts or 40 cashews per bag. What is the region of constraint for the number of nuts per bag?

72. You wish to study at least 15 hours (over a 4-day span) for your upcoming statistics and biology tests. You need to study a minimum of 6 hours for each test. The maximum you wish to study for statistics is 10 hours and for biology is 8 hours. What is the region of constraint for the numbers of hours you should study for each test?

Find the minimum and maximum values of the given functions, subject to the given constraints.

73. Objective Function:

$f(x, y) = 6x + 10y$

Constraints:

$\begin{cases} x \geq 0,\ y \geq 0 \\ 2x + 5y \leq 10 \end{cases}$

74. Objective Function:

$f(x, y) = 5x + 2y$

Constraints:

$\begin{cases} x \geq 0,\ y \geq 0 \\ x + y \leq 10 \\ x + 2y \geq 10 \\ 2x + y \geq 10 \end{cases}$

75. Objective Function:

$f(x, y) = 5x + 4y$

Constraints:

$\begin{cases} x \geq 0,\ y \geq 0 \\ 2x + 3y \leq 12 \\ 3x + y \leq 12 \\ x + y \geq 2 \end{cases}$

76. Objective Function:

$f(x, y) = 70x + 82y$

Constraints:

$\begin{cases} x \geq 0,\ y \geq 0 \\ x \leq 10,\ y \leq 20 \\ x + y \geq 5 \\ x + 2y \leq 18 \end{cases}$

77. Krueger's Pottery manufactures two kinds of hand-painted pottery: a vase and a pitcher. There are three processes to create the pottery: throwing (forming the pottery on the potter's wheel), baking, and painting. No more than 90 hours are available per day for throwing, only 120 hours are available per day for baking, and no more than 60 hours per day are available for painting. The vase requires 3 hours for throwing, 6 hours for baking, and 2 hours for painting. The pitcher requires 3 hours for throwing, 4 hours for baking, and 3 hours for painting. The profit for each vase is \$25 and the profit for each pitcher is \$30. How many of each piece of pottery should be produced a day to maximize profit? What would the maximum profit be if Krueger's produced this amount?

78. Pranas produces bionic arms and legs for those that are missing a limb. Pranas can produce at least 20, but no more than 60 arms in a week due to the lab limitations. They can produce at least 15, but no more than 40 legs in a week. To keep their research grant, the company must produce at least 50 limbs per week. It costs $450 to produce the bionic arm and $550 to produce the bionic leg. How many of each should be produced per week to minimize the cost? What would the minimum cost be if Pranas produced this amount?

Section 11.8

Use graphing to approximate the real solution(s) of the following systems, and then verify that your answers are correct.

79. $\begin{cases} (x-2)^2 + y = 2 \\ x - y = 2 \end{cases}$

80. $\begin{cases} x^2 + y^2 = 25 \\ -x - y = 5 \end{cases}$

Solve the following systems of nonlinear equations. Be sure to check for nonreal solutions.

81. $\begin{cases} x^2 + 2y^2 = 1 \\ x^2 = y \end{cases}$

82. $\begin{cases} x^2 + y^2 = 25 \\ 2x^2 - y^2 = 23 \end{cases}$

83. $\begin{cases} y = (x-1)^2 \\ y + 8 = (x+1)^2 \end{cases}$

Draw the graph and determine whether the ordered pairs are solutions to the system of inequalities.

84. $\begin{cases} y^2 \leq 9 - x^2 \\ y < |x| \\ y > -|x| \end{cases}$ **a.** $(2,5)$ **b.** $(7,8)$ **c.** $(5,0)$ **d.** $(3,4)$

Graph the following systems of inequalities.

85. $\begin{cases} y \leq \sin x \\ y > -\sin x \end{cases}$

86. $\begin{cases} y \leq \sqrt{x+1} \\ y > x^2 - 1 \end{cases}$

87. $\begin{cases} x^2 y \leq 1 \\ 2y \leq x^2 + 2 \\ y < 16x^2 \end{cases}$

88. The product of two positive integers is 144, and their sum is 25. What are the integers?

89. Stephen and Scott were driving the same 72-mile route, and they departed at the same time. After 30 minutes, Stephen was 6 miles ahead of Scott. If it took Scott one more hour than Stephen to reach their destination, how fast were they each driving?

CHAPTER 12 REVIEW EXERCISES

Section 12.1

Determine the first five terms of each sequence whose n^{th} term is defined as follows.

1. $a_n = (-3)^n$

2. $a_n = (-1)^n \sqrt[3]{n}$

3. $a_1 = -3, a_{n-1} = a_n + 1$ for $n \geq 2$

4. $a_n = \dfrac{n!}{n^n}$

Find a possible formula for the general n^{th} term of each sequence. Answers may vary.

5. $-7, -1, 5, 11, 17, \ldots$

6. $\dfrac{1}{2}, \dfrac{3}{4}, \dfrac{9}{8}, \dfrac{27}{16}, \dfrac{81}{32}, \ldots$

7. $0, 3, 8, 15, 24, 35, \ldots$

8. $\dfrac{3}{2}, \dfrac{5}{3}, \dfrac{7}{4}, \dfrac{9}{5}, \dfrac{11}{6}, \ldots$

9. $-2, -4, -12, -48, -240, \ldots$

10. $2, 6, 12, 20, 30, \ldots$

Translate each expanded sum that follows into summation notation, and vice versa. Then evaluate the sum.

11. $\sum_{i=3}^{8} (-2i + 3)$

12. $\sum_{i=2}^{7} (-2)^{i-1}$

13. $8 + 27 + 64 + \cdots + 343$

14. $\sum_{i=1}^{6} (2i - 3)$

15. $\sum_{i=1}^{5} -4(2^i)$

16. $8 + 18 + 32 + \cdots + 200$

Find a formula for the n^{th} partial sum S_n of each series. If the series is finite, determine the sum. If the series is infinite, determine if it converges or diverges, and if it converges, determine the sum.

17. $\sum_{i=1}^{80} \left(\dfrac{1}{i+1} - \dfrac{1}{i+2} \right)$

18. $\sum_{i=1}^{\infty} \left(\dfrac{1}{i+1} - \dfrac{1}{i+2} \right)$

19. $\sum_{i=1}^{\infty} \left(3^i - 3^{i+1} \right)$

Determine the first five terms of each generalized Fibonacci sequence.

20. $a_1 = -2,\ a_2 = 5,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

21. $a_1 = -10,\ a_2 = -12,$ and $a_n = a_{n-2} + a_{n-1}$ for $n \geq 3$

Section 12.2

Find the explicit formula for the general n^{th} term of each arithmetic sequence.

22. $5, 2, -1, -4, -7, \ldots$

23. $a_2 = 14$ and $a_4 = 19$

24. $a_7 = -43$ and $d = -9$

25. $a_1 = 2, a_4 = 11$

26. $a_9 = \dfrac{13}{2}, d = \dfrac{3}{4}$

27. $-5, 4, 13, 22, 31, \ldots$

Use the given information about each arithmetic sequence to answer the question.

28. Given that $a_1 = -2$ and $a_4 = -20$, what is a_{25}?

29. Given that $a_3 = 17$ and $a_7 = 29$, what is a_{89}?

30. In the sequence $8, 19, 30, \ldots,$ which term is 668?

31. In the sequence $6, 1, -4, \ldots,$ which term is -169?

32. In the sequence $\frac{8}{3}, \frac{10}{3}, 4, \ldots,$ which term is $\frac{56}{3}$?

Find the value of the partial sum of each arithmetic sequence.

33. $\sum_{i=1}^{97} (2i - 7)$

34. $\sum_{i=1}^{60} (-4i + 3)$

35. Sylvia suspects that she has an ant infestation in her apartment. The first day she noticed them, she saw 10 ants in her kitchen. Each day she notices 4 more ants than on the previous day. If she doesn't call an exterminator, how many ants would she see on the fifteenth day?

Section 12.3

Find the explicit formula for the general n^{th} term of each geometric sequence.

36. $2, 8, 32, 128, 512, \ldots$

37. $3, \frac{3}{5}, \frac{3}{25}, \frac{3}{125}, \frac{3}{625}, \ldots$

38. $18, -6, 2, -\frac{2}{3}, \frac{2}{9}, \ldots$

39. $a_1 = 6$ and $a_4 = 384$

40. $a_2 = 20$ and $a_6 = 320$

41. $a_1 = 8$ and $a_4 = \frac{1}{8}$

Given the two terms of a geometric sequence, find the common ratio and first five terms of the sequence.

42. $a_1 = 4$ and $a_4 = 108$

43. $a_4 = \frac{5}{3}$ and $a_6 = \frac{20}{27}$

Use the given information about each geometric sequence to answer the question.

44. Given that $a_2 = \frac{3}{5}$ and $a_4 = \frac{1}{15}$, what is the common ratio r?

45. Given that $a_1 = 3$ and $a_4 = -24$, what is the common ratio r?

46. Given that $a_5 = -16$ and $a_6 = -4$, what is a_{11}?

Each of the following sums is a partial sum of a geometric sequence. Use this fact to evaluate the sums.

47. $\sum_{i=3}^{9} 3\left(\frac{1}{2}\right)^i$

48. $5 + 10 + \cdots + 20,480$

Determine if each of the following infinite geometric series converges. If so, find the sum.

49. $\sum_{i=0}^{\infty} -3\left(\frac{3}{4}\right)^i$

50. $\sum_{i=1}^{\infty} \left(-\frac{5}{4}\right)^i$

51. $\sum_{i=1}^{\infty} \frac{2}{5}\left(\frac{5}{7}\right)^i$

Section 12.4

Use the Principle of Mathematical Induction to prove the following statements.

52. $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$

53. $5+8+11+\cdots+(3n+2)=\frac{n(3n+7)}{2}$

54. $1\cdot 3+2\cdot 4+3\cdot 5+\cdots+n(n+2)=\frac{n(n+1)(2n+7)}{6}$

55. For all natural numbers n, $11^n - 7^n$ is divisible by 4.

56. For all natural numbers n, $7^n - 1$ is divisible by 3.

Section 12.5

Use the Multiplication Principle of Counting and the permutation and combination formulas to answer the following questions.

57. A license plate must contain 4 numerical digits followed by 3 letters. If the first digit cannot be 0 or 1, how many different license plates can be created?

58. How many different 7-digit phone numbers do not contain the digits 6 or 7?

59. In how many different orders can the letters in the word "aardvark" be arranged?

60. In how many different ways can first place, second place, and third place be awarded in a 10-person shot put competition?

61. At a meeting of 21 people, a president, vice president, secretary, treasurer, and recruitment officer are to be chosen. How many different ways can these positions be filled?

62. A college admissions committee selects 4 out of 12 scholarship finalists to receive merit-based financial aid. How many different sets of 4 recipients can be chosen?

Use the Binomial and Multinomial Theorems to expand each of the following expressions.

63. $(1-2y)^5$

64. $(x+2)^7$

65. $(5x^2-2y)^5$

66. $(x+2y+z)^3$

Section 12.6

Apply the formulas for the probabilities of intersection and union to the following sets and determine **a.** $P(E\cap F)$ and **b.** $P(E\cup F)$. Let $n(S)$ equal the size of the sample space.

67. $n(S)=9,\ E=\{3,5,7\},\ F=\{1,2,3,4\}$

68. $n(S)=6,\ E=\{A,B\},\ F=\{X,Y,Z\}$

69. $n(S)=7,\ E=\{\alpha,13\},\ F=\{\alpha,\beta,13,14\}$

70. $n(S)=8,\ E=\{a,4,m,7\},\ F=\{m,3,s\}$

71. $n(S)=10,\ E=\{3,4,X,Y,5,Z\},\ F=\{5,6,7\}$

72. A card is drawn from a standard 52-card deck. Find the probability of drawing

a. a seven or a club.

b. a face card but not a red queen.

c. a black three or a spade.

73. What is the probability of being dealt a five-card hand (from a standard 52-card deck) that contains only face cards?

74. There is a 10% chance of rain each individual day for an entire week. What is the probability that it will rain at least once during this seven- day period?

CHAPTER 13 REVIEW EXERCISES

Section 13.1

Construct and simplify the difference quotient at c with increment h.

1. $f(x) = 5x - 7$

2. $g(x) = 3x^2 + x$

3. $h(x) = \dfrac{1}{2x+2}$

4. $f(x) = 9x^2 + 5$

Use the difference quotient of each function for one appropriate value of c to determine the average rate of change of the function over each of the given intervals.

5. $f(x) = 5x - 7$, interval $= [1, 1.1]$

6. $g(x) = 3x^2 + x$, interval $= [0.9, 1]$

7. $h(x) = \dfrac{1}{2x+2}$, interval $= [-0.01, 0]$

8. $f(x) = 9x^2 + 5$, interval $= [2, 2.01]$

Use difference quotients to approximate the slope of the tangent to the graph of the function at the given point. Use at least five different h-values that are decreasing in magnitude. (Answers will vary.)

9. $f(x) = 3x + 2;\quad (0, 2)$

10. $g(x) = 2 - x^2;\quad (1, 1)$

11. $h(x) = \sqrt{x-1};\quad (2, 1)$

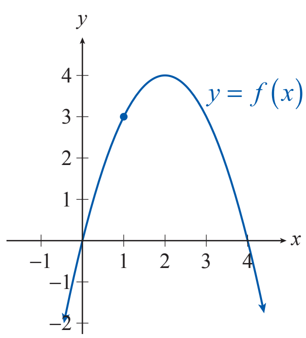
12. $k(x) = \sin x;\quad (0, 0)$

13. A pellet is shot vertically upward from an initial height of 6 feet. Its height after t seconds is given by $h(t) = 6 + 608t - 16t^2$ feet. Use difference quotients to answer the questions below.
- **a.** What will be the pellet's height at the end of the first second?
- **b.** What is the average velocity of the pellet during the first two seconds?
- **c.** Estimate the instantaneous velocity at $t = 0$ seconds.
- **d.** Estimate the instantaneous velocity at $t = 2$ seconds.
- **e.** When will the velocity be 0?

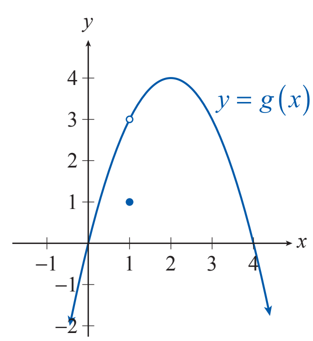
Section 13.2

Use the graph of the function to find the indicated (possibly one-sided) limits, if they exist.

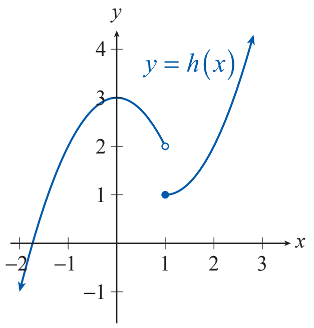
14. $\lim_{x \to 1} f(x)$



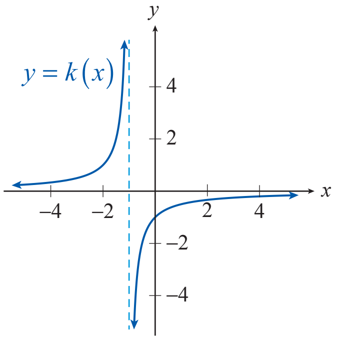
15. $\lim_{x \to 1} g(x)$



16. $\lim_{x \to 1^-} h(x)$



17. $\lim_{x \to -1^+} k(x)$



Create a table of values to estimate the value of the indicated limit without graphing the function. Choose the last *x*-value so that it is no more than 0.001 units from the given *c*-value.

18. $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$

19. $\lim_{x \to 0} x^x$

20. $\lim_{x \to 0} \frac{\sin(2x)}{4x}$

21. $\lim_{x \to 0} \left(2x \sin\left(\frac{1}{4x}\right)\right)$

22. Use one-sided limit notation to describe the behavior of $f(x) = \frac{1}{x-1}$ near $x = 1$.

Section 13.3

Find a $\delta > 0$ that satisfies the limit claim corresponding to $\varepsilon = 0.01$; that is, such that $0 < |x - c| < \delta$ would imply $|f(x) - L| < 0.01$.

23. $\lim_{x \to 0} (3 - 2x) = 3$

24. $\lim_{x \to 4} \sqrt{x} = 2$

Give the formal definition of the limit claim. Then use the definition to prove the claim.

25. $\lim_{x \to 1} (3x + 1) = 4$

26. $\lim_{x \to 1} x^2 = 1$

27. $\lim_{x \to 1} \sqrt{x} = 1$

28. $\lim_{x \to 2} \frac{2}{x} = 1$

Section 13.4

Use algebra and/or appropriate limit laws to evaluate the given limit (one-sided limit where indicated). If the limit is unbounded, use the symbol ∞ or $-\infty$ in your answer.

29. $\lim_{x \to 3} (2x^2 - 3x + 5)$

30. $\lim_{x \to -2} \left(\frac{x^3}{4} + 2x^2 - x + 1\right)$

31. $\lim_{x \to 3} \sqrt{x^3 + 2x^2 + 4}$

32. $\lim_{x \to -2} \frac{2x + 1}{x^2 - x}$

33. $\lim_{t \to 1} \left(\frac{3t + 5t^3}{t^2 + 1}\right)^{\frac{3}{2}}$

34. $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$

35. $\lim_{x \to -5} \frac{x + 5}{x^2 - 25}$

36. $\lim_{x \to 5^-} \frac{x + 5}{x^2 - 25}$

37. $\lim_{x\to 1^+} \frac{x^2+1}{x^4-1}$

38. $\lim_{x\to 1} \frac{x^2-1}{x^4-1}$

39. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$

40. $\lim_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$

41. $\lim_{x\to 0^-} \frac{2|x|}{x}$

42. $\lim_{x\to -2^+} \sqrt{4-x^2}$

43. $\lim_{x\to 2^-} ([\![x]\!]+2x)$

44. $\lim_{x\to 1^+} ([\![x]\!]x)$

45. If $f(x)=x^2$, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Use the Squeeze Theorem to prove the limit claim.

46. $\lim_{x\to 0}\left(x\cos\left(\frac{1}{x}\right)\right)=0$

47. $\lim_{x\to \infty} \frac{\sin x}{\ln x}=0$

Section 13.5

48. Sketch a graph of a function (a formula is not necessary) that is not continuous at $x=0$ from either direction, but both of its one-sided limits exist at $x=0$.

49. Sketch a graph of a function that is left-continuous at $x=0$, but its right-hand limit at $x=0$ doesn't exist.

Find and classify the discontinuities (if any) of the function as removable or nonremovable.

50. $f(x)=\frac{x-9}{\sqrt{x}-3}, \quad x\ge 0$

51. $g(x)=\frac{\sqrt{x}+2}{x-4}, \quad x\ge 0$

52. $h(x)=\frac{1}{\sqrt{x^2+1}}$

53. $t(x)=2+2[\![x]\!]$

54. $G(x)=\frac{x}{\sqrt{x+1}-1}, \quad x\ge -1$

55. $k(x)=|x-3|+|x+1|$

Use the ε-δ definition to prove that the function is continuous.

56. $f(x)=3x-1$

57. $g(x)=2x^2$

58. Find the values of a and b such that f is continuous on the entire real line.

$$f(x)=\begin{cases} -1 & \text{if } x\le -3 \\ ax+b & \text{if } -3<x<2 \\ x^2 & \text{if } x\ge 2 \end{cases}$$

59. Use the Intermediate Value Theorem to prove that the equation $2x^5 + x + 1 = 0$ has a solution on the interval $[-1,1]$.

60. Use the Intermediate Value Theorem to show that the graphs of $f(x) = x^3$ and $g(x) = e^{-x}$ intersect.

Section 13.6

Find the equation of the tangent line to the graph of $f(x)$ at the given point.

61. $f(x) = x^2 + x; \quad (1,2)$

62. $f(x) = \sqrt{x}; \quad (4,2)$

Use the definition (also called the *limit process*) to find the derivative function f' of the given function f. Find all x-values (if any) where the tangent line is horizontal.

63. $f(x) = 2x - x^2$

64. $f(x) = \dfrac{3}{x-2}$

For Exercises 65–66, sketch the graph of a function f possessing the given characteristics. (A formula is useful, but not necessary.)

65. f is continuous at 0, $f(0) = 1$, $f'(x) < 0$ for $x < 0$, $f'(x) > 0$ for $x > 0$, and $f'(0)$ does not exist

66. $g(1) < 0$, $g'(1) > 0$, and $g(2) > 0$, but $g'(2) < 0$

67. Prove that if $f(x)$ is a quadratic function, then $f'(x)$ is linear.

68. A small object is thrown upward with an initial velocity of 12 m/s from the top of a 15 m high building.

a. How high does it go and when does it reach the ground?
b. What is the speed of impact?

(**Hint:** Use $h(t) = -5t^2 + 12t + 15$ as the position function, where h is in meters, t in seconds.)

69. The owner of a small toy manufacturer has determined that he can sell x toys if the price is $p = D(x) = 0.2x + 30$ dollars. The total cost as a function of x is given by $C(x) = 0.1x^2 + 15x + 247.5$ dollars.

a. Find the profit function $P(x)$.
b. Find any break-even points.
c. Find the marginal profit function.