CHAPTER 8 REVIEW EXERCISES

Section 8.1

Use trigonometric identities to simplify the expressions. There may be more than one correct answer.

1. $\cot x \sec x$

2. $(\csc^2 x - 1)\cos^2\left(\frac{\pi}{2} - x\right)$

3. $\dfrac{\tan(-y)}{\cot(\pi + y)}$

4. $\dfrac{\tan^2 \alpha}{\csc^2\left(\frac{\pi}{2} - \alpha\right)} + \dfrac{\cos \alpha}{\sec(\alpha + 2\pi)}$

5. $\sin^2\theta\sec^2\theta - \csc^2\left(\frac{\pi}{2} - \theta\right) + \sin(-\theta)$

6. $\sin\left(\frac{\pi}{2} - x\right)\cos(x + 2\pi) + \sin(-x)\sec\left(\frac{\pi}{2} - x\right)$

Verify the following trigonometric identities.

7. $(\tan x + \sec x)(\sec x - \tan x) = 1$

8. $\cos^2 x \tan^2 x = 1 - \dfrac{1}{\sec^2 x}$

9. $\dfrac{\cos\left(\frac{\pi}{2} - t\right)}{\tan(-t)} = -\cos t$

10. $5 + \tan^2 y = 4 + \sec^2 y$

11. $\tan(\theta + \pi) = -\dfrac{\sec(\theta + 2\pi)}{\csc(-\theta)}$

12. $-\tan\left(\frac{\pi}{2} - x\right)\tan(-x) - \tan^2 x \sin^2\left(\frac{\pi}{2} - x\right) = \cos^2 x$

Use the suggested substitution to rewrite the given expression as a trigonometric expression. Assume $0 \le \theta \le \frac{\pi}{2}$.

13. $\sqrt{16 + x^2}$, $\tan\theta = \frac{x}{4}$

14. $\sqrt{64 - 16x^2}$, $2\sin\theta = x$

15. $\sqrt{25x^2 - 100}$, $\csc\theta = \frac{x}{2}$

16. $\sqrt{9x^2 + 36}$, $x = 2\tan\theta$

Section 8.2

Use the sum and difference identities to determine the exact value of each of the following expressions.

17. $\cos\left(\frac{\pi}{2} + \frac{5\pi}{3}\right)$

18. $\cos 255°$

19. $\sin(-15°)$

20. $\sin\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)$

21. $\tan\left(\pi - \frac{2\pi}{3}\right)$

22. $\tan 105°$

23. Suppose that $\sin\alpha = \frac{8}{17}$ and $\sin\beta = \frac{3}{5}$ and the terminal sides of both α and β are in quadrant II. Find $\tan(\alpha - \beta)$.

24. Suppose that $\sin\alpha = \frac{5}{13}$ and $\cos\beta = \frac{4}{5}$, the terminal side of α is in quadrant II, and the terminal side of β is in quadrant IV. Find $\sin(\alpha - \beta)$.

Use the sum and difference identities to rewrite each of the following expressions as a trigonometric function of one angle, and then evaluate the result.

25. $\sin 175° \cos 35° + \cos 175° \sin 35°$

26. $\dfrac{\tan\left(\frac{9\pi}{8}\right) - \tan\left(\frac{3\pi}{8}\right)}{1 + \tan\left(\frac{9\pi}{8}\right)\tan\left(\frac{3\pi}{8}\right)}$

Use the sum and difference identities to verify the following identities.

27. $\sin x = \cos\left(\frac{\pi}{2} - x\right)$

28. $\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2\sin\alpha\sin\beta$

Express each of the following as an algebraic function of *x*.

29. $\cos\left(\sin^{-1} x + \tan^{-1} x\right)$

30. $\cos\left(\cos^{-1}(2x) + \tan^{-1}(2x)\right)$

Express each of the following functions in terms of a single sine function.

31. $f(x) = \sqrt{2}\sin x - \sqrt{2}\cos x$

32. $h(\alpha) = \sqrt{3}\sin(4\alpha) - \cos(4\alpha)$

Section 8.3

Use the given information to determine $\cos(2x)$, $\sin(2x)$, and $\tan(2x)$ if possible.

33. $\tan x = \frac{4}{3}$ and $\sin x$ is positive

34. $\sin x = \frac{-1}{\sqrt{10}}$ and $\tan x$ is positive

Verify the following trigonometric identities.

35. $\cos(3x) = 4\cos^3 x - 3\cos x$

36. $\dfrac{\sin(4x)}{4} = \sin x \cos x - 2\sin^3 x \cos x$

Use a power-reducing identity to rewrite the given expression as directed.

37. Rewrite $\sin^3 x \cos^2 x$ in terms containing only the first powers of sine and cosine.

38. Rewrite $\tan^2 x \sin^3 x$ in terms containing only the first powers of sine and cosine.

Determine the exact value of each of the following expressions.

39. $\tan\left(\frac{5\pi}{12}\right)$

40. $\cos(157.5°)$

41. $\tan 15°$

42. $\sin\left(-\frac{5\pi}{8}\right)$

Use the product-to-sum identities to rewrite the given expression as a sum or difference.

43. $\cos(x+y)\sin(x-y)$

44. $\cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)$

45. $\sin 165° \cos 15°$

46. $\sin(4x)\sin(3x)$

Use the sum-to-product identities to rewrite the given expression as a product.

47. $\sin(5\alpha)-\sin(3\alpha)$

48. $\cos 225° + \cos 15°$

49. $\cos\left(\frac{3\pi}{4}\right)-\cos\left(\frac{2\pi}{3}\right)$

50. $\sin\left(\frac{5\pi}{6}\right)+\sin\left(\frac{\pi}{3}\right)$

Section 8.4

Use the trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equations.

51. $8\cos^2 x + 1 = 7$

52. $2\sin^2 x = \sin x$

53. $-\sin^2 x + 4\cos x + 1 = 0$

54. $\tan^3 x = \tan x$

55. $-2\sin^2 x = -\cos x - 1$

56. $\sin x + \cos x \cot x = -2$

Use trigonometric identities, algebraic methods, and inverse trigonometric functions, as necessary, to solve the following trigonometric equations on the interval $[0, 2\pi)$.

57. $3\tan^2 x + 9 = 10$

58. $\sin^2 x = 3 - 2\sin x$

Determine if the value given is a solution to the trigonometric equation. If the value of *x* is not a solution, give all solutions to the equation.

59. $4\sin^2 x = 3; \quad x = \frac{5\pi}{3} + 2n\pi$

60. $\frac{1}{2}\csc x + 1 = 2; \quad x = \frac{3\pi}{4} + 2n\pi$

61. $\tan(2x)\cos x = -\frac{\sqrt{3}}{2}; \quad x = \frac{\pi}{6} + 2n\pi$

62. $\sin x + \cos(2x) = 1; \quad x = \frac{5\pi}{6} + 2n\pi$

Solve the following equations on the interval $[0°, 360°)$. Give exact answers when appropriate; otherwise, round your answers to one decimal place.

63. $\cos^2 x \sin x = \sin x$

64. $2\cos^2 x + 7\cos x = 4$