CHAPTER 5 REVIEW EXERCISES

Section 5.1

Verify that the given values of x solve the corresponding polynomial equations.

1. $4x^3 - 5x^2 = -3x + 18;\ x = 2$

2. $x^2 - 6x = -13;\ x = 3 + 2i$

3. $x^3 + x = 6x^2 - 164;\ x = 5 - 4i$

4. $x^3 + (1+4i)x = (7-2i)x^2 - 2i + 36;\ x = -2i$

Solve the following polynomial equations by factoring and/or using the quadratic formula, making sure to identify all the solutions.

5. $x^4 - 7x^2 + 10 = 0$

6. $x^5 - x^3 - 2x = 0$

7. $x^4 + 4 = 4x^2$

8. $6x^2 + 8x = -x^3$

9. $x^4 + x^3 = x^2$

10. $x^2 + 4x + 7 = 0$

For each of the following polynomial functions, determine the behavior of its graph as $x \to \pm\infty$ and identify the x- and y-intercepts. Use this information to then sketch the graph of each polynomial.

11. $f(x) = (x+2)(x-1)(x-3)$

12. $f(x) = (x-2)^2(x+1)^2$

13. $g(x) = x^2 - 5x + 4$

14. $h(x) = -x^3 - 7x^2 - 10x$

Solve the following polynomial inequalities.

15. $2x^2 + 15 \le 11x$

16. $(x-3)^2(x+1)^2 > 0$

17. $(x-4)(x+2)(x^2-1) \le 0$

18. $x^3 - 2x^2 - 8x \ge 0$

19. $x^2(x-2)(1-x) < 0$

20. $-3x^2 + 7x - 2 > 0$

21. A manufacturer has determined that the revenue from the sale of x video games is given by $r(x) = -x^2 + 12x$. The cost of producing x video games is $C(x) = 120 - 22x$. Given that profit is revenue minus cost, what value(s) for x will give the company a nonnegative profit?

Section 5.2

Use polynomial long division to rewrite each of the following fractions in the form $q(x) + \dfrac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

22. $\dfrac{8x^4 - 6x^3 + 2x^2 + 3x + 4}{2x^2 - 1}$

23. $\dfrac{11x^2 + 2x - 5}{x - 3}$

24. $\dfrac{x^4 - 3x^2 + x - 8}{x^2 + 3x + 2}$

25. $\dfrac{2x^5 - 4x^3 - x^2 + x - 2}{x^2 - x}$

26. $\dfrac{2x^3 + ix^2 - 12x - 4 + i}{2x + i}$

Use synthetic division to determine if the given value for c is a zero of the corresponding polynomial. If not, determine $p(c)$.

27. $p(x) = 6x^5 - 23x^4 - 95x^3 + 70x^2 + 204x - 72;\ \ c = 1$

28. $p(x) = 48x^4 + 10x^3 - 51x^2 - 10x + 3;\ \ c = \frac{1}{6}$

29. $p(x) = 18x^5 - 87x^4 + 110x^3 - 28x^2 - 16x + 3;\ \ c = \frac{2}{3}$

Use synthetic division to rewrite each of the following fractions in the form $q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

30. $\dfrac{x^4 - 2x^3 - x^2 + x - 21}{x - 3}$

31. $\dfrac{-x^4 - x^3 - x^2 + 2x + 69}{x + 3}$

32. $\dfrac{x^5 + 2x^4 + 3x^3 + 6x^2 - 5x + 13}{x + 2}$

33. $\dfrac{-x^4 + 8x^3 - 6x^2 - 4x + 2}{x - 1}$

34. $\dfrac{x^4 + (4 - 2i)x^3 - (1 + 8i)x^2 + (3 + 2i)x - 6i}{x - 2i}$

Construct a polynomial function with the stated properties.

35. Second-degree, zeros of -2 and 6, and goes to ∞ as $x \to \infty$.

36. Fourth-degree and a single x-intercept of -4 and y-intercept $(0, 128)$.

37. Third-degree, zeros of ± 2 and 3 and passing through the point $(4, 24)$.

Section 5.3

List all of the potential rational zeros of the following polynomials. Then use polynomial division and the quadratic formula, if necessary, to identify the actual zeros.

38. $f(x) = x^4 + 3x^3 - 3x^2 - 11x - 6$

39. $g(x) = 2x^3 - 11x^2 + 18x - 9$

40. $h(x) = 2x^3 + 2x^2 - 9x + 9$

41. $p(x) = x^4 + 8x^3 + 22x^2 + 24x + 9$

Using the Rational Zero Theorem or your answers to the preceding problems, solve the following polynomial equations.

42. $2x^4 - 6x^2 = -6x^3 + 22x + 12$

43. $2x^3 - 9x^2 + 18x = 9 + 2x^2$

44. $2x^3 + 9 = 9x - 2x^2$

45. $x^4 - x^5 = -x^5 - 8x^3 - 22x^2 - 24x - 9$

Use Descartes' Rule of Signs to determine the possible numbers of positive and negative real zeros of each of the following polynomials.

46. $f(x) = 2x^4 - 3x^3 - x^2 + 3x + 10$

47. $g(x) = x^6 - 4x^5 - 2x^4 + x^3 - 6x^2 - 11x + 6$

Use synthetic division to identify integer upper and lower bounds of the real zeros of the following polynomials.

48. $f(x) = 2x^3 - 11x^2 + 3x + 36$

49. $g(x) = 4x^3 - 16x^2 - 79x - 35$

Using your answers to the preceding problems, polynomial division, and the quadratic formula, if necessary, find all of the zeros of the following polynomials.

50. $f(x) = 2x^3 - 11x^2 + 3x + 36$

51. $g(x) = 4x^3 - 16x^2 - 79x - 35$

Use the Intermediate Value Theorem to show that each of the following polynomials has a real zero between the indicated values.

52. $f(x) = 2x^4 - 6x^3 + x - 5;\ -2 \text{ and } 0$

53. $f(x) = -x^3 + 3x^2 + x - 3;\ 2 \text{ and } 4$

Find all of the real zeros of the following functions.

54. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

55. $g(x) = x^3 - 4x^2 + 9x - 36$

56. $f(x) = x^3 + 6x^2 + 11x + 6$

57. $f(x) = x^3 - 7x^2 + 13x - 3$

Solve the following equations.

58. $x^4 - 2x^3 + 10x^2 = 9(2x - 1)$

59. $2x^3 = 7x^2 - 4x - 4$

60. $-8 = 3x^3 + 4x^2 + 6x$

Section 5.4

Throughout these exercises, a graphing utility may be helpful in identifying zeros and in checking your graphs.

Sketch the graph of each factored polynomial.

61. $f(x) = (x+4)^2(x-1)$

62. $g(x) = x(x-3)(x+4)^3$

Factor each of the following polynomials completely, and then sketch the graph of each one.

63. $f(x) = x^3 - 3x^2 + x - 3$

64. $f(x) = x^5 - x^4 - 2x^3 - x^2 + x + 2$

Solve each polynomial equation.

65. $3x^5 + x^4 + 5x^3 = x^2 + 28x + 20$

66. $8x^5 + 12x^4 - 18x^3 - 35x^2 = 18x + 3$

67. $x^5 + 3x^4 + 3x^3 + 9x^2 = 4(x+3)$

Factor each of the following polynomials completely, making use of the given zero.

68. $f(x)=14x^4-109x^3+296x^2-321x+70;\ 2+i$ is a zero

69. $f(x)=x^4-5x^3+19x^2-125x-150;\ -5i$ is a zero

70. $f(x)=2x^4+3x^3-7x^2+8x+6;\ 1+i$ is a zero

71. $f(x)=4x^3+10x^2-x+15;\ -3$ is a zero

Construct polynomial functions with the stated properties.

72. Fourth-degree, only real coefficients, $\frac{1}{2}$ and $1+2i$ are two of the zeros, y-intercept is -30, leading coefficient is 2.

73. Fifth-degree, only real coefficients, -1 is a zero of multiplicity 3, $\sqrt{6}$ is a zero, y-intercept is -6, leading coefficient is 1.

74. Fifth-degree, only real coefficients, 1 is a zero of multiplicity 3, $\sqrt{3}$ is a zero, y-intercept is 3, leading coefficient is 1.

Section 5.5

Find equations for the vertical asymptotes, if any, for each of the following rational functions.

75. $f(x)=\dfrac{4}{2x-5}$

76. $f(x)=\dfrac{x^2-3x+2}{x-1}$

77. $f(x)=\dfrac{x^2-1}{x-x^2}$

78. $f(x)=\dfrac{x^2-x-6}{x^2-6x+9}$

Find equations for the horizontal or oblique asymptotes, if any, for each of the following rational functions.

79. $f(x)=\dfrac{2x^3+5x^2-1}{x^2-2x}$

80. $f(x)=\dfrac{x^2-x+8}{3x^2-7}$

81. $f(x)=\dfrac{x^2-9}{x+3}$

82. $f(x)=\dfrac{x^2+2x-3}{(x+1)^3}$

Sketch the graphs of the following rational functions.

83. $f(x)=\dfrac{2x}{x+1}$

84. $f(x)=\dfrac{4x^2}{x^2+3x}$

85. $f(x)=\dfrac{x^2+2}{x+2}$

86. $f(x)=\dfrac{x+1}{x^2-4}$

Solve the following rational inequalities.

87. $\dfrac{7}{x+3}\geq\dfrac{2x}{x+3}$

88. $\dfrac{x}{x^2-5x+6}\leq\dfrac{3}{x^2-5x+6}$

89. $\dfrac{x-4}{x+3}<\dfrac{x}{x-2}$

90. $\dfrac{x-2}{x+3}<2$