CHAPTER 13 REVIEW EXERCISES

Section 13.1

Construct and simplify the difference quotient at c with increment h.

1. $f(x) = 5x - 7$

2. $g(x) = 3x^2 + x$

3. $h(x) = \dfrac{1}{2x+2}$

4. $f(x) = 9x^2 + 5$

Use the difference quotient of each function for one appropriate value of c to determine the average rate of change of the function over each of the given intervals.

5. $f(x) = 5x - 7$, interval $= [1, 1.1]$

6. $g(x) = 3x^2 + x$, interval $= [0.9, 1]$

7. $h(x) = \dfrac{1}{2x+2}$, interval $= [-0.01, 0]$

8. $f(x) = 9x^2 + 5$, interval $= [2, 2.01]$

Use difference quotients to approximate the slope of the tangent to the graph of the function at the given point. Use at least five different h-values that are decreasing in magnitude. (Answers will vary.)

9. $f(x) = 3x + 2; \quad (0, 2)$

10. $g(x) = 2 - x^2; \quad (1, 1)$

11. $h(x) = \sqrt{x-1}; \quad (2, 1)$

12. $k(x) = \sin x; \quad (0, 0)$

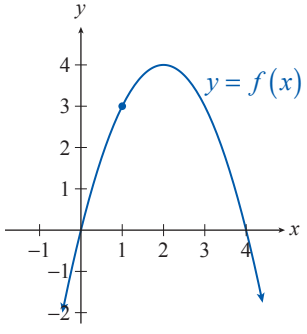
13. A pellet is shot vertically upward from an initial height of 6 feet. Its height after t seconds is given by $h(t) = 6 + 608t - 16t^2$ feet. Use difference quotients to answer the questions below.

- **a.** What will be the pellet's height at the end of the first second?
- **b.** What is the average velocity of the pellet during the first two seconds?
- **c.** Estimate the instantaneous velocity at $t = 0$ seconds.
- **d.** Estimate the instantaneous velocity at $t = 2$ seconds.
- **e.** When will the velocity be 0?

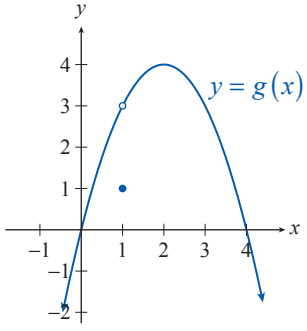
Section 13.2

Use the graph of the function to find the indicated (possibly one-sided) limits, if they exist.

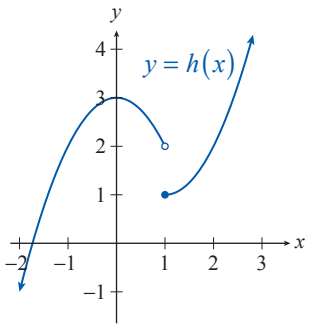
14. $\lim\limits_{x \to 1} f(x)$



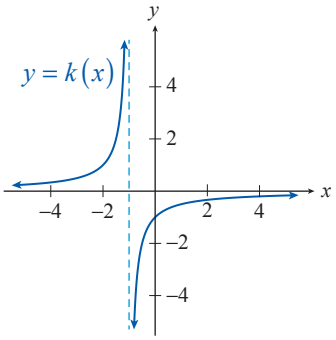
15. $\lim\limits_{x \to 1} g(x)$



16. $\lim_{x\to 1^-} h(x)$



17. $\lim_{x\to -1^+} k(x)$



Create a table of values to estimate the value of the indicated limit without graphing the function. Choose the last *x*-value so that it is no more than 0.001 units from the given *c*-value.

18. $\lim_{x\to 1} \dfrac{x^3-1}{x-1}$

19. $\lim_{x\to 0} x^x$

20. $\lim_{x\to 0} \dfrac{\sin(2x)}{4x}$

21. $\lim_{x\to 0}\left(2x\sin\left(\dfrac{1}{4x}\right)\right)$

22. Use one-sided limit notation to describe the behavior of $f(x)=\dfrac{1}{x-1}$ near $x=1$.

Section 13.3

Find a $\delta>0$ that satisfies the limit claim corresponding to $\varepsilon=0.01$; that is, such that $0<|x-c|<\delta$ would imply $|f(x)-L|<0.01$.

23. $\lim_{x\to 0}(3-2x)=3$

24. $\lim_{x\to 4}\sqrt{x}=2$

Give the formal definition of the limit claim. Then use the definition to prove the claim.

25. $\lim_{x\to 1}(3x+1)=4$

26. $\lim_{x\to 1} x^2=1$

27. $\lim_{x\to 1}\sqrt{x}=1$

28. $\lim_{x\to 2}\dfrac{2}{x}=1$

Section 13.4

Use algebra and/or appropriate limit laws to evaluate the given limit (one-sided limit where indicated). If the limit is unbounded, use the symbol ∞ or $-\infty$ in your answer.

29. $\lim_{x\to 3}(2x^2-3x+5)$

30. $\lim_{x\to -2}\left(\dfrac{x^3}{4}+2x^2-x+1\right)$

31. $\lim_{x\to 3}\sqrt{x^3+2x^2+4}$

32. $\lim_{x\to -2}\dfrac{2x+1}{x^2-x}$

33. $\lim_{t\to 1}\left(\dfrac{3t+5t^3}{t^2+1}\right)^{\frac{3}{2}}$

34. $\lim_{x\to 4}\dfrac{x^2-16}{x-4}$

35. $\lim_{x\to -5}\dfrac{x+5}{x^2-25}$

36. $\lim_{x\to 5^-}\dfrac{x+5}{x^2-25}$

37. $\lim\limits_{x\to 1^+}\dfrac{x^2+1}{x^4-1}$

38. $\lim\limits_{x\to 1}\dfrac{x^2-1}{x^4-1}$

39. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+1}-2}{x-3}$

40. $\lim\limits_{x\to 0}\dfrac{\dfrac{1}{2+x}-\dfrac{1}{2}}{x}$

41. $\lim\limits_{x\to 0^-}\dfrac{2|x|}{x}$

42. $\lim\limits_{x\to -2^+}\sqrt{4-x^2}$

43. $\lim\limits_{x\to 2^-}\left(\llbracket x\rrbracket+2x\right)$

44. $\lim\limits_{x\to 1^+}\left(\llbracket x\rrbracket x\right)$

45. If $f(x)=x^2$, find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

Use the Squeeze Theorem to prove the limit claim.

46. $\lim\limits_{x\to 0}\left(x\cos\left(\dfrac{1}{x}\right)\right)=0$

47. $\lim\limits_{x\to\infty}\dfrac{\sin x}{\ln x}=0$

Section 13.5

48. Sketch a graph of a function (a formula is not necessary) that is not continuous at $x=0$ from either direction, but both of its one-sided limits exist at $x=0$.

49. Sketch a graph of a function that is left-continuous at $x=0$, but its right-hand limit at $x=0$ doesn't exist.

Find and classify the discontinuities (if any) of the function as removable or nonremovable.

50. $f(x)=\dfrac{x-9}{\sqrt{x}-3},\quad x\geq 0$

51. $g(x)=\dfrac{\sqrt{x}+2}{x-4},\quad x\geq 0$

52. $h(x)=\dfrac{1}{\sqrt{x^2+1}}$

53. $t(x)=2+2\llbracket x\rrbracket$

54. $G(x)=\dfrac{x}{\sqrt{x+1}-1},\quad x\geq -1$

55. $k(x)=|x-3|+|x+1|$

Use the ε-δ definition to prove that the function is continuous.

56. $f(x)=3x-1$

57. $g(x)=2x^2$

58. Find the values of a and b such that f is continuous on the entire real line.

$$f(x)=\begin{cases}-1 & \text{if } x\leq -3\\ ax+b & \text{if } -3<x<2\\ x^2 & \text{if } x\geq 2\end{cases}$$

59. Use the Intermediate Value Theorem to prove that the equation $2x^5 + x + 1 = 0$ has a solution on the interval $[-1,1]$.

60. Use the Intermediate Value Theorem to show that the graphs of $f(x) = x^3$ and $g(x) = e^{-x}$ intersect.

Section 13.6

Find the equation of the tangent line to the graph of $f(x)$ at the given point.

61. $f(x) = x^2 + x; \quad (1,2)$

62. $f(x) = \sqrt{x}; \quad (4,2)$

Use the definition (also called the *limit process*) to find the derivative function f' of the given function f. Find all x-values (if any) where the tangent line is horizontal.

63. $f(x) = 2x - x^2$

64. $f(x) = \dfrac{3}{x-2}$

For Exercises 65–66, sketch the graph of a function f possessing the given characteristics. (A formula is useful, but not necessary.)

65. f is continuous at 0, $f(0) = 1$, $f'(x) < 0$ for $x < 0$, $f'(x) > 0$ for $x > 0$, and $f'(0)$ does not exist

66. $g(1) < 0$, $g'(1) > 0$, and $g(2) > 0$, but $g'(2) < 0$

67. Prove that if $f(x)$ is a quadratic function, then $f'(x)$ is linear.

68. A small object is thrown upward with an initial velocity of 12 m/s from the top of a 15 m high building.

a. How high does it go and when does it reach the ground?
b. What is the speed of impact?

(Hint: Use $h(t) = -5t^2 + 12t + 15$ as the position function, where h is in meters, t in seconds.)

69. The owner of a small toy manufacturer has determined that he can sell x toys if the price is $p = D(x) = 0.2x + 30$ dollars. The total cost as a function of x is given by $C(x) = 0.1x^2 + 15x + 247.5$ dollars.

a. Find the profit function $P(x)$.
b. Find any break-even points.
c. Find the marginal profit function.