CHAPTER 10 REVIEW EXERCISES

Section 10.1

Find the center, foci, and vertices of the ellipse that each equation describes.

1. $(x-3)^2+4(y+1)^2=16$

2. $9x^2+4y^2+18x-16y+9=0$

Sketch the graphs of the following ellipses and determine the coordinates of the foci.

3. $\dfrac{(x+1)^2}{16}+\dfrac{(y-2)^2}{9}=1$

4. $x^2+9y^2-6x+18y=-9$

5. $3x^2+y^2=27$

6. $25x^2+4y^2-200x+300=0$

In each of the following exercises, an ellipse is described by either a picture or by the properties it possesses. Find the equation, in standard form, for each ellipse.

7. Center at $(-1,4)$, major axis is vertical and of length 8, foci $\sqrt{7}$ units from the center.

8. Foci at $(1,2)$ and $(7,2)$, $e=\dfrac{1}{2}$.

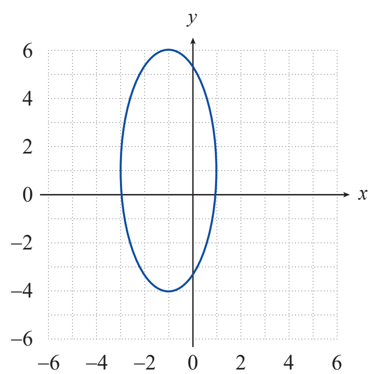
9. Vertices at $\left(\dfrac{7}{2},-1\right)$ and $\left(\dfrac{1}{2},-1\right)$, $e=0$.

10. Vertices at $(1,-8)$ and $(1,2)$, minor axis of length 6.

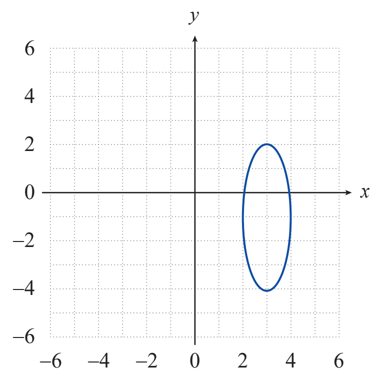
11. Foci at $(0,0)$ and $(4,0)$, major axis of length 8.

12. Center at $(0,4)$, $a=2c$, and vertices at $(-4,4)$ and $(4,4)$.

13.



14.



For exercises 15 and 16, use the fact that the area A of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is $A=\pi\cdot a\cdot b$ and $a+b=30$.

15. Write the area of the ellipse as a function of a.

16. Find the equation of an ellipse with an area of 200π square inches.

Section 10.2

Graph the following parabolas and determine the focus and directrix of each.

17. $(y+1)^2 = -12(x+3)$

18. $y^2 - 8y + 2x + 14 = 0$

19. $y^2 + 2y = 4x - 1$

20. $x + \frac{1}{4}y^2 = 0$

21. $2y + 4x^2 = 8$

22. $y^2 - 4y + 2x + 24 = 0$

Find the equation, in standard form, for the parabola with the given properties or with the given graph.

23. Vertex at $(-2, 3)$, directrix is the line $y = 2$.

24. Vertex at $(5, -3)$, focus at $(5, 1)$.

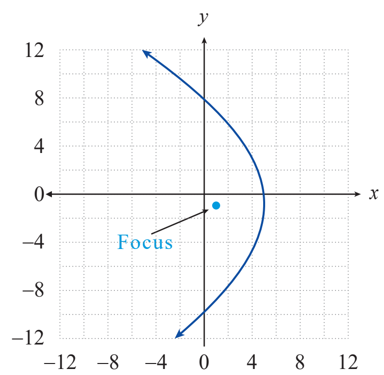
25. Focus at $(3, -1)$, directrix is the line $x = 2$.

26. Focus at $(1, -2)$, directrix is the x-axis.

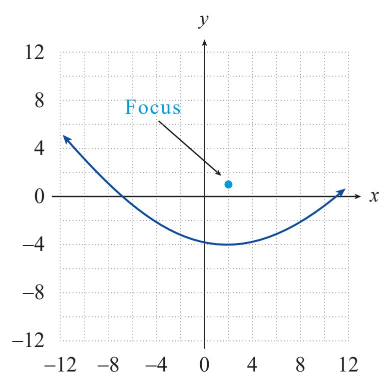
27. Vertex at $(2, -1)$, directrix is the line $x = -2$.

28. Symmetric with respect to the x-axis, focus at $(-3, 0)$, and $p = 4$.

29.



30.



31. A motorcycle headlight is made by placing a strong light bulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 5y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?

Section 10.3

Sketch the graphs of the following hyperbolas, using asymptotes as guides. Determine the coordinates of the foci in each case.

32. $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$

33. $9x^2 - 4y^2 + 54x - 8y + 41 = 0$

34. $x^2 - y^2 = 1$

35. $\frac{y^2}{25} - \frac{x^2}{144} = 1$

Find the center, foci, and vertices of the hyperbola that each equation describes.

36. $(x+1)^2 - 4(y-2)^2 = 36$

37. $x^2 - 9y^2 + 36y - 72 = 0$

38. $y^2 - 4x^2 - 2y - 32x = 67$

39. $\dfrac{(y-3)^2}{4} - \dfrac{(x-3)^2}{49} = 1$

Find the equation, in standard form, for the hyperbola with the given properties or with the given graph.

40. Vertices at $(4,-1)$ and $(-2,-1)$ and foci at $(5,-1)$ and $(-3,-1)$.

41. Asymptotes of $y = \pm\frac{5}{2}(x+1) - 2$ and vertices at $(-3,-2)$ and $(1,-2)$.

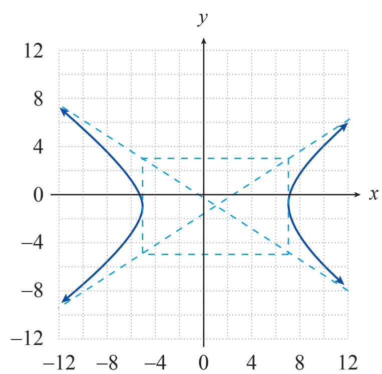
42. Foci at $(-1,-2)$ and $(-1,8)$ and asymptotes of $y = \pm\left(\frac{3}{4}x + \frac{3}{4}\right) + 3$.

43. Asymptotes of $y = \pm(3x-6) + 2$ and vertices at $(2,-1)$ and $(2,5)$.

44. Vertices at $(\pm 3, 0)$ and foci at $(\pm 5, 0)$.

45. Foci at $\left(-1, 7 \pm \sqrt{13}\right)$ and asymptotes of $y = \pm\frac{2}{3}(x+1) + 7$.

46.



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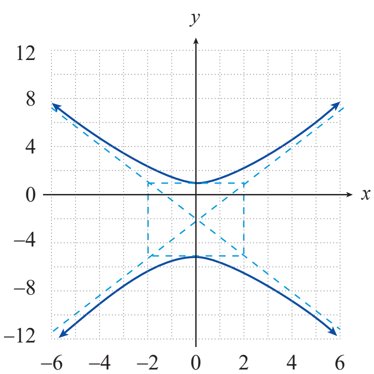

Section 10.4

Find the $x'y'$-coordinates of each point for the given rotation angle θ.

48. $(-8,7), \quad \theta = \frac{\pi}{4}$

49. $(22,86), \quad \theta = \frac{\pi}{3}$

50. $(4.6,-8.9), \quad \theta = 53°$

51. $\left(2\sqrt{3}, 6\sqrt{3}\right), \quad \theta = 30°$

Use the discriminant to classify each of the following conic sections. Then determine the angle θ that will allow you to convert the equation and eliminate the *xy*-term. Finally, sketch the graph of the conic section.

52. $xy - 6 = 0$

53. $10x^2 + 2\sqrt{3}xy + 12y^2 - 100y = 0$

54. $10\sqrt{3}x^2 + 42xy - 4\sqrt{3}y^2 = 187\sqrt{3}$

55. $x^2 + 2xy + y^2 + x - y = 0$

Section 10.5

Identify each conic section and find the equation for its directrix.

56. $r=\frac{5}{4-8\sin\theta}$

57. $r=\frac{7}{4+4\sin\theta}$

58. $r=\frac{4}{6-3\cos\theta}$

59. $r=\frac{7}{5+2\cos\theta}$

Construct a polar equation for each conic section with the focus at the origin and the given eccentricity and directrix.

	Conic	Eccentricity	Directrix
60.	Hyperbola	$e=4$	$y=3$
61.	Ellipse	$e=\frac{1}{4}$	$x=16$
62.	Parabola	$e=1$	$y=-7$
63.	Hyperbola	$e=9$	$x=\frac{1}{3}$