Looking Ahead

The following example incorporates some of the skills you learned in this section, including order of operations, evaluating exponents, and finding a least common multiple in order to find a common denominator.

Example Preview

Evaluate the following expression, using the correct order of operations.

$$4+1\cdot 5\div 8+(-1)^3$$

Solution

Because there are no grouping symbols in this expression, the first step is to calculate exponents and roots.

$$\begin{aligned} 4+1\cdot 5\div 8+(-1)^3 &= 4+1\cdot 5\div 8-1 && \text{Simplify the term with an exponent.} \\ &= 4+\frac{5}{8}-1 && \text{Perform multiplications and divisions from left to right.} \\ &= \frac{32}{8}+\frac{5}{8}-\frac{8}{8} && \text{Find a common denominator.} \\ &= \frac{29}{8} && \text{Add.} \end{aligned}$$

1.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Nine squared is equal to eighteen.

2. $2^7 = 128$

3. 7^0 is undefined.

4. According to the order of operations, multiplication is always performed before division.

5. A number that is divisible by 10 is also divisible by 2 and 5.

6. 6801 is divisible by 9.

7. 7605 is divisible by 10.

8. 5,187,042 is divisible by 3.

9. A prime number has exactly 1 factor.

10. A composite number has 2 or more factors.

11. 231 is a prime number.

12. All the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

13. The LCM of 15 and 25 is 50.

14. The first five multiples of 9 are 9, 18, 27, 36, and 45.

15. When given larger numbers, the most efficient way to find the LCM is to use the prime factorization method.

Practice

For each exponential expression **a.** identify the base, **b.** identify the exponent, and **c.** evaluate the exponential expression.

16. 2^3

17. 4^0

Simplify.

18. $30 \div 2 - 11 + 2(5-1)^3$

Using the tests for divisibility, determine which of 2, 3, 4, 5, 6, 9, and 10 (if any) will divide exactly into each given number.

19. 105

20. 150

21. 331

22. 1234

Determine whether each number is prime or composite. If the number is composite, find at least three factors of the number.

23. 47

24. 63

Find the prime factorization of each number. Use the tests for divisibility for 2, 3, 4, 5, 6, 9, and 10 whenever they help to find beginning factors.

25. 125

26. 150

27. Find the LCM of 3, 4, and 8.

28. **For** 14, 35, and 49, **a.** find the LCM and **b.** state how many times each number divides into the LCM.

For each equation, find the missing numerator that will make the fractions equivalent.

29. $\frac{5}{8} = \frac{?}{24}$

30. $\frac{5}{12} = \frac{?}{108}$

Applications

Solve.

31. ***Purchases:*** Robert is purchasing shirts for his weekend soccer team. The shirts he wants to buy are normally \$25 each but are on sale for \$10 off. His team has a total of 11 players. How much will he spend to buy the shirts?

a. If you simplify the expression \$25 – \$10 · 11 using the order of operations, will you get the correct answer? If not, explain what is wrong with the expression.

b. What is the answer? If necessary, write the corrected expression to get the correct results when following the order of operations.

32. ***Fundraising:*** You are on a team that is participating in a charity walk with a goal to raise \$12,400. Each team member agrees to raise the same amount of money. If the possible team sizes are 5, 6, 9, or 10 members, which team sizes allow the goal amount to be evenly split between the team members? How much money would each team member raise for each team size that can evenly split the goal amount?

33. ***Time:*** A company is working on a project that will take 440 hours of work to complete. The manager in charge of the project has the option to have 4, 6, or 8 people work on the project. If the manager wants to evenly divide the work between the team members, which team size will evenly split the work hours? How many hours would each team member spend on the project for each team size that evenly splits the work hours?

34. ***Inventory:*** Twenty-four pencils are to be distributed evenly between the members of a group. What are the possible group sizes if each person in the group is to receive the same number of pencils?

35. ***Baking:*** A chocolatier makes 72 specialty truffles. She wants to sell packages that each have the same number of truffles. What are her options for the number of truffles that can be in a package?

36. ***Security:*** Three security guards meet at the front gate for coffee before they walk around inspecting buildings at a manufacturing plant. The guards take 15, 20, and 30 minutes, respectively, for the inspection trip.

a. If they start at the same time, in how many minutes will they meet again at the front gate for coffee?

b. How many trips will each guard have made?

Writing & Thinking

37. Give one example where addition should be completed before multiplication.

38. **a.** If a number is divisible by both 3 and 5, then it will be divisible by 15. Give two examples.

b. However, a number might be divisible by 3 and not by 5. Give two examples.

c. Also, a number might be divisible by 5 and not 3. Give two examples.

39. Are all odd numbers also prime numbers? Explain your answer.

40. Explain the difference between factors of a number and multiples of that number.

41. Explain, in your own words, why each number in a set divides evenly into the LCM of that set of numbers.

42. Explain why simply multiplying two numbers together will not necessarily find the LCM of those numbers. Give an example of when it would find the LCM and an example when it would not.

Example Preview

If Sarah were to paint her living room alone, it would take 5 hours. Her sister Rachel could do the job in 8 hours. How many hours would it take them working together?

Solution

The rate of work for Sarah is $\frac{1}{5}$, while the rate of work for her sister Rachel is $\frac{1}{8}$. If we let x denote the time needed to paint the living room when both sisters are working together, the sum of the two individual rates must equal $\frac{1}{x}$. So, we need to solve the equation $\frac{1}{5}+\frac{1}{8}=\frac{1}{x}$. In this case, the LCD is $40x$.

The equation can be solved as follows.

$$\begin{aligned} 40x\cdot\frac{1}{5}+40x\cdot\frac{1}{8}&=40x\cdot\frac{1}{x} \\ 8x+5x&=40 \\ 13x&=40 \\ x&=\frac{40}{13} \end{aligned}$$

It would take them $\frac{40}{13}$, or a little over 3 hours to paint the living room together.

1.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. To find $\frac{1}{2}$ of $\frac{2}{9}$ requires multiplication.

2. $\frac{3}{4}\cdot\frac{9}{10}=\frac{27}{40}$

3. The statement $\frac{1}{3}\cdot\frac{2}{5}=\frac{2}{5}\cdot\frac{1}{3}$ is an example of the associative property of multiplication.

4. The product of a nonzero number and its reciprocal is undefined.

5. The reciprocal of 1 is undefined.

6. The result of $\frac{1}{3} \div \frac{1}{6}$ is 2.

7. The reciprocal of 12 is $\frac{12}{1}$.

Practice

Multiply and reduce to lowest terms. (**Hint:** Factor before multiplying.)

8. $\frac{0}{3} \cdot \frac{5}{7}$

9. $\frac{1}{3} \cdot \frac{3}{4}$

10. $\left(-\frac{1}{5}\right)\left(-\frac{4}{7}\right)$

11. $\frac{5}{16} \cdot \frac{16}{15}$

12. $\frac{9}{10} \cdot \frac{35}{40} \cdot \frac{25}{15}$

Divide and reduce to lowest terms.

13. $\frac{2}{3} \div \frac{3}{4}$

14. $0 \div \frac{5}{6}$

15. $\frac{5}{6} \div 0$

16. $\frac{14}{15} \div \frac{21}{25}$

Applications

Solve.

17. ***Recipes:*** A recipe calls for $\frac{3}{4}$ cups of flour. How much flour should be used if only half of the recipe is to be made?

18. ***Demographics:*** A study showed that $\frac{3}{5}$ of the students in an elementary school were left-handed. If the school had an enrollment of 600 students, how many were left-handed?

19. ***Geology:*** The floor of the Atlantic Ocean is spreading apart at an average rate of $\frac{3}{50}$ of a meter per year. How long will it take for the ocean floor to spread 12 meters?

20. ***Airplane Capacity:*** An airplane is carrying 180 passengers. This is $\frac{9}{10}$ of the capacity of the airplane.

    a. Is the capacity of the airplane more or less than 180?

    b. If you were to multiply 180 times $\frac{9}{10}$, would the product be more or less than 180?

    c. What is the capacity of the airplane?

Writing & Thinking

21. If two fractions are between 0 and 1, can their product be more than 1? Explain.

22. Explain the process of multiplying two fractions. Give an example of a product that cannot be reduced.

23. Explain why the number 0 has no reciprocal.

24. Is division a commutative operation? Explain briefly and give three examples using fractions to help justify your answer.

Looking Ahead

The following example shows one method for solving rational equations. This method requires the subtraction of rational expressions, which follows the same procedure as the subtraction of fractions.

Example Preview

Solve the following rational equation and simplify your answer.

$$\frac{x}{x+2}-\frac{1}{x-4}=1$$

Solution

To solve this equation, assume no denominator is 0. This means $x \neq -2, 4$. Begin by subtracting the rational expressions on the left side of the equation. Note that the LCD is $(x+2)(x-4)$. Change each expression into an equivalent rational expression with that denominator and subtract.

$$\frac{x}{x+2}\cdot\frac{x-4}{x-4}-\frac{1}{x-4}\cdot\frac{x+2}{x+2}=1$$

$$\frac{x^2-4x-(x+2)}{(x+2)(x-4)}=1$$

$$\frac{x^2-4x-x-2}{x^2-2x-8}=1$$

$$\frac{x^2-5x-2}{x^2-2x-8}=1$$

Next, multiply both sides by the denominator and solve the resulting equation.

$$x^2-5x-2=x^2-2x-8$$

$$-5x-2=-2x-8$$

$$-3x=-6$$

$$x=2$$

Since 2 is not equal to –2 or 4, the solution is $\{2\}$.

1.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The final step in adding fractions is to reduce, if possible.

2. The process for finding the LCD is the same as the process for finding the LCM.

3. LCD represents the Least Common Digit.

4. When subtracting fractions, simply subtract the numerators and the denominators.

5. Subtraction of fractions requires that the fractions have the same denominators.

Practice

Add and reduce to lowest terms.

6. $\frac{3}{25}+\frac{12}{25}$

7. $\frac{2}{7}+\frac{4}{21}+\frac{1}{3}$

Subtract and reduce to lowest terms.

8. $-\frac{7}{15}+\frac{3}{5}$

9. $\frac{1}{4}+\left(-\frac{1}{20}\right)+\frac{8}{15}$

10. $\frac{7}{8}-\frac{5}{8}$

11. $\frac{9}{14}-\frac{2}{21}$

12. $2-\frac{9}{16}$

13. $-\frac{5}{12}-\left(-\frac{1}{6}\right)$

Applications

Solve.

14. ***Cooking:*** A recipe calls for the following spices: $\frac{1}{2}$ teaspoon of turmeric, $\frac{1}{4}$ teaspoon of ginger, and $\frac{1}{8}$ teaspoon of cumin. What is the total quantity of these three spices?

15. ***Postage:*** Three pieces of mail weigh $\frac{1}{2}$ ounce, $\frac{1}{5}$ ounce, and $\frac{3}{10}$ ounce. What is the total weight of the letters?

Writing & Thinking

16. Give an example of a situation where you might add or subtract fractions (other than in class).

17. Explain how finding the LCM relates to LCDs.

Multiplying both sides of the equation by the LCD results in the following.

$$(z+9)\frac{z^2}{z+9}=(z+9)\frac{-13z-36}{z+9}$$
$$z^2=-3z-36$$
$$z^2+13z+36=0$$

The resulting quadratic equation can be solved by factoring.

$$z^2+13z+36=0$$
$$(z+9)(z+4)=0$$
$$z=-4,\ \cancel{-9}$$

Since it was determined earlier that –9 must be excluded as a solution to this rational equation, the solution is –4.

1.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A proportion is a statement that two ratios are being multiplied.

2. Cross canceling is used to determine if a proportion is true.

3. In order to solve the proportion $\frac{16}{36.8}=\frac{x}{27.6}$ we construct the equation $36.8x=441.6$.

4. When using proportions to solve a word problem, there is only one correct way to set up the proportion.

5. The proportions $\frac{36\text{ tickets}}{\$540}=\frac{x\text{ tickets}}{\$75}$ and $\frac{x\text{ tickets}}{36\text{ tickets}}=\frac{\$75}{\$540}$ will yield the same answer.

Practice

Determine whether each proportion is true or false.

6. $\frac{3}{6}=\frac{4}{8}$

7. $\frac{1}{3}=\frac{33}{100}$

Solve each proportion.

8. $\frac{5}{4}=\frac{x}{8}$

9. $\frac{3.5}{2.6}=\frac{10.5}{B}$

Applications

Solve.

10. ***Concrete:*** The quality of concrete is based on the ratio of bags of cement to cubic yards of gravel. One batch of concrete consists of 27 bags of cement mixed into 9 cubic yards of gravel, while a second has 15 bags of cement mixed with 5 cubic yards of gravel. Determine whether the ratio of cement to gravel is the same for both batches.

11. ***Grading:*** An English teacher must read and grade 27 essays. If the teacher takes 20 minutes to read and grade 3 essays, how much time will he need to grade all 27 essays?

Writing & Thinking

12. In your own words, clarify how you can know that a proportion is set up correctly or not.

To Change a Percent to a Fraction or a Mixed Number

1. Write the percent as a fraction with ______________________________

2. Reduce the ____________________

PROCEDURE

Looking Ahead

Your review of percents will be helpful in the following example, which involves calculating the tip at a restaurant.

Example Preview

Marvin decides to leave a 15% tip after eating dinner at Fresh Catchery. If the bill is \$27.32, how much should he pay?

Solution

First, find the tip amount by finding 15% of \$27.32. Converting 15% to a decimal gives 0.15. The tip amount is $(0.15)(\$27.32) = \$4.098$, which rounds to \$4.10.

The total cost is thus \$27.32 + \$4.10 = \$31.42. Marvin should pay \$31.42.

1.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a decimal number is less than 1, then the equivalent percent will be less than 100%.

2. It is not possible to have a percent greater than 100%.

3. A decimal number that is between 0.01 and 0.10 is between 10% and 100%.

4. To change from a percent to a decimal, simply omit the percent sign.

5. Fractions that have denominators other than 100 cannot be changed to a percent.

6. The fraction $\frac{1}{5}$ is equivalent to $\frac{1}{5}\%$.

Practice

Change each fraction to a percent.

7. $\frac{20}{100}$

8. $\frac{125}{100}$

Change each decimal number to a percent.

9. 0.02

10. 2.3

Change each percent to a decimal number.

11. 7%

12. 179%

Change each fraction or mixed number to a percent. If necessary, round to the nearest tenth of a percent.

13. $\frac{3}{4}$

14. $5\frac{3}{10}$

Change each percent to a fraction or mixed number and reduce, if possible.

15. 120%

16. 12.5%

Applications

Solve.

17. ***Interest:*** A savings account is offering an interest rate of 0.04 for the first year after opening the account. Change 0.04 to a percent.

18. ***Sales Tax:*** Suppose that sales tax is figured at 7.25%. Change 7.25% to a decimal.

19. ***Exam Grades:*** Out of a possible total of 240 points on an exam, David received 204 points. What percent of the exam did David get correct?

20. ***College Degrees:*** To receive a Bachelor of Science (BS) degree at Bluefield State College, the student must complete a total of 128 credit hours, of which 41 of these credits must be general education Core Skills courses. What percent of the total curriculum is dedicated to general education courses? [1]

Writing & Thinking

21. Describe the relationship between percent and the number 100.

22. Describe a situation where more than 100% is possible. Describe a situation where it is impossible to have more than 100%.

1 Source: 2010–2011 Bluefield, WV State College Catalogue, p.76

23. Justify why mixed numbers are a larger percentage than proper fractions alone. (Consider the value of 100%.)

24. Describe the process to change a percent to a fraction or mixed number.

1.R.6 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. On a number line, smaller numbers are always to the left of larger numbers.

2. The absolute value of a negative number is a positive number.

3. All whole numbers are also integers.

4. Zero is a positive number.

Practice

Graph each set of real numbers on a real number line.

5. $\{-3, -2, 0, 1\}$

6. $\left\{-2, -1, -\frac{1}{3}, 2\right\}$

List the numbers in the set $A = \left\{-7, -\sqrt{6}, -2, -\frac{5}{3}, -1.4, 0, \frac{3}{5}, \sqrt{5}, \sqrt{11}, 4, 5.9, 8\right\}$ that are described in each exercise.

7. Whole numbers

8. Rational numbers

Determine whether each statement is true or false. If a statement is false, rewrite it in a form that is a true statement. (There may be more than one way to correct a statement.)

9. $0 = -0$

10. $|-8| \geq 4$

Applications

Solve. Represent each quantity with a signed integer.

11. ***Oceans:*** The Alvin is a manned deep-ocean research submersible that has explored the wreck of the Titanic. The operating depth of the Alvin is 4500 meters below sea level.

12. ***Oceans:*** The Mariana trench is the deepest known location on the Earth's ocean floor. The deepest known part of the Mariana Trench is approximately 11 kilometers below sea level.

Writing & Thinking

13. Explain, in your own words, how an expression such as $-y$ might represent a positive number.

14. Compare and contrast absolute value with opposites.

Looking Ahead

Being able to accurately add both negative and positive real numbers is a fundamental skill needed to solve linear equations in one variable. We want to be able to find the value of the variable that makes the proposed equation a true statement. You may need to add both negative and positive numbers to both sides of the equation in order to balance it.

Example Preview

Solve the following linear equation.

$$-13 = -3u - 16$$

Solution

$$\begin{aligned} -13 &= -3u - 16 \\ -13 + 16 &= -3u - 16 + 16 \\ 3 &= -3u \\ \frac{3}{-3} &= \frac{-3u}{-3} \\ -1 &= u \end{aligned}$$

1.R.7 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The sum of a positive number and a negative number is always positive.

2. When adding two numbers with unlike signs, the result uses the sign of the number with the larger absolute value.

3. The sum of two positive numbers can equal zero.

Practice

Add. Reduce any fractions to lowest terms.

4. $8+(-3)$

5. $2+(-8)$

6. $-\frac{1}{6}+\frac{7}{15}$

7. $3.2+(-1.2)+(-2.5)$

Add. Be sure to find the absolute values first.

8. $13+|-5|$

Applications

Solve.

9. ***Profit:*** For 2017, a business reports a profit of $45,000 during the first quarter, a loss of $8000 during the second quarter, a loss of $2000 during the third quarter, and a profit of $15,000 during the fourth quarter.

a. Write an addition expression to represent the total profit made by the company in 2017. Do not simplify.

b. Simplify the expression from Part **a.**

10. ***Oceans:*** A submarine dives to a depth of 250 feet below the surface. It rises 75 feet before diving an additional 100 feet. What is the final depth of the submarine?

Writing & Thinking

11. Describe, in your own words, how the sum of the absolute values of two numbers might be 0. (Is this even possible?)

12. Describe in your own words the conditions under which the sum of two integers will be 0.

Looking Ahead

Reviewing the main ideas related to subtraction of real numbers will allow you to understand and simplify problems involving the addition and subtraction of complex numbers.

Example Preview

Simplify the following expression.

$$(12-18i)-(5-12i)$$

Solution

$$\begin{aligned}(12-18i)-(5-12i) &= (12-5)+(-18-(-12))i \\ &= 7-6i\end{aligned}$$

1.R.8 Exercises

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The sum of a number and its additive inverse is the number itself.

2. The additive inverse of negative seven is seven.

3. We can think of addition of numbers as accumulating numbers.

4. The expression "15 – 7" can be thought of as "fifteen plus negative seven."

Practice

Find the additive inverse (opposite) of each real number.

5. 11

6. –3.4

Subtract. Reduce fractions to lowest terms.

7. $-8-(-11)$

8. $\frac{7}{15}-\frac{2}{15}$

Perform the indicated operation to find the net change in value.

9. $-6+(-4)-5$

10. $-11.3+5.3-7.9$

Applications

Solve.

11. ***Temperature:*** At 2 p.m. the temperature was 76 °F. At 8 p.m. the temperature was 58 °F. What was the change in temperature?

12. ***Real Estate:*** A couple sold their house for $135,000. They paid the realtor $8100, and other expenses of the sale came to $800. If they owed the bank $87,000 for the mortgage, what were their net proceeds from the sale?

Writing & Thinking

13. Explain, in your own words, how to find the difference between a positive and a negative number.

14. What is the additive inverse of 0? Why?

1.R.9 Exercises

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a negative number is divided by a positive number, the result will be a negative number.

2. The product of zero and a number is zero.

3. If two numbers have the same sign, both the product and the quotient of the two numbers will be negative.

4. The mean of a set of numbers is always positive.

Practice

Multiply. Reduce fractions to lowest terms.

5. $12 \cdot 4$

6. $(-7)(-16)(0)$

Divide. Reduce fractions to lowest terms. Round answers with decimals to the nearest tenth.

7. $\frac{-20}{-10}$

8. $\frac{-5.6}{7}$

Applications

Solve.

9. ***Mean:*** Find the mean of the following set of integers: –10, 15, 16, –17, –34, and –42.

10. ***Animals:*** According to the US Fish and Wildlife Service, migratory birds are imported at a value of about \$19 each. Suppose that about 800,000 live birds are imported each year. What is the total value of these imported birds?

Writing & Thinking

11. If you multiply an odd number of negative numbers together, do you think that the product will be positive or negative? Explain your reasoning.

12. Explain the conditions under which the quotient of two numbers is 0.

2.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. **a.** Every square is a rectangle.

b. Every rectangle is a square.

2. **a.** Every parallelogram is a rectangle.

b. Every rectangle is a parallelogram.

3. A trapezoid has only one pair of parallel lines.

4. The $(b+c)$ in the trapezoid area formula represents the sum of the lengths of the base and the corners.

5. The height of a triangle is the distance between the base and the vertex opposite the base.

6. Every radius on a circle has the same length.

7. The length of the diameter of a circle is half of the length of the radius.

8. To find the volume of a can of corn, the formula $V = \pi r^2 h$ would be used.

9. The area of the paper label on a can of peaches is an example of surface area.

10. To find the volume of a rectangular solid, the areas of each surface are added together.

Match each formula for perimeter to its corresponding geometric figure.

11. **a.** Square **A.** $P = 2l + 2w$

b. Parallelogram **B.** $P = 4s$

c. Rectangle **C.** $P = 2b + 2a$

d. Trapezoid **D.** $P = a + b + c$

e. Triangle **E.** $P = a + b + c + d$

Match each formula for volume to its corresponding geometric figure.

12. **a.** Rectangular solid **A.** $V = \frac{4}{3}\pi r^3$

b. Rectangular pyramid **B.** $V = \frac{1}{3}\pi r^2 h$

c. Right circular cylinder **C.** $V = lwh$

d. Right circular cone **D.** $V = \pi r^2 h$

e. Sphere **E.** $V = \frac{1}{3}lwh$

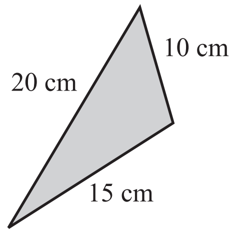
Practice

Calculate the perimeter of each figure.

13. A parallelogram with sides of length 15 cm and 7 cm.

14. A square with sides of length $4\frac{1}{2}$ km.

15.



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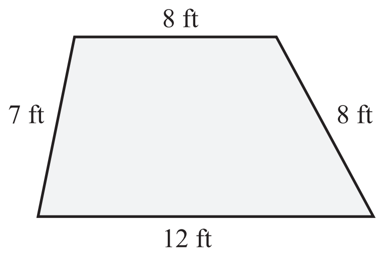

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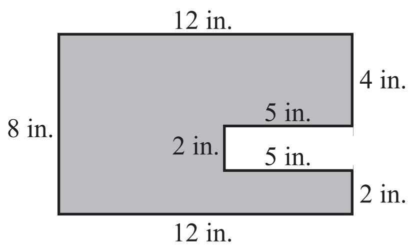
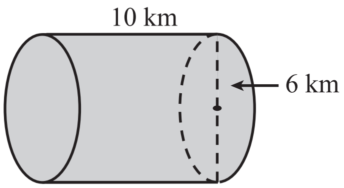


Calculate the volume of the solid. Use π ≈ 3.14

18.



19. A rectangular solid with length 5 in., width 2 in., and height 7 in.

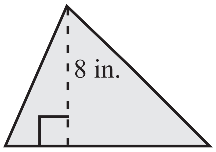
20. A right circular cone 3 mm high with a 2 mm radius.

Calculate the area of each figure.

21. A square with sides of length 9 ft.

22. A parallelogram with height 2.3 ft and base 11.9 ft.

23.



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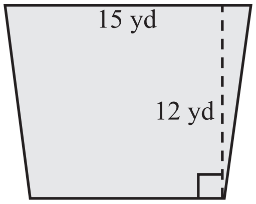
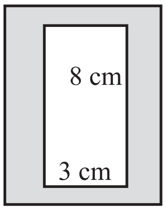


25.



Calculate **a.** the perimeter and **b.** the area of each figure. Use π ≈ 3.14.

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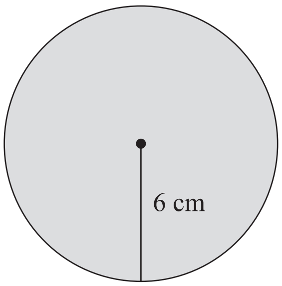

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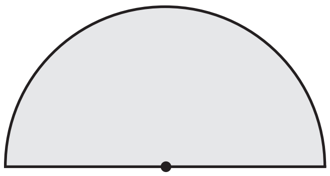

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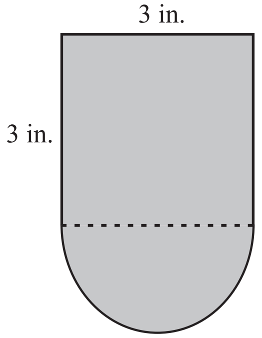
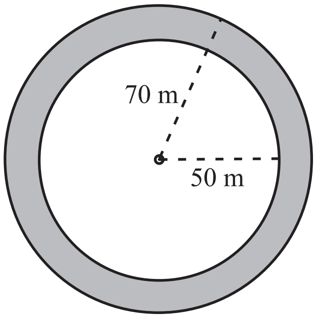


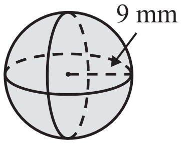
Calculate the area of the shaded portion of the figure. Use $\pi \approx 3.14$.

29.



Calculate the surface area of each solid. Use $\pi \approx 3.14$.

30.



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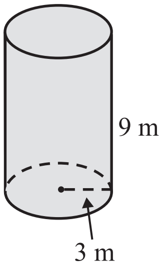


Applications

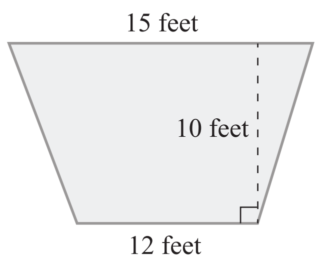
Solve. Use $\pi \approx 3.14$.

32. ***Construction:*** The Pentagon near Washington, D.C., is a five-sided building where each outside wall is 921 feet.

a. What is the perimeter of the building?

b. If it takes a person 0.00341 minutes to walk 1 foot, how long will it take the person to walk completely around the building? Round your answer to the nearest tenth of a minute.

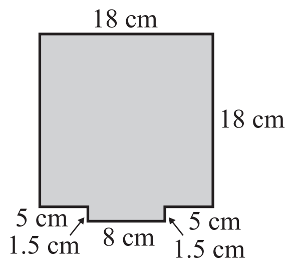
33. ***Construction:*** The main stage at a theater is in the shape of a trapezoid. The owner of the theater is planning to install a new specially designed flooring system on the stage. The stage is 12 feet wide in the front and 15 feet wide in the back. The stage is 10 feet deep. How much flooring will the manager need?



34. ***Technology:*** A square electronics circuit board is 18 centimeters on each side. On the center of one of the edges is an 8 by 1.5 centimeter rectangular lip for plugging in.

a. What is the total perimeter of the circuit board, including the lip?

b. What is the area of the circuit board?

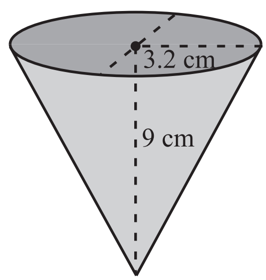


35. ***Sales:*** Papa Luigi's sells a 9-inch diameter pizza for $8.

a. Determine the area of the pizza to the nearest tenth.

b. Determine the price per square inch to the nearest cent per square inch.

36. ***Geometry:*** Disposable paper drinking cups like those used at water coolers are often cone-shaped. Find the volume of such a cup that is 9 cm high with a 3.2 cm radius. Express the answer to the nearest milliliter.



Writing & Thinking

37. List the steps and formulas you would use to find the volume of an ice cream cone (assuming the ice cream itself forms a perfect half sphere).

38. Explain what the value of $(b+c)$ represents in the formula for the area of a trapezoid.

39. Explain why $2\pi r$ is equivalent to πd.

40. Propose a method for calculating the area of a semicircle and justify your method.

Looking Ahead

Your review of the Pythagorean Theorem will be helpful in determining the equation for a circle given its center and a point on the circle. Recall that all the points on a circle lie at a specific distance from its center. This distance is the radius r of the circle.

Example Preview

Find the standard form of the equation for the circle with the following properties.

Center (11,2), passes through (2,–10)

Solution

The standard form of the equation for a circle is

$$(x - h)^2 = (y - k)^2 = r^2$$

where the center is (h,k) and the radius has length r. Since every point on a circle is the same distance r from the circle's center, we can conclude that the radius r equals the distance from the center, (11,2), to the point given in the problem, (2,–10). We use the distance formula to find this value.

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2-(11))^2 + (-10-(2))^2} \\ &= \sqrt{81+144} \\ &= 15 \end{aligned}$$

Now we substitute this and the values given in the problem and simplify to obtain the following equation.

$$\begin{aligned} (x-11)^2 + (y-2)^2 &= (15)^2 \\ (x-11)^2 + (y-2)^2 &= 225 \end{aligned}$$

2.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. 49 is a perfect square.

2. In the expression $\sqrt{81}$, the number 9 is the radicand.

3. The Pythagorean Theorem can be used to find the length of the longest side of a right triangle if the lengths of the two legs are known.

4. The Pythagorean Theorem works for any type of triangle.

Practice

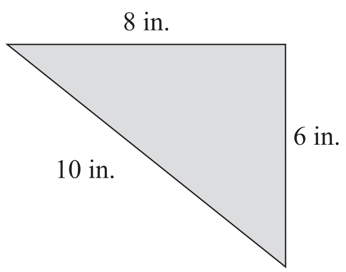
Evaluate each expression.

5. $\sqrt{36}$

6. $\sqrt{225}$

Use the Pythagorean Theorem to determine whether or not each triangle is a right triangle.

7.



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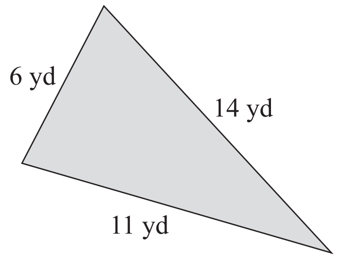
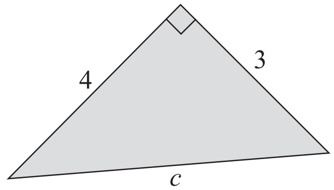


Find the hypotenuse for each right triangle accurate to the nearest hundredth.

9.

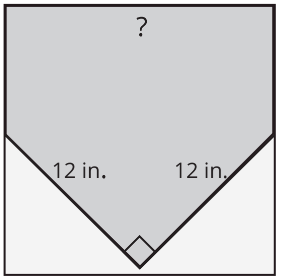


Applications

Solve.

10. ***Safety:*** The base of a fire engine ladder is 30 feet from a building and reaches to a third floor window 50 feet above ground level. Find the length of the ladder to the nearest hundredth of a foot.

11. ***Baseball:*** The shape of home plate in the game of baseball can be created by cutting off two triangular pieces at the corners of a square, as shown in the figure. If each of the triangular pieces has a hypotenuse of 12 inches and legs of equal length, what is the length of one side of the original square, to the nearest tenth of an inch?



Writing & Thinking

12. Explain the connection between a perfect square and its square root. Give an example.

Looking Ahead

Now that you have reviewed and mastered evaluating radicals, you can apply this skill to problems involving more complicated expressions such as the one in the following example.

Example Preview

Solve the following equation.

$$\sqrt{x^2 - 28} = 6$$

Solution

$$\sqrt{x^2 - 28} = 6$$

Begin by squaring both sides of the equation to eliminate the square root.

$$x^2 - 28 = 36$$

Now, since there is only an x^2 variable, we can "extract roots" by adding 28 to both sides and then take the square root of both sides.

$$x^2 = 64$$
$$x = \pm\sqrt{64}$$
$$x = \pm 8$$

Since both of these do check in the original equation, the correct answers are $x = 8$ and $x = -8$.

2.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a number is squared and the principal square root of the result is found, that square root is always equal to the original number.

2. There is no real number that can be a square root of a negative number.

3. The index is the number underneath the radical sign.

4. The cube root of –27 is a real number.

Practice

Simplify the following square roots and cube roots.

5. $\sqrt{49}$

6. $\sqrt{289}$

7. $\sqrt[3]{1000}$

8. $\sqrt[3]{\frac{27}{64}}$

9. $\sqrt{0.04}$

Applications

Solve.

10. ***Area:*** The area of a square tile is 16 square inches.

a. How long are the sides of the square tile?

b. How many tiles would be needed for a four-foot-long and four-inch-high backsplash in a newly designed bathroom?

11. ***Volume:*** The volume of a child's building block is 64 cubic centimeters.

a. Assuming the building block is a perfect cube, find the length of each side of the block.

b. If a child stacks 5 blocks directly on top of each other, find the height of the structure that is created.

Writing & Thinking

12. Discuss, in your own words, why the square root of a negative number is not a real number.

13. Discuss, in your own words, why the cube root of a negative number is a negative number.

Looking Ahead

In the following example, you will find the square root of an expression that contains variables raised to exponents that are not all even.

Example Preview

Simplify the following radical expression.

$$\sqrt{14y^{14}z}$$

Solution

This expression can be simplified as follows.

$$\begin{aligned}\sqrt{14y^{14}z} &= \sqrt{14}\cdot\sqrt{y^{14}}\cdot\sqrt{z}\\ &= \sqrt{14}\cdot\sqrt{z}\cdot\left|y^7\right|\\ &= \sqrt{14}\cdot\left|y^7\right|\\ &= \left|y^7\right|\sqrt{14}\end{aligned}$$

2.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Any variable term with an exponent of 5 has a perfect cube factor within that variable term.

2. The simplest form of a radical expression can be found by using prime factorization.

3. If x is a real number, then $\sqrt{x^2} = x$.

4. The term $7b\sqrt[3]{6c^2}$ is in simplified form.

Practice

Simplify each of the following radical expressions. Assume that all variables represent positive real numbers.

5. $\sqrt{162}$

6. $\sqrt{\frac{32}{49}}$

7. $\sqrt{24x^{11}y^2}$

8. $\sqrt[3]{56}$

9. $\sqrt[3]{-8x^8}$

Applications

Use the following two formulas associated with electricity

$I = \sqrt{\dfrac{P}{R}}$ — *P* = power (in watts)

I = current (in amperes)

E = voltage (in volts)

$E = \sqrt{PR}$ — *R* = resistance (in ohms, Ω)

10. ***Electricity:*** What is the current in amperes of a light bulb that produces 150 watts of power and has a 25 Ω resistance?

11. ***Electricity:*** If a light bulb has a resistance of 30 Ω and produces 90 watts of power, what is its current in amperes?

Writing & Thinking

12. Under what conditions is the expression $\sqrt{a}$ not a real number?

13. Explain why the expression $\sqrt[3]{y}$ is a real number regardless of whether $y > 0$, $y < 0$, or $y = 0$.

Looking Ahead

Now that you have reviewed the main ideas around the Cartesian coordinate system, you will apply these skills to more advanced problems. For example, the next exercise involves identifying key features of a circle and then graphing the circle on the Cartesian plane.

Example Preview

Consider the following equation of a circle. Find the center (h,k), the radius r, and graph the circle.

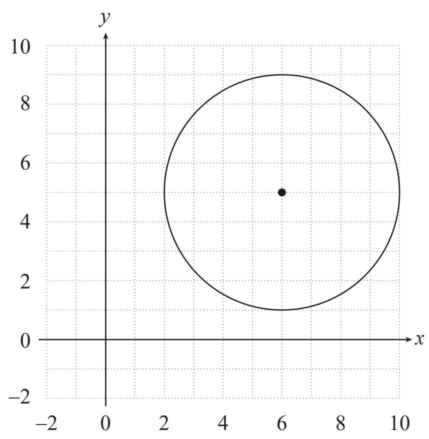
$$(x-6)^2+(y-5)^2=16$$

Solution

The standard form of the equation of a circle is $(x-h)^2+(y-k)^2=r^2$.

The center is $(h,k)=(6,5)$ and the radius is $r=4$ because $r^2=16$.

Plot the center of the circle at $(6,5)$ and size the circle such that the points on the circle are 4 units away from the center.



2.R.5 Exercises

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The graph of every ordered pair that has a positive x-coordinate and a negative y-coordinate can be found in Quadrant IV.

2. To find the y-value that corresponds with $x = 2$, substitute 2 for x into the given equation and solve for y.

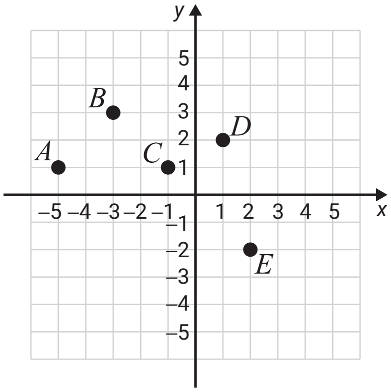
3. If $(-7, 3)$ is a solution of $y = 3x + 24$, then $(-7, 3)$ satisfies $y = 3x + 24$.

4. If point $A = (0, 4)$, then point A lies on the x-axis.

Practice

List the set of ordered pairs corresponding to the points on the graph.

5.



Plot each set of ordered pairs and label the points.

6. $\{A(4, -1), B(3, 2), C(0, 5), D(1, -1), E(1, 4)\}$

Determine the missing coordinate in each of the ordered pairs so that the point will satisfy the equation given.

7. $x - 2y = 2$

a. $(0, \_\_)$

b. $(4, \_\_)$

c. $(\_\_, 0)$

d. $(\_\_, 3)$

Complete the tables so that each ordered pair will satisfy the given equation. Plot the resulting sets of ordered pairs.

8. $y = 2x - 3$

x	*y*
0	
	–1
–2	
	3

Determine which, if any, of the ordered pairs satisfy the given equation.

9. $2x - 3y = 7$

a. $(1, 3)$

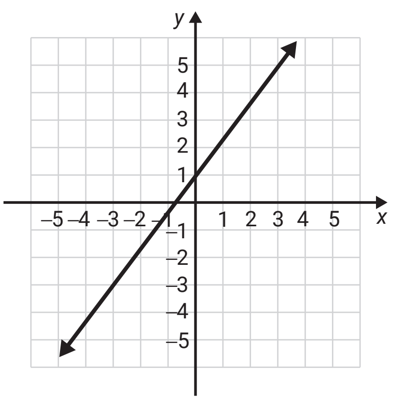
b. $\left(\frac{1}{2}, -2\right)$

c. $\left(\frac{7}{2}, 0\right)$

d. $(2, 1)$

The graph of a line is shown. List any three points on the line. (There is more than one correct answer.)

10.



Applications

Solve.

11. ***Exchange Rates:*** At one point in 2017, the exchange rate from US dollars to Euros was $E = 0.85D$ where E is Euros and D is dollars.

a. Make a table of ordered pairs for the values of D and E if D has the values \$100, \$200, \$300, \$400, and \$500.

b. Plot the points corresponding to the ordered pairs.

12. ***Temperature:*** Given the equation $F = \frac{9}{5}C + 32$ where C is temperature in degrees Celsius and F is the corresponding temperature in degrees Fahrenheit:

a. Make a table of ordered pairs for the values of C and F if C has the values −20°, −10°, −5°, 0°, 5°, 10°, and 15°.

b. Plot the points corresponding to the ordered pairs.

2.R.6 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If an equation of the form $ax + b = c$ uses decimal or fractional coefficients, the addition and multiplication principles of equality cannot be used.

2. The first step in solving $2x + 3 = 9$ is to add 3 to both sides.

3. To solve an equation that has been simplified to $4x = 12$, you need to multiply both sides by $\frac{1}{4}$, or divide both sides by 4.

4. When solving a linear equation with decimal coefficients, one approach is to multiply both sides in such a way to give integer coefficients before solving.

Practice

Solve each equation.

5. $3x + 11 = 2$

6. $-5x + 2.9 = 3.5$

7. $\frac{2}{5} - \frac{1}{2}x = \frac{7}{4}$

8. $\frac{y}{3} - \frac{2}{3} = 7$

Applications

Solve.

9. ***Music:*** The tickets for a concert featuring the new hit band, Flying Sailor, sold out in 2.5 hours. If there were 35,000 tickets sold, solve the equation $35,000 - 2.5x = 0$ to find the number of tickets sold per hour.

10. ***Movies:*** All snacks (candy, popcorn, and soda) cost \$3.50 each at the local movie theater. Admission tickets cost \$7.50 each. After a long week, Carlos treats himself to a night at the movies. His movie night budget is \$25 and he spends all his movie money. Solve the equation $\$3.50x + \$7.50 = \$25.00$ to determine how many snacks Carlos can buy.

Writing & Thinking

11. Find the error(s) made in solving each equation and give the correct solution.

a.
$$\begin{aligned} \frac{1}{3}x+4&=9 \\ 3\cdot\frac{1}{3}x+4&=3\cdot 9 \\ x+4&=27 \\ x+4-4&=27-4 \\ x&=23 \end{aligned}$$

b.
$$\begin{aligned} 5x+3&=11 \\ (5x-3)+(3-3)&=11-3 \\ 2x+0&=8 \\ \frac{2x}{2}&=\frac{8}{2} \\ x&=4 \end{aligned}$$

Looking Ahead

Now that you have reviewed solving linear equation of the form $ax+b=cx+d$, you will be able to apply your skills to solving multi-step problems involving multiple equations like the following example.

Example Preview

Express the following equations in slope-intercept form and determine if the two lines are perpendicular.

$$8-(3y+4x)=6(x-y) \quad \text{and} \quad 3y+2=8+10x$$

Solution

$$\begin{aligned} 8-(3y+4x)&=6(x-y) \\ 8-3y-4x&=6x-6y \\ -3y+6y&=6x+4x-8 \\ 3y&=10x-8 \\ y&=\frac{10}{3}x-\frac{8}{3} \end{aligned}$$

$$\begin{aligned} 3y+2&=8+10x \\ 3y&=10x+8-2 \\ 3y&=10x+6 \\ y&=\frac{10}{3}x+2 \end{aligned}$$

These two lines have slopes of $\frac{10}{3}$ and $\frac{10}{3}$, respectively. Since both of these lines have the same slope, these two lines are parallel and not perpendicular.

2.R.7 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Every linear equation has exactly one solution.

2. If a linear equation simplifies to a statement that is always true, then the original equation is called an identity.

3. If an equation has no solution, it is called an identity.

4. The most general form of a linear equation is $ax + b = cx + d$.

Practice

Solve each equation.

5. $3x + 2 = x - 8$

6. $2(z + 1) = 3z + 3$

7. $x - 0.1x + 0.8 = 0.2x + 0.1$

8. $0.6x - 22.9 = 1.5x - 18.4$

Determine whether each equation is a conditional equation, an identity, or a contradiction.

9. $-2x + 13 = -2(x - 7)$

10. $3x + 9 = -3(x - 3) + 6x$

Applications

Solve.

11. ***Event Planning:*** Caitlyn and Steve are planning their wedding reception and must decide between two catering halls. The first site, A Wedding Space, rents for \$800 for one day and charges \$50 per person for dinner. The second venue, A Wedding Place, costs \$1000 to rent for one day and charges \$40 per person for the same dinner. Solve the equation $800 + 50x = 1000 + 40x$ to determine how many guests they can invite so that the cost they pay will be the same at both wedding catering halls.

12. ***Personal Finance:*** The value of a new car depreciates at a rate of about \$250 per month. Suppose a car originally costs \$30,000. The car was bought with a \$1000 down payment and a loan with 0% financing for 60 months with payments of \$200 a month. Solve the equation $30{,}000 - 250t = 29{,}000 - 200t$ to determine how many months it will take for the value of the vehicle to equal the amount owed on the loan?

Writing & Thinking

13. Answer each question.

a. Simplify the expression $3(x+5)+2(x-7)$.

b. Solve the equation $3(x+5)+2(x-7)=31$.

c. How are the methods you used to answer questions **a.** and **b.** similar? How are they different?

2.R.8 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If only one endpoint is included in an interval, it is called a half-open interval.

2. When both sides of a linear inequality are multiplied by a negative constant, the sense of the inequality should stay the same.

3. To check the solution set of a linear inequality, every solution in the solution set must be checked in the original inequality.

4. The infinity symbol ∞ does not represent a specific number.

Practice

Graph each interval on a real number line and tell what type of interval it is.

5. $x \leq -3$

6. $-1.5 \leq x < 3.2$

Solve each inequality and graph the solution set. Write each solution set using interval notation.

7. $x + 1 > 5$

8. $-2x \geq 6$

9. $4x - 7 \geq 9$

10. $5x + 6 \geq 2x - 2$

Applications

Solve.

11. ***Test Scores:*** A statistics student has grades of 82, 95, 93, and 78 on four hour-long exams. He must average 90 or higher to receive an A for the course. What scores can he receive on the final exam and earn an A if:

a. The final is equivalent to a single hour-long exam (100 points maximum)?

b. The final is equivalent to two hourly exams (200 points maximum)?

12. ***Postage:*** Allison is going to the post office to buy 34¢ stamps and 3¢ adjustment stamps. Since the current postage rate is 49¢, she will need 5 times as many 3¢ adjustment stamps as 34¢ stamps. If she has $12.25 to spend, what is the largest number of 34¢ stamps she can buy?

Writing & Thinking

13. **a.** Write a list of three situations where inequalities might be used in daily life.

b. Illustrate theses situations with algebraic inequalities and appropriate numbers.

2.R.9 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. All radical equations will have two solutions.

2. If no true statements result when all possible solutions are checked, then there is no solution.

3. When solving equations with radicals, you should only have to raise both sides of the equation to a power one time.

4. A radical expression set equal to a negative value, such as $\sqrt{x+2} = -4$, has no real solution.

Practice

Solve the following equations. Be sure to check your answers in the original equation.

5. $\sqrt{8x+1} = 5$

6. $5 + \sqrt{x+5} - 2x = 0$

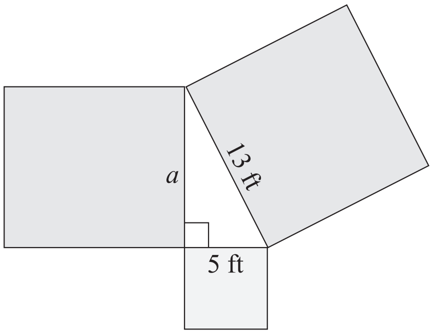
7. $\sqrt{2x-5} = \sqrt{3x-9}$

8. $\sqrt{x} + \sqrt{x-3} = 3$

Applications

Solve.

9. ***Landscaping:*** A landscaper is designing a pond in the shape of a right triangle that has a square flower patch along each edge. She knows two of the flower patches will have side lengths of 5 ft and 13 ft and that the remaining flower patch must have a side length a which satisfies the equation $13=\sqrt{a^2+5^2}$. What is the side length of the remaining flower patch?



3.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If the domain of a linear function is not explicitly stated, the implied domain is the set of all values of x that produce real values for y.

2. A relation is a function in which each domain element has exactly one corresponding range element.

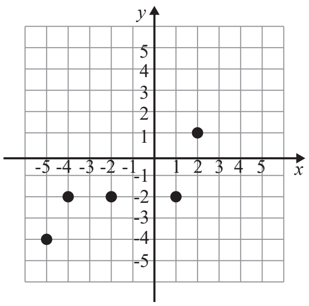
3. In a function, the range elements can have more than one corresponding domain element.

4. If $s = \{(1, -6), (3, 5), (4, 0), (1, 2)\}$, then s is a function.

Practice

List the sets of ordered pairs that correspond to the points. State the domain and range and indicate if the relation is a function.

5.

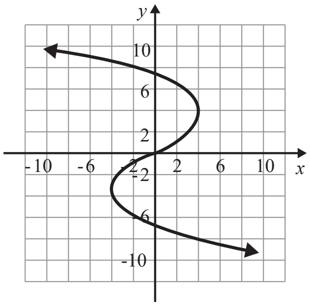


Graph the relation. State the domain and range and indicate which of the relation is a function.

6. $h = \{(1, -5), (2, -3), (-1, -3), (0, 2), (4, 3)\}$

Use the vertical line test to determine whether the graph represents a function. State the domain and range using interval notation.

7.



State the domain of the function.

8. $h(x)=\dfrac{7}{3x}$

Find the values of the function as indicated.

9. $F(x)=6x^2-10$

a. $F(0)$

b. $F(-4)$

c. $F(4)$

Applications

Solve.

10. ***Nursing:*** A nurse hangs a 1000-milliliter IV bag which is set to drip at 120 milliliters per hour. Create a model of this situation to represent the amount of IV solution left in the bag after x hours.

a. The y-intercept is the amount of IV solution in the bag initially (time = 0). What is the y-intercept?

b. The slope is equal to the rate that the IV solution is dispensed per hour. What is the slope? (**Hint:** Consider whether the amount of IV solution in the bag is increasing or decreasing and how this would affect the slope.)

c. Write an equation in slope-intercept form to model this situation.

d. Write the equation from Part **c.** using function notation.

e. State the domain and range of the function.

f. State any additional restrictions that should be made on the domain for it to make sense in the context of this problem.

g. How much IV solution is left in the bag after 5 hours?

3.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The order in which the values are given is particularly important when working with subtraction and division problems.

2. "More than" and "increased by" are key phrases specifying the operation of subtraction.

3. Division is indicated by the phrase "five less than a number."

4. Key phrases for parentheses can be used to limit ambiguity in English phrases.

Practice

Write the algebraic expressions described by the English phrases. Choose your own variable.

5. six added to a number

6. twenty decreased by the product of four and a number

7. eighteen less than the quotient of a number and two

Translate each pair of English phrases into algebraic expressions. Notice the differences between the algebraic expressions and the corresponding English phrases.

8. **a.** six less than a number

b. six less a number

9. **a.** six less than four times a number

b. six less four times a number

Write the algebraic expression described by the English phrase using the given variables.

10. the cost of purchasing a fishing rod and reel if the rod costs x dollars and the reel costs $8 more than twice the cost of the rod

Translate each algebraic expression into an equivalent English phrase. (There may be more than one correct translation.)

11. $-9x$

12. $\frac{9}{x+3}$

Writing & Thinking

13. Explain why translating addition and multiplication problems from English into algebra may be easier than changing subtraction or division problems. (Consider the properties previously studied.)

14. Explain the difference between $5(n+3)$ and $5n+3$ when converting from algebra to English.

3.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If an odd integer is divided by 2, the remainder will be 1.

2. To find 3 consecutive odd integers, you could use n, $n + 1$, and $n + 3$.

3. Odd integers are integers that are divisible by 1.

4. Even integers are consecutive if each is 2 more than the previous even integer.

Practice

Read each problem carefully, translate the various phrases into algebraic expressions, set up an equation, and solve the equation.

5. Five less than a number is equal to 13 decreased by the number. Find the number.

6. Twice a number increased by 3 times the number is equal to 4 times the sum of the number and 3. Find the number.

7. Find three consecutive integers whose sum is 93.

8. Find three consecutive odd integers such that the sum of twice the first and three times the second is 7 more than twice the third.

Applications

Solve.

9. ***Expenses:*** A collect call from a landline in Ohio to another landline in Ohio has a connection fee of \$2.75 and a charge of \$0.36 per minute. Mr. Anderson made a collect call which cost the receiver of the call \$9.95. This situation can be modeled by $\$9.95 = \$2.75 + \$0.36m$.

a. The unknown value is represented by the variable m in the equation. What is the unknown value in this situation?

b. Solve the equation for the variable.

c. What does the answer to Part **b.** mean? Write a complete sentence.

10. ***Event Planning:*** Robin is in charge of purchasing desserts for a dinner party that her nonprofit organization is throwing. She decides to buy a cake and several specialty cupcakes from Barbara's Bombtastic Bakery. She needs to buy one 8-inch round cake which costs \$19.50. She has \$45 to spend and will spend the leftover amount on cupcakes, which are \$8.50 for a box of 4. How many boxes of cupcakes can Robin purchase?

a. What is the unknown value in this problem? Let the variable c represent this unknown value.

b. Write an equation to represent this situation.

c. Solve the equation for the variable.

d. What does the answer to Part **c.** mean? Write a complete sentence.

Writing & Thinking

11. **a.** How would you represent four consecutive odd integers?

b. How would you represent four consecutive even integers?

c. Are these representations the same? Explain.

Looking Ahead

The skills you have reviewed in this section are the foundation to the process for finding x-intercepts of a quadratic function, as the following example shows.

Example Preview

Find the x-intercepts, if any, of the graph of the following function.

$$f(x) = -3x^2 - 6x$$

Solution

Since the x-intercepts are the points on the x-axis where $f(x) = 0$, we need to solve the following equation.

$$-3x^2 - 6x = 0$$

At this point, we need to recognize that both of the terms on the left side of this quadratic equation have a common factor of $-3x$. So, this quadratic equation can be solved in the following manner.

$$\begin{aligned} -3x^2 - 6x &= 0 \\ -3x(x+2) &= 0 \\ -3x = 0,\ x+2 &= 0 \\ x = 0, \quad x &= -2 \end{aligned}$$

Therefore, the parabola, which is the graph represented by this quadratic function, crosses the x-axis at $(-2,0)$ and $(0,0)$.

3.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When finding the GCF of a polynomial, you need to consider only the coefficients.

2. An expression is factored completely if none of its factors can be factored.

3. One way to find the GCF of a set of numbers is to use the prime factorization of each number.

4. Binomials cannot be factored out of algebraic expressions.

Practice

Find the GCF for each set of terms.

5. $\{25, 30, 75\}$

6. $\{8a^3, 16a^4, 20a^2\}$

Factor each polynomial by finding the GCF (or $-1 \cdot$ GCF).

7. $14x + 21$

8. $10x^2y - 25xy$

Factor each of the polynomials by grouping. If a polynomial cannot be factored, write "not factorable."

9. $3x + 3y + ax + ay$

10. $10xy - 2y^2 + 7yz - 35xz$

Applications

Solve.

11. ***Projectile Motion:*** A circus performer is shot vertically into the air with an initial velocity of 48 feet per second. The height of the performer above the ground in feet can be described by the polynomial $48x - 16x^2$ after x seconds.

a. Find the height of the circus performer after 2 seconds.

b. Factor the polynomial $48x - 16x^2$.

c. Use the factored form of the polynomial from Part **b.** to find the height of the circus performer after 2 seconds.

d. Are the answers from Parts **a.** and **c.** the same? Explain why or why not.

Writing & Thinking

12. Explain why the GCF of $-3x^2 + 3$ is 3 and not -3.

3.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. In a trinomial such as $x^2 - 5x + 4$, one would need to find two factors of 4 whose sum is negative 5.

2. In factoring a trinomial with leading coefficient 1, if the constant term is negative, then both factors must be negative.

3. The first step in factoring a trinomial is to look for a common monomial factor.

4. For a trinomial with leading coefficient 1, if no pair exists whose product is the constant and whose sum is the middle term's coefficient, then the trinomial is not factorable.

Practice

Completely factor each trinomial. If a trinomial cannot be factored, write "not factorable."

5. $x^2 - 6x - 27$

6. $a^2 + a + 2$

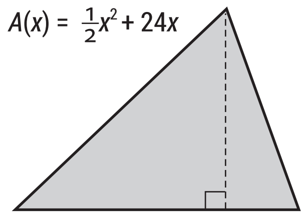
7. $y^2 - 14y + 24$

8. $2a^4 + 24a^3 + 54a^2$

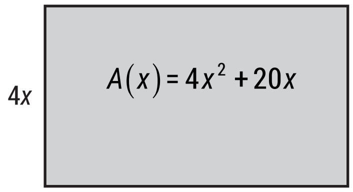
Applications

Solve.

9. ***Triangles:*** The area of a triangle is $\frac{1}{2}$ the product of its base and its height. If the area of the triangle shown is given by the function $A(x) = \frac{1}{2}x^2 + 24x$, find representations for the lengths of its base and its height (where the base is longer than the height).



10. ***Rectangles:*** The area of the rectangle shown is given by the polynomial function $A(x) = 4x^2 + 20x$. If the width of the rectangle is $4x$, what is the length?



Writing & Thinking

11. Discuss, in your own words, how the sign of the constant term determines what signs will be used in the factors when factoring trinomials.

We then set each factor equal to zero and solve for t.

$$\begin{aligned} 4t - 13 &= 0 & \qquad 4t + 13 &= 0 \\ 4t &= 13 & 4t &= -13 \\ t &= \frac{13}{4} = 3.5 & t &= \frac{-13}{4} = -3.5 \end{aligned}$$

Since time in this instance cannot be negative, the correct answer is $t = 3.5$ seconds. It took the rock 3.5 seconds to fall 196 feet to the bottom of the cliff.

3.R.6 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A trinomial is factorable if the middle term is the difference of the inner and outer products of two binomials.

2. The trial-and-error method of factoring a trinomial follows the same steps as the FOIL method of multiplication.

3. The first step in the *ac*-method of factoring is to rewrite the middle term.

4. Factoring can be checked by multiplying the factors and verifying that the product matches the original polynomial.

Practice

Completely factor each polynomial. If a polynomial cannot be factored, write "not factorable."

5. $6x^2 + 11x + 5$

6. $-x^2 + 3x - 2$

7. $x^2 + 8x + 64$

8. $9x^2 - 3x - 20$

9. $12x^2 - 38x + 20$

10. $5a^2 - 7a + 2$

Writing & Thinking

11. It is true that $2x^2 + 10x + 12 = (2x + 6)(x + 2) = (2x + 4)(x + 3)$. Explain how the trinomial can be factored in two ways. Is there some kind of error?

12. It is true that $5x^2 - 5x - 30 = (5x - 15)(x + 2)$. Explain why this is not the completely factored form of the trinomial.

Setting the four linear factors equal to zero and solving for x yields:

$$x+2=0 \quad x-2=0 \quad x+3=0 \quad x-3=0$$
$$x=-2 \quad x=2 \quad x=-3 \quad x=3$$

This gives us $\{\pm 2, \pm 3\}$ as the solution set of the equation.

3.R.7 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. You should always start by checking the number of terms when factoring a polynomial.

2. If a trinomial is to be factored, the trial-and-error or *ac*-methods can be used.

3. If there are four terms in a polynomial, it cannot be factored.

Practice

Completely factor each of the given polynomials. If a polynomial cannot be factored, write "not factorable."

4. x^2-100

5. $x^2+10x+25$

6. $x^2+16x+64$

7. $20x^2-21x-54$

8. $2y^2+6yz+5y+15z$

9. $x^3 + 125$

10. $a^2 + 2a + 24$

11. $x^2 + 9x - 36$

12. $64 + 49t^2$

13. $4x^2 - 14x + 6$

14. $200x + 20x^2 - 4x^3$

3.R.8 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When solving quadratic equations by factoring, it is important that all of the coefficients are integers.

2. The standard form for a quadratic equation is $ax^2 + bx = c$.

3. Not all quadratic equations can be solved by factoring.

4. All quadratic equations have two distinct solutions.

Practice

Solve each equation by factoring.

5. $x^2 - 11x + 18 = 0$

6. $9x^2 + 63x + 90 = 0$

7. $(x-5)(x+3) = 9$

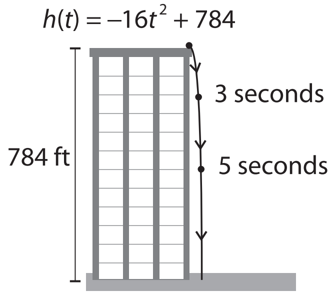
8. Find a polynomial equation with integer coefficients that has $x = 5$ and $x = 7$ as roots.

Applications

Solve.

9. ***Falling Objects:*** A ball is dropped from the top of a building that is 784 feet high. The height of the ball above ground level is given by the polynomial function $h(t) = -16t^2 + 784$ where t is measured in seconds.

   a. How high is the ball after 3 seconds? 5 seconds?

   b. How far has the ball traveled in 3 seconds? 5 seconds?

   c. When will the ball hit the ground? Explain your reasoning in terms of factors.

10. ***Falling Objects:*** A tennis ball is dropped from a building. The position of the ball after t seconds is given by the polynomial function $s(t) = -4.9t^2 + 490$, where s is the height in meters of the ball.

    a. Find $s(0)$. What does this value represent in the context of this problem?

    b. How high is the tennis ball 2 seconds after it has been dropped?

    c. How long before the tennis ball hits the ground?

Writing & Thinking

11. When solving equations by factoring, one side of the equation must be 0. Explain why this is so.

12. In solving the equation $(x+5)(x-4) = 6,$ why can't we just put one factor equal to 3 and the other equal to 2? Certainly $3 \cdot 2 = 6.$

Because the y-intercept is -10, the coefficient a must be chosen so that $f(0)=-10$. To make this calculation simpler, begin my multiplying out $(x-(2+i))(x-(2-i))$.

$$\begin{aligned}(x-(2+i))(x-(2-i)) &= (x-2-i)(x-2+i)\\ &= x^2-2x+ix-2x+4-2i-ix+2i-i^2\\ &= x^2-2x\cancel{+ix}-2x+4\cancel{-2i}\cancel{-ix}\cancel{+2i}-(-1)\\ &= x^2-2x-2x+4+1\\ &= x^2-4x+5\end{aligned}$$

Now, substitute $f(0)=-10$ and solve for a.

$$\begin{aligned}f(x) &= a(x^2-4x+5)(x+2)\\ f(0) &= a((0)^2-4(0)+5)((0)+2)\\ -10 &= a(5)(2)\\ -10 &= 10a\\ a &= -1\end{aligned}$$

Thus, the simplified polynomial function is as follows.

$$\begin{aligned}f(x) &= (-1)(x^2-4x+5)(x+2)\\ &= -(x^3+2x^2-4x^2-8x+5x+10)\\ &= -x^3+2x^2+3x-10\end{aligned}$$

3.R.9 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Regardless of the value of the exponent, the only possible values for any power of i are i and $-i$.

2. The product $\sqrt{a}\cdot\sqrt{b}$ can be rewritten as $\sqrt{ab}$ as long as a and b are real numbers.

3. When i is squared, the product is 1.

4. The conjugate of $4 - 5i$ is $4 + 5i$.

Practice

Perform the indicated operations and write each result in standard form.

5. $-4i(6-7i)$

6. $(2+7i)(6+i)$

7. $\frac{5}{4i}$

8. $\frac{6+i}{3-4i}$

Simplify the following powers of *i* and write each result in standard form. Assume *k* is a positive integer.

9. i^{13}

10. i^{-3}

Find the indicated products and simplify.

Writing & Thinking

11. Explain why the product of every complex number and its conjugate is a nonnegative real number.

12. What condition is necessary for the conjugate of a complex number, $a + bi$, to be equal to the reciprocal of this number?

Looking Ahead

The ability to find the zeros of a quadratic equation using the quadratic formula are applied especially when the quadratic equation is not easily factored, as shown in the following example.

Example Preview

Solve the following polynomial equation.

$$9x^2 - 49x + 39 = 0$$

Solution

The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where the general quadratic equation is of the form $ax^2 + bx + c = 0$ and $a \neq 0$.

$$9x^2 - 49x + 39 = 0$$

$$x = \frac{-(-49) \pm \sqrt{(49)^2 - 4(9)(39)}}{2(9)}$$

$$x = \frac{49 \pm \sqrt{2401 - 1404}}{18}$$

$$x = \frac{49 \pm \sqrt{997}}{18}$$

This gives us $\frac{49 + \sqrt{997}}{18}$ and $\frac{49 - \sqrt{997}}{18}$ as the two solutions of the equation.

3.R.10 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The quadratic formula will always work when solving quadratic equations.

2. If the discriminant is a perfect square, the quadratic equation is factorable.

3. When using the quadratic formula, if the discriminant is greater than zero, there are infinite solutions.

4. If the discriminant is less than zero, there is no real solution.

Practice

Find the discriminant and determine the nature of the solutions of each quadratic equation.

5. $x^2 + 6x - 8 = 0$

6. $x^2 - 8x + 16 = 0$

Solve each of the quadratic equations using the quadratic formula.

7. $x^2 + 4x - 4 = 0$

8. $x^2 - 2x + 7 = 0$

9. $3x^2 - 7x + 4 = 0$

Applications

Solve.

10. ***Throwing Objects:*** An orange is thrown down from the top of a building that is 300 feet tall with an initial velocity of 6 feet per second. The distance of the object from the ground can be calculated using the equation $d = 300 - 6t - 16t^2$, where t is the time in seconds after the orange is thrown.

a. On a balcony, a cup is sitting on a table located 100 feet from the ground. If the orange is thrown with the right aim to fall into the cup, how long will the orange fall? Round to the nearest hundredth. (**Hint:** The distance is 100 feet.)

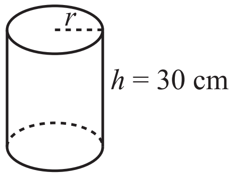
b. If the orange misses the cup and falls to the ground, how long will it take for the orange to splatter on the sidewalk? (**Hint:** What is the height of the orange when it hits the ground?)

c. Approximately how much longer would it take for the orange to fall to the sidewalk than it would for the orange to fall into the cup?

Writing & Thinking

11. Find an equation of the form $Ax^4 + Bx^2 + C = 0$ that has the four roots ± 2 and ± 3. Explain how you arrived at this equation.

12. The surface area of a right circular cylinder can be found using the following formula: $S = 2\pi r^2 + 2\pi rh$, where r is the radius of the cylinder and h is the height. Estimate the radius of a circular cylinder of height 30 cm and surface area 300 cm^2. Explain how you used your knowledge of quadratic equations.



Now You Try It!

Use the space provided to work out the solution to the next example.

Example A Using the Order of Operations with Real Numbers

Simplify: $(1-3)^2+|9-4^2|-1^3$

Looking Ahead

Your review of the order of operations will be helpful in evaluating functions. Evaluating the function $g(x)$ in the following example involves exponents, multiplication, and subtraction, which must be done in the proper order.

Example Preview

Determine $g(x+a)-g(x)$ for the following function.

$$g(x)=5x^2-4x$$

Solution

To determine $g(x+a)-g(x)$, we substitute the values in for every occurrence of x.

$$\begin{aligned} g(x+a)-g(x) &= \left(5(x+a)^2-4(x+a)\right)-\left(5x^2-4x\right) \\ &= 5\left(x^2+2ax+a^2\right)-4(x+a)-\left(5x^2-4x\right) && \text{Apply the exponent.} \\ &= 5x^2+10ax+5a^2-4x-4a-5x^2+4x && \text{Multiply across the parentheses.} \\ &= 10ax+5a^2-4a && \text{Add and subtract.} \end{aligned}$$

4.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If there are no grouping symbols, multiplication should always be performed before addition.

2. When following the rules for order of operations, powers indicated by exponents should be evaluated last.

3. The square root symbol is a grouping symbol.

4. A well-known mnemonic device for remembering the rules for order of operations is SADMEP.

Practice

Simplify.

5. **a.** $24 \div 4 \cdot 6$

b. $24 \cdot 4 \div 6$

6. $15 \div (-3) \cdot 3 - 10$

7. $3^2 \div (-9) \cdot (4 - 2^2) + 5(-2)$

8. $14 \cdot 3 \div (-2) - 6(4)$

9. $|16 - 20| + (-10)^2 + 5^2$

10. $6(13 - 15)^2 \cdot 8 \div 2^2 + 3(-1)$

11. $8 - 9\left[(-39) \div (-13) + 7(-2) - (-2)^2\right]$

Applications

Solve.

12. ***Discounts:*** The Matthews family, a family of 4, is planning a trip to New York City. During their visit, they want to see the Broadway play *Matilda*. The tickets cost \$102 each. The Matthews purchase the tickets online and the website charges a service fee of \$7.50 per ticket. The website is running a sale where the Matthews can get 10% off of their entire purchase.

    a. Write an expression to describe how much of a discount the Matthews will receive on their purchase.

    b. What is the final purchase price of the tickets?

13. ***Banking:*** Dennis overdrew his checking account and ended up with a balance of –\$42. The bank charged a \$35 overdraft fee and an additional \$5 fee for every day the account was overdrawn. Dennis left his account overdrawn for 3 days.

    a. Write an expression to show the balance of Dennis's checking account after 3 days.

    b. Simplify the expression in Part **a.** to find the balance of Dennis's checking account after 3 days.

Writing & Thinking

14. Explain, in your own words, why the following expression cannot be evaluated.

$$\left(24-2^4\right)+6\left(3-5\right)\div\left(3^2-9\right)$$

15. Consider any number between 0 and 1. If you square this number, will the result be larger or smaller than the original number? Is this always the case? Explain.

Looking Ahead

The skills you learned related to simplifying expressions will be very helpful when combining functions through addition, subtraction, multiplication, division, and composition. The following example shows how a combination of two functions is easier to understand once simplified.

Example Preview

Consider the following functions.

$$f(x) = x^3 + 3 \text{ and } g(x) = 4x$$

Find the formula for $(f+g)(x)$ and simplify your answer. Then find the domain for $(f+g)(x)$.

Solution

From the definitions of addition, subtraction, multiplication, and division of functions, we know that

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= (x^3+3)+(4x)\\ &= x^3+4x+3.\end{aligned}$$

The domain of $(f+g)(x)$ is the entire set of real numbers because both $f(x)$ and $g(x)$ are defined for all real numbers.

4.R.2 Exercises

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A variable that does not appear to have an exponent has an exponent of 1.

2. In the term $-9x$, nine is being subtracted from x.

3. In the term "12a," 12 is the constant.

4. Like terms have the same coefficients.

Practice

Identify the like terms in each list of terms.

5. $-5, 3, 7x, 8, 9x, 3y$

Simplify each expression by combining like terms.

6. $8x + 7x$

7. $3x - 5x + 12x$

8. $13x + 12x^2 + 15x - 35 - 41 - 2x^2$

Simplify each expression and then evaluate the expression for $y = 3$ and $a = -2$.

9. $5y + 4 - 2y$

10. $\frac{3a + 5a}{-2} + 12a$

Applications

Solve.

11. ***Profit:*** An apartment management company owns a property with 100 units. The company has determined that the profit made per month from the property can be calculated using the equation $P = -10x^2 + 1500x - 6000$, where x is the number of units rented per month. How much profit does the company make when 80 units are rented?

12. ***Physics:*** A ball is thrown upward from an initial height of 96 feet with an initial velocity of 16 feet per second. After t seconds, the height of the ball can be described by the expression $-16t^2 + 16t + 96$. What is the height of the ball after 3 seconds?

Writing & Thinking

13. Discuss like and unlike terms and give an example of each.

14. Explain the difference between -13^2 and $(-13)^2$.

Looking Ahead

Now that you have reviewed how to multiply polynomials, you will see how this idea can be used to perform operations with functions.

Example Preview

Find the formula for $(fg)(x)$ for the following functions.

$$f(x) = x^2 + 5 \quad \text{and} \quad g(x) = x^3 - 2$$

Solution

$$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) \\ &= (x^2 + 5)(x^3 - 2) \\ &= x^6 - 2x^2 + 5x^3 - 10 \\ &= x^6 + 5x^3 - 2x^2 - 10\end{aligned}$$

4.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The distributive property can only be used to multiply a monomial and a polynomial.

2. The product of $(a + b)$ and $(c + d)$ is $ac + bd$.

3. The FOIL method is a way to remember one specific order that the distributive property can be applied.

Practice

Multiply and simplify, if necessary.

4. $-3x^2(2x^3+5x)$

5. $-4x^3(x^5-2x^4+3x)$

6. $(x+4)(x-3)$

7. $(y+3)(y^2-y+4)$

Applications

Solve.

8. ***Advertising:*** A graphic artist is designing a poster to advertise an upcoming event. The only restrictions regarding the poster size is that it must have a length of $3x$ inches and a width of $2x + 5$ inches. Find a simplified expression for the area of the poster.

9. ***Shipping:*** Armon works for a company that ships artwork worldwide. The size of each item varies, but all of the art is on square canvases. Armon's job is to make the wooden shipping crates for each piece of art. In order to protect the artwork, each crate must be 10 inches deep. The crate must also be 10 inches wider and 12 inches taller than the artwork. Letting x represent the length of one side of the artwork, find the volume of the rectangular shipping crate.

Writing & Thinking

10. We have seen how the distributive property is used to multiply polynomials.

Show how the distributive property can be used to find the product

$$\begin{array}{r} 75 \\ \times\, 93 \\ \hline \end{array}$$

(**Hint:** 75 = 70 + 5 and 93 = 90 + 3)

4.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When dividing polynomials, any remainder must be of smaller degree than the divisor.

2. The first step in the division algorithm is to align the polynomials in ascending order.

3. To aid in organization and clarity when dividing polynomials, it is best to fill in any missing powers with ones.

4. The process followed when dividing two polynomials is called the division algorithm with polynomials.

Practice

Express each quotient as a sum (or difference) of fractions and simplify, if possible.

5. $\dfrac{8y^3 - 16y^2 + 24y}{8y}$

6. $\dfrac{20y^5 - 14y^4 + 21y^3 + 42y^2}{4y^2}$

Divide by using the division algorithm. Write the answers in the form $\boldsymbol{Q + \frac{R}{D}}$, where the degree of ***R*** < the degree of ***D***.

7. $\dfrac{x^2 - 2x - 20}{x + 4}$

8. $\dfrac{21x^3 + 41x^2 + 13x + 5}{3x + 5}$

9. $\dfrac{x^4 - 3x^3 + 2x^2 - x + 2}{x - 3}$

10. $\dfrac{x^3 - 27}{x - 3}$

Applications

Solve.

11. ***Geometry:*** A moving company uses a box that has a volume of $x^3 - 2x^2 - 13x - 10$ cubic inches.

a. If the height of the box is $x + 2$, what is the area of the base of the box?

b. If the height of the box is $x + 1$, what is the area of the base of the box?

Writing & Thinking

12. Suppose that a polynomial is divided by $(3x-2)$ and the answer is given as $x^2+2x+4+\frac{20}{3x-2}$.

What was the original polynomial? Explain how you arrived at this conclusion.

Looking Ahead

Your review of rational expressions and how to simplify them will help you with identifying the vertical asymptotes, if they exist, of a given rational function.

Example Preview

Find equations for the vertical asymptotes, if any, for the following rational function.

$$f(x) = \frac{-6x^2 + 4x + 2}{-2x + 2}$$

Solution

First, we need to reduce the function by removing any common factors in the numerator and denominator.

$$\begin{aligned} f(x) &= \frac{-6x^2 + 4x + 2}{-2x + 2} \\ &= \frac{\cancel{(-2x+2)}(3x+1)}{\cancel{(-2x+2)}} \\ &= 3x + 1 \end{aligned}$$

Now, the vertical asymptotes of *f* are at the zeros of the denominator.

Since no denominator remains after factoring the common terms, there are no vertical asymptotes for *f*. (Note that the domain of *f* excludes $x = 1$. This means the graph has a "hole" at this *x*-value instead of a vertical asymptote.)

4.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A simplified rational expression cannot have any common factors other than 1 and −1 in both the numerator and denominator.

2. The difference between a rational number and a rational expression is that a rational expression generally has polynomials in the numerator and/or denominator.

3. While a rational number cannot have a zero denominator, a rational expression can have a zero denominator.

4. If a denominator is $x + 5$, it is defined for all values except 5.

Practice

Reduce each expression to lowest terms. State any restrictions on the variable(s).

5. $\frac{9x^2y^3}{12xy^4}$

6. $\frac{2x-8}{16-4x}$

7. $\frac{xy-3y+2x-6}{y^2-4}$

8. $\frac{x^2+10x+24}{2x^2+x-28}$

9. $\frac{x}{x^2-3x}$

10. Evaluate $\frac{3y-4}{y^2+25}$ for $y=3$

Applications

Solve.

11. ***Event Planning:*** The cost of renting a party room with tables, chairs, and simple decorations is \$200 plus \$15 per person attending.

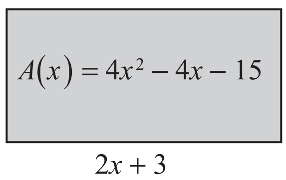
a. Write a rational expression that represents the total price per person for renting the party room, where x is the number of people attending.

b. What is the price per person to rent the party room if 10 people are attending?

c. Determine which values of the variable will make the rational expression from Part **a.** undefined.

d. Considering the context of the given problem, are there any additional restrictions on the variable? If so, explain why these restrictions are in place.

12. ***Rectangles:*** The area of a rectangle (in square feet) is represented by the polynomial function $A(x) = 4x^2 - 4x - 15$. If the length of the rectangle is $(2x + 3)$ feet, find a representation for the width.



Writing & Thinking

13. a. Define the term rational expression.

b. Give an example of a rational expression that is undefined for $x = -2$ and $x = 3$ and has a value of 0 for $x = 1$. Explain how you determined this expression.

c. Give an example of a rational expression that is undefined for $x = -5$ and never has a value of 0. Explain how you determined this expression.

Solution

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}\\ &= \frac{\frac{x^2-25}{x}}{\frac{x+5}{2x}}\\ &= \frac{x^2-25}{x}\cdot\frac{2x}{x+5}\\ &= \frac{\cancel{(x+5)}(x-5)2\cancel{x}}{\cancel{x}\cancel{(x+5)}}\\ &= \frac{2(x-5)}{1}\\ &= 2x-10\end{aligned}$$

4.R.6 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The reciprocal of $\frac{x}{x+3}$ is $\frac{-x-3}{x}$.

2. Dividing rational expressions is similar to dividing fractions.

3. There are no restrictions on the denominator $12x^2$.

4. Because $\frac{4x^2}{16x}$ reduces to $\frac{x}{4}$, there are no restrictions on the denominator.

Practice

Perform the indicated operations and reduce to lowest terms. Assume that no denominator has a value of 0.

5. $\frac{x^2-9}{x^2+2x}\cdot\frac{x+2}{x-3}$

6. $\frac{2x^2+x-3}{x^2+4x}\cdot\frac{2x+8}{x-1}$

7. $\frac{x-1}{6x+6}\div\frac{2x-2}{x^2+x}$

8. $\frac{x+3}{x^2+3x-4}\div\frac{x+2}{x^2+x-2}$

Applications

Solve

9. ***Carpentry:*** Erik is building a cubby bookshelf, that is, a bookshelf divided into storage holes (cubbies) instead of shelves. He wants the height of the bookshelf to be $x^2 - 3x - 10$ and the width to be $x^2 + 5x + 6$. Each cubby hole in the bookshelf will have a height of $x + 3$ and a width of $x - 5$.

a. Write a rational expression to determine how many cubbies high the bookshelf will be.

b. Write a rational expression to determine how many cubbies wide the bookshelf will be.

c. Multiply the rational expressions from Parts **a.** and **b.** (and reduce to lowest terms) to obtain a rational expression that gives the total number of cubbies in the entire bookshelf.

Looking Ahead

Now that you have reviewed how to simplify complex rational expressions, you will be able to apply these skills to more advanced topics like the quotient of rational functions. The following example shows you how to apply the skills of this section to such a problem.

Example Preview

Find the formula for $(g \circ f)(x)$ for the following functions.

$$f(x) = \frac{1}{x} \quad \text{and} \quad g(x) = \frac{x-2}{5}$$

Solution

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g\left(\frac{1}{x}\right) \\ &= \frac{\left(\frac{1}{x}\right) - 2}{5} \\ &= \frac{\frac{1}{x} - 2}{5} \cdot \frac{x}{x} \\ &= \frac{1-2x}{5x}\end{aligned}$$

4.R.7 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When simplifying complex fractions, the answer should always be reduced to lowest terms.

2. Complex fractions are those fractions in which only the denominator consists of one or more fractions itself.

3. Sometimes finding the LCM of all denominators is an important first step for simplifying complex fractions.

4. The LCM of the denominators of $\frac{2}{x-6}$ and $\frac{x}{6}$ is 6.

Practice

Simplify the following complex fractions.

5. $\dfrac{\frac{2x}{3y^2}}{\frac{5x^2}{6y}}$

6. $\dfrac{\frac{x+3}{2x}}{\frac{2x-1}{4x^2}}$

7. $\dfrac{\frac{3}{x}+\frac{5}{2x}}{\frac{1}{x}+4}$

8. $\dfrac{\frac{7}{x}-\frac{14}{x^2}}{\frac{1}{x}-\frac{4}{x^3}}$

Simplify the following complex algebraic expressions.

9. $\dfrac{\frac{1}{x+1} - \frac{3}{2x} \cdot \frac{4x}{x+1}}{}$

10. $\frac{x}{x-1} - \frac{3}{x-1} \cdot \frac{x+2}{x}$

Applications

Solve.

11. ***Investing:*** The average percent yield (APY) of an annuity is the annual interest rate earned in a given year that accounts for the effects of compounding. The APY acts as the interest rate for a simple interest account and is larger than the stated interest rate on the compound interest account. The formula to calculate the APY on an annuity after 2 years is

$$\text{APY} = \left(1 + \frac{r}{2}\right)^2 - 1,$$

where r is the stated interest rate.

a. Simplify the expression for APY and write as a single rational expression.

b. Using the original formula, calculate the APY for an annuity whose interest rate is 6%. Do not round.

c. Using the expression in Part **a.**, calculate the APY for an annuity whose interest rate is 6%. Do not round.

d. Does the result from Part **c.** match the result from Part **b.**? Explain why or why not.

e. How much larger is the APY than the interest rate?

f. Why do you think the APY is larger than the interest rate? Write a complete sentence.

Writing & Thinking

12. Some complex fractions involve the sum (or difference) of complex fractions. Beginning with the outermost denominator, simplify each of the following expressions.

a. $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+1}}}$

b. $2-\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-1}}}$

c. $x+\cfrac{1}{x+\cfrac{1}{x+\cfrac{1}{x+1}}}$

Looking Ahead

The following example requires you to use the skills you learned to rewrite a number with a different base using negative exponents. Once the expressions on opposite sides of the equation have the same base, they can only be equal if their exponents are equal. This leads to a simple linear equation in one variable that can easily be solved.

Example Preview

Solve the following exponential equation.

$$3^{3x-5}=\frac{1}{9}$$

Solution

Solving this exponential equation involves rewriting the terms on both sides of the equation with the same base. Once this is done, the exponents can be equated, and the subsequent equation can be solved for x.

This exponential equation can be solved in the following manner.

$$3^{3x-5}=\frac{1}{9}$$

$$3^{3x-5}=\left(\frac{1}{3}\right)^2$$

$$3^{3x-5}=3^{-2}$$

$$3x-5=-2 \qquad \text{Exponents must be equal with the same base.}$$

$$3x=3$$

$$x=1$$

6.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a constant does not have an exponent written, it is assumed that the exponent is 0.

2. If a is a nonzero real number and n is an integer, then $a^{-n}=-a^n$.

3. Since the product rule is stated for integer exponents, the rule is also valid for 0 and negative exponents.

4. When using the quotient rule, you should subtract the smaller exponent from the larger exponent.

Practice

Simplify each expression. The final form of the expressions with variables should contain only positive exponents. Assume that all variables represent nonzero numbers.

5. $y^3 \cdot y^8$

6. $\dfrac{y^7}{y^2}$

7. $x^{-3} \cdot x^0 \cdot x^2$

8. $\dfrac{10^4 \cdot 10^{-3}}{10^{-2}}$

9. $(9x^2y^3)(-2x^3y^4)$

10. $\dfrac{-8x^{-2}y^4}{4x^2y^{-2}}$

Applications

Solve.

11. ***Computers:*** Rylee wants to move all her files to a new hard drive that has 2^{12} GB of storage on it. She wants to designate the same amount of storage for each of 2^4 projects. How much storage should be assigned to each project? Write your answer as a power of two.

12. ***Bacteria:*** Trey is studying patterns in bacteria. For a positive test result in his experiment, bacteria must grow in population at a minimum rate of 3^2 in 24 hours. If the initial population of the bacteria is 3^5 and his final measurement after 24 hours is 3^8, should he mark the test as positive or negative?

6.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Taking the reciprocal of a fraction changes the sign of any exponent in the fraction.

2. For an exponent to refer to –7 as the base, –7 must be in parentheses.

3. When simplifying an expression with exponents, the rules for exponents must be used in a specific order or the answer will vary.

4. The expression -8^2 simplifies to –64.

Practice

Use the rules for exponents to simplify each of the expressions. Assume that all variables represent nonzero real numbers.

5. $\left(2^{-3}\right)^{-2}$

6. $-3\left(7xy^2\right)^0$

7. $-2\left(3x^5y^{-2}\right)^{-3}$

8. $\left(\frac{x}{2}\right)^3$

9. $\left(\frac{2x^2y}{y^3}\right)^{-4}$

10. $\left(\frac{5a^4b^{-2}}{6a^{-4}b^3}\right)^{-2}\left(\frac{5a^3b^4}{2^{-2}a^{-2}b^{-2}}\right)^3$

6.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The same rules for exponents apply to both integer exponents and rational exponents.

2. If the cube root of 7 were to be converted into exponential notation it would be $\sqrt[3]{7}$.

3. Any expression to the power 0, such as $\left(\sqrt[4]{x}\right)^0$, is equal to 1.

4. The expression $y^{\frac{1}{2}}$ can be rewritten in radical notation as $\sqrt{y^2}$.

Practice

5. Use radical notation to write an expression that is equivalent to $8^{\frac{1}{3}}$.

6. Use exponential notation to write an expression that is equivalent to $\sqrt{3}$.

Simplify each numerical expression.

7. $100^{-\frac{1}{2}}$

8. $64^{\frac{2}{3}}$

9. Simplify $\dfrac{a^{\frac{1}{2}} \cdot a^{-\frac{3}{4}}}{a^{-\frac{1}{2}}}$. Assume that all variables represent positive real numbers.

10. Simplify $\dfrac{\sqrt[4]{y^3}}{\sqrt[6]{y}}$ by first changing it into an equivalent expression with rational exponents. Rewrite the answer in simplified radical form. Assume that all variables represent positive real numbers.

Applications

Solve.

11. ***Area:*** The width of a rectangle is $\sqrt[3]{64^2}$ ft and the length is $216^{\frac{2}{3}}$ ft. What is the area of the rectangle?

12. ***Amusement Parks:*** An amusement park is creating signs to indicate the velocity of the roller coaster car on certain hills of the most popular rides. A roller coaster car gains kinetic energy as it goes down a hill. The velocity, or speed, of an object in kilometers per hour (km/h) can be determined by $V = \left(\frac{2k}{m}\right)^{\frac{1}{2}}$, where k is the kinetic energy of the object in joules (J) and m is the mass of the object in kilograms (kg).

a. For the most popular roller coaster, the car has a mass of 300 kg and the car has a kinetic energy of 375,000 J on the first hill. What velocity does the car obtain on the first hill?

b. For the second most popular roller coaster, the car has a mass of 350 kg and the car has a kinetic energy of 70,000 on the first hill. What velocity does the car obtain on the first hill?

Writing & Thinking

13. Is $\sqrt[5]{a} \cdot \sqrt{a}$ the same as $\sqrt[5]{a^2}$? Explain why or why not.

14. Assume that x represents a positive real number. Describe what kind of number the exponent n must be for x^n to mean

a. a product.

b. a quotient.

c. 1.

d. a radical expression.

Looking Ahead

Being familiar with the main properties of logarithms and logarithmic functions allows you to tackle logarithmic and exponential equations without the need of a calculator. In the example below, you will use many of those skills to determine the value of x in an exponential equation.

Example Preview

Solve the following exponential equation.

$$e^{2x} + 4e^{x} - 45 = 0$$

Solution

This equation is in quadratic form. First, solve for e^x by factoring. Then, solve for x by converting from exponential to logarithmic form.

$$(e^x - 5)(e^x + 9) = 0$$
$$e^x = 5, -9$$
$$x = \ln(5), \ln(-9)$$

Since $\ln(-9)$ is undefined, the only solution is $\ln(5)$.

6.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so that the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Exponential functions of the form $y = b^x$ are one-to-one functions and have inverses.

2. The exponent of an exponential function is the base of its inverse logarithmic function.

3. Exponents are logarithms.

4. The logarithm of the base is always 1.

Practice

5. Express $7^2 = 49$ in logarithmic form.

6. Express $\log_5 125 = 3$ in exponential form.

Solve by first changing each equation to exponential form.

7. $\log_5 \frac{1}{125} = x$

8. $\log_x 121 = 2$

9. $\log_8 8^{3.7} = x$

Graph the function and its inverse on the same set of axes.

10. $f(x) = 2^x$

Writing & Thinking

11. Discuss, in your own words, the symmetrical relationship of the graphs of the two logarithmic functions $y = \log_{10} x$ and $y = -\log_{10} x$.

Next, we know from the definitions of the six basic trigonometric that tangent and cotangent are the two functions that use the adjacent leg and the opposite, so we will use one of these to find the missing value.

Using the figure, we can determine the following.

$$\tan\left(77^{\circ}\right)=\frac{opp}{33},$$

$$opp=33\tan\left(77^{\circ}\right)\approx 143\text{ feet}$$

7.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The sum of the measures of two complementary angles is equal to the measure of one right angle.

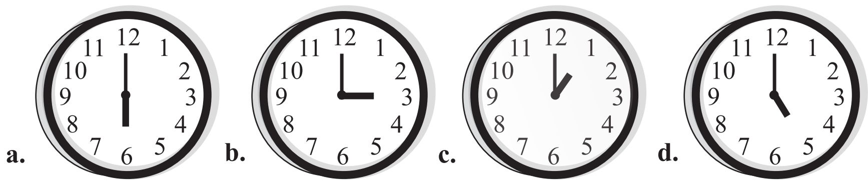
2. The sum of the measures of complementary angles is greater than the sum of the measures of supplementary angles.

3. Adjacent angles are two angles that share a side.

4. If two lines in a plane are not parallel, then they are perpendicular.

Practice

5. Name the type of angle formed by the hands on a clock.



a. at six o’clock

b. at three o’clock

c. at one o'clock

d. at five o'clock

6. Assume that $\angle 1$ and $\angle 2$ are complementary.

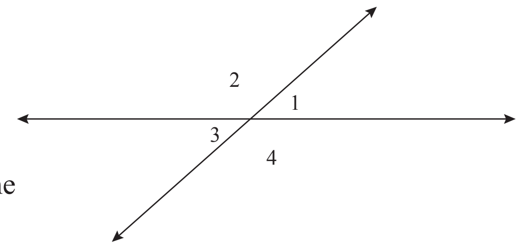
a. If $m\angle 1 = 15°$, what is $m\angle 2$?

b. If $m\angle 1 = 3°$, what is $m\angle 2$?

c. If $m\angle 1 = 45°$, what is $m\angle 2$?

d. If $m\angle 1 = 75°$, what is $m\angle 2$?

7. The figure shows two intersecting lines.



a. If $m\angle 1 = 30°$, what is $m\angle 2$?

b. Is $m\angle 3 = 30°$? Give a reason for your answer other than the fact that $\angle 1$ and $\angle 3$ are vertical angles.

c. Name two pairs of congruent angles.

d. Name four pairs of adjacent angles.

Writing & Thinking

8. Explain, in your own words, the relationships between vertex, ray, angle, and line.

Now we see that

$$\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

and

$$\csc\frac{\pi}{4}=\frac{1}{\sin\frac{\pi}{4}}=\frac{1}{\frac{\sqrt{2}}{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

7.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A triangle with sides of 4 inches, 4 inches, and 3 inches is an isosceles triangle.

2. A triangle with three angles that each measure less than 90 degrees is an acute triangle.

3. Similar triangles have corresponding sides that are equal.

4. If $\triangle ABC \cong \triangle DEF$, then the measure of angle C equals the measure of angle D.

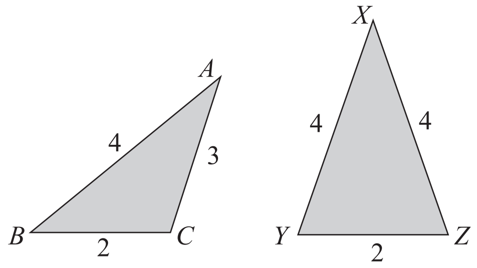
5. If $\triangle ABC \sim \triangle DEF$, then $AC = DF$.

6. Congruent triangles have corresponding angles that are equal.

Practice

Determine whether each pair of triangles is similar. If the pair of triangles is similar, explain why and indicate the similarity by using the ~ symbol.

7.



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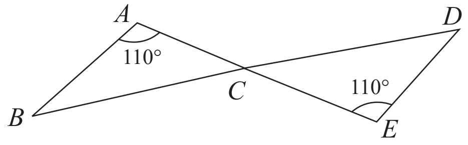
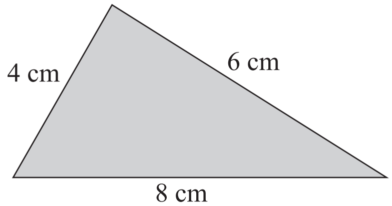
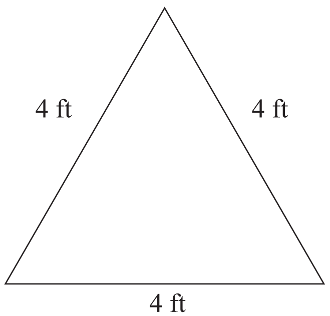


Classify each triangle in the most precise way possible, given the indicated lengths of its sides and/or measures of its angles.

9.

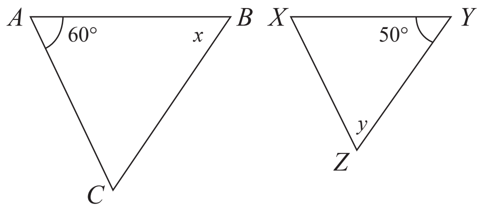


10.

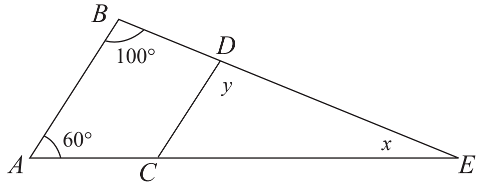


Find the values for *x* and *y*.

11. $\triangle ABC \sim \triangle XYZ$

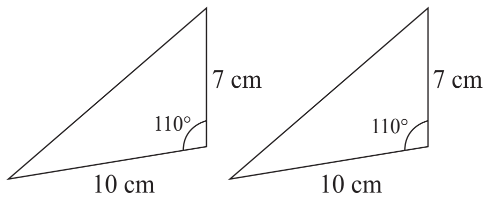


12. $\triangle ABE \sim \triangle CDE$

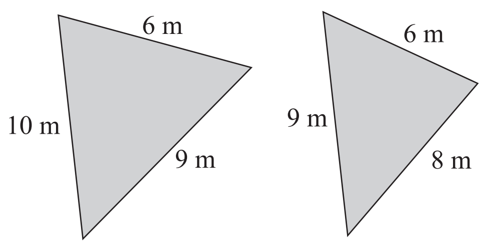


Determine whether each pair of triangles is congruent. If the pair of triangles is congruent, state the property that confirms that they are congruent.

13.



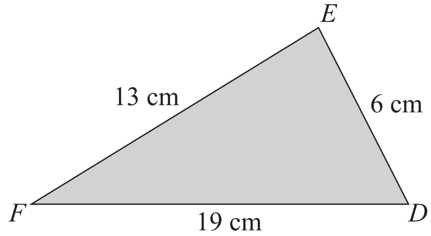
14.



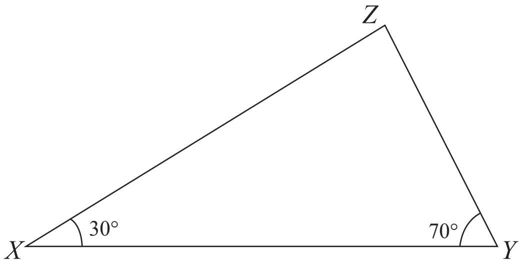
Applications

Solve.

15. Suppose the lengths of the sides of $\triangle DEF$ are as shown in the figure. Is this possible? Explain your reasoning.



16. In the triangle shown, $m\angle X = 30°$ and $m\angle Y = 70°$.



a. What is $m\angle Z$?

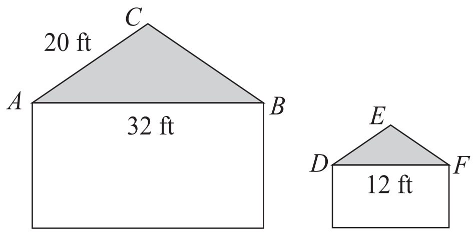
b. What kind of triangle is $\triangle XYZ$?

c. Which side is opposite $\angle X$?

d. Which sides include $\angle X$?

e. Is $\triangle XYZ$ a right triangle?

17. ***Construction:*** A child's playhouse is built to look like a smaller version of the family house, where the ends of the roofs have similar proportions. The width of the main house (*AB*) is 32 feet and the length from the peak to the gutter of the roof for one of the sides is 20 feet. If the width of the playhouse (*DF*) is 12 feet, what is the length from the peak to the gutter (*DE*) of the playhouse roof?



18. ***Holiday Decorating:*** Your neighbors are hanging their holiday lights. The ladder they are currently using is 12 feet long and when leaned up against the house just reaches the top of their 8-foot tall porch. How long of a ladder will they need to reach the top of their chimney which is at a height of 32 feet? (Assume that both ladders are placed such that they make the same angle with the ground.)

Writing & Thinking

19. Determine the errors in the following statements. Assume $\triangle ABC \sim \triangle DEF$.

a. Corresponding angles are congruent. This means $m\angle A = m\angle D$, $m\angle B = m\angle F$, and $m\angle C = m\angle E$.

b. Corresponding sides are the same length.

10.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When two binomials are in the form of the sum and difference of the same term, the product will be a trinomial.

2. When the two binomials being multiplied together are the same, the product will be a trinomial.

3. Perfect square trinomials result from squaring a binomial sum or a binomial difference.

4. When finding the product of two binomials that are in the form of the sum and difference of the same two terms, the FOIL method and the difference of two squares formula will produce different results.

Practice

Find each product and identify any that are either the difference of two squares or a perfect square trinomial.

5. $(x-7)^2$

6. $(x+12)(x-12)$

7. $(x+4)(x+4)$

8. $(3x+7)^2$

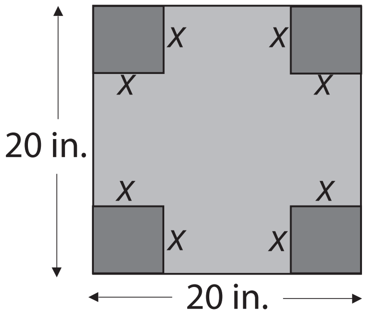
9. $(3x-2)(3x-2)$

10. $(5x-9)(5x+9)$

Applications

Solve.

11. ***Geometry:*** A square is 20 inches on each side. A square x inches on each side is cut from each corner of the square.



a. Represent the area of the remaining portion of the square in the form of a polynomial function $A(x)$.

b. Represent the perimeter of the remaining portion of the square in the form of a polynomial function $P(x)$.

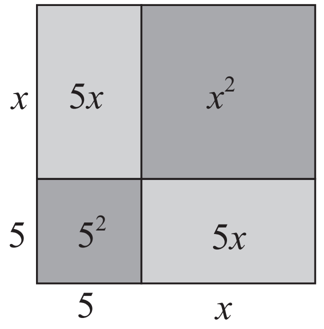
12. ***Probability:*** In the case of binomial probabilities, if x is the probability of success in one trial of an event, then the expression $f(x)=15x^4(1-x)^2$ is the probability of 4 successes in 6 trials where $0 \le x \le 1$.

a. Represent the expression $f(x)$ as a single polynomial by multiplying the polynomials.

b. If a fair coin is tossed, the probability of heads occurring is $\frac{1}{2}$. That is, $x = \frac{1}{2}$. Find the probability of 4 heads occurring in 6 tosses.

Writing & Thinking

13. A square with sides of length $(x+5)$ can be broken up as shown in the diagram. The sums of the areas of the interior rectangles and squares is equal to the total area of the square: $(x+5)^2$. Show how this fits with the formula for the square of a sum.



10.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The expression $x^2 + 20x + 100$ is a perfect square trinomial.

2. When factoring polynomials, always look for a common monomial factor first.

3. The sum of two squares, $(x^2 + a^2)$, is factorable.

Practice

Completely factor each of the given polynomials. If a polynomial cannot be factored, write "not factorable."

4. $25 - z^2$

5. $y^2 - 16y + 64$

6. $x^2 + 64y^2$

7. $2x^2 - 128$

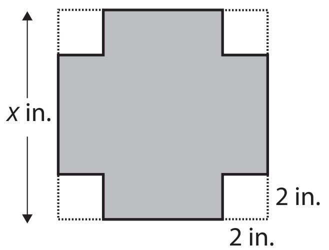
8. $25x^2 + 30x + 9$

9. $9x^2 - y^2$

Solve.

10. **a.** Represent the area of the shaded region of the square shown as the difference of two squares.

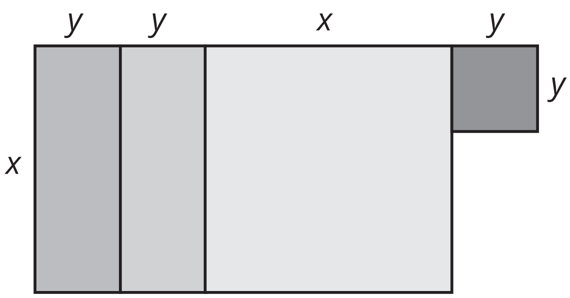
b. Use the factors of the expression in Part **a.** to draw (and label the sides of) a rectangle that has the same area as the shaded region.

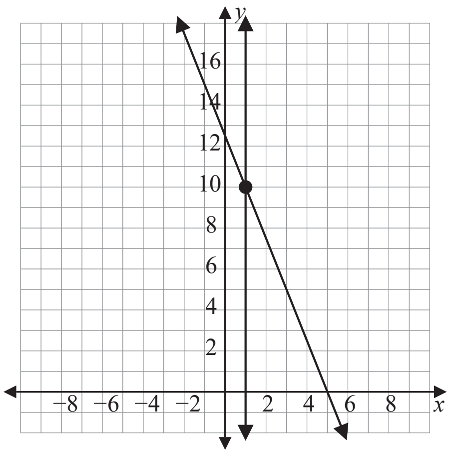


Writing & Thinking

11. a. Show that the sum of the areas of the rectangles and squares in the figure is a perfect square trinomial.

b. Rearrange the rectangles and squares in the form of a square and represent its area as the square of a binomial.





The two lines appear to intersect at the point (1,10). This can be checked by substituting 1 for x and 10 for y in both equations.

b. We can solve either equation for either variable and the choice will not affect the final answer. We will solve for y in the first equation.

$$\begin{aligned} -5x - 2y &= -25 \\ -2y &= 5x - 25 \\ y &= \frac{-5x + 25}{2} \end{aligned}$$

We know from the second equation:

$$\begin{aligned} 5x &= 5 \\ x &= 1 \end{aligned}$$

To find the numerical value for y, we substitute 1 for x into the expression we obtained for y in the first step.

$$y = \frac{-5(1) + 25}{2} = 10$$

Therefore, the solution to the system of equations is (1,10).

11.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. To check a solution, substitute it into one of the equations. If the solution satisfies one equation it will satisfy all of the equations.

2. A system of equations with graphs that are parallel lines has exactly one solution.

3. A system of equations with graphs that intersect at one point has exactly one solution.

4. A system of equations with graphs that are the same line has infinitely many solutions.

Practice

Determine which of the given points, if any, lie on both of the lines in the systems of equations by substituting each point into both equations.

5. $\begin{cases} 2x+4y-6=0 \\ 3x+6y-9=0 \end{cases}$

a. $(1, 1)$

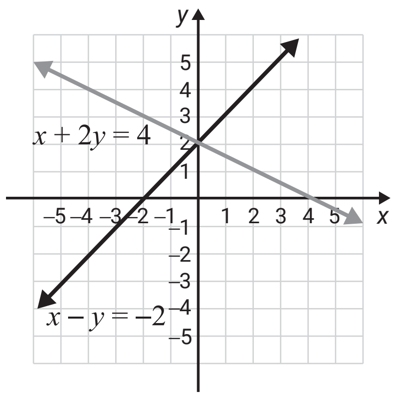
b. $(2, 0)$

c. $\left(0, \frac{3}{2}\right)$

d. $(-1, 3)$

The graphs of the lines represented by system of equations are given. Determine the solution of the system by looking at the graph. Check your solution by substituting into both equations.

6. $\begin{cases} x+2y=4 \\ x-y=-2 \end{cases}$



Solve each system of equations by graphing.

7. $\begin{cases} x - 2y = 4 \\ x = 4 \end{cases}$

8. $\begin{cases} 2x + y = 0 \\ 4x + 2y = -8 \end{cases}$

Applications

Each of the following applications has been modeled using a system of equations. Solve the system graphically.

9. ***Swimming Pools:*** OSHA recommends that swimming pool owners clean their pool decks with a solvent composed of a 12% chlorine solution and a 3% chlorine solution. Fifteen gallons of the solvent consists of 6% chlorine. How much of each of the mixing solutions were used?
Let x = the number of gallons of the 12% solution
and y = the number of gallons of the 3% solution.
The corresponding modeling system is $\begin{cases} x + y = 15 \\ 0.12x + 0.03y = 0.06(15) \end{cases}$

10. ***School Supplies:*** A student bought a calculator and a textbook for a course in algebra. He told his friend that the total cost was \$170 (without tax) and that the calculator cost \$20 more than twice the cost of the textbook. What was the cost of each item?
Let x = the cost of the calculator
and y = the cost of the textbook.
The corresponding modeling system is $\begin{cases} x + y = 170 \\ x = 2y + 20 \end{cases}$

Writing & Thinking

11. Explain, in your own words, why the answer to a consistent system of linear equations can be written as an ordered pair.

Finally, to find x, we substitute our values for y and z into any of the original equations that contain x.

$$x+4\left(\frac{47}{14}\right)=10$$
$$x=\frac{-24}{7}$$

There is only one solution for this system of equations: $\left(\frac{-24}{7},\frac{47}{14},\frac{-27}{14}\right)$.

11.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The method of substitution reduces the problem from one of solving two equations in two variables to solving one equation in one variable.

2. The method of substitution is most often used when one of the equations is impossible to graph.

3. The method of substitution is more accurate than the graphing method.

4. When using the method of substitution, you should always solve the first equation for x.

Practice

Use the method of substitution to solve each system.

5. $\begin{cases} x + y = 6 \\ y = 2x \end{cases}$

6. $\begin{cases} 3x - 7 = y \\ 2y = 6x - 14 \end{cases}$

7. $\begin{cases} 4x = y \\ 4x - y = 7 \end{cases}$

8. $\begin{cases} 3y + 5x = 5 \\ y = 3 - 2x \end{cases}$

Applications

Each of the following applications has been modeled using a system of equations. Use the method of substitution to solve each system.

9. ***Rectangles:*** The perimeter of a rectangle is 50 meters and the length is 5 meters longer than the width. Find the dimensions of the rectangle.
Let x = the length and y = the width.
The corresponding modeling system is $\begin{cases} 2x+2y=50 \\ x-y=5 \end{cases}$

10. ***Health & Fitness:*** A fitness center manager is trying to decide whether to charge an enrollment fee of \$25 with a monthly rate of \$50 or an enrollment fee of \$100 with a monthly rate of \$25. After how many months would it be more profitable for the manager to choose the lower enrollment fee and the higher monthly rate? Round up to the nearest month.
The corresponding modeling system is $\begin{cases} y=50x+25 \\ y=25x+100 \end{cases}$

Writing & Thinking

11. Explain the advantages of solving a system of linear equations

a. by graphing.

b. by substitution.

Now that we have our starting matrix, the solution for this system of equations can be found in the following manner.

$$\begin{bmatrix} 1 & 3 & 0 & | & -2 \\ 0 & 1 & -5 & | & 21 \\ -2 & -2 & 2 & | & 0 \end{bmatrix} \xrightarrow{2R_1+R_3} \begin{bmatrix} 1 & 3 & 0 & | & -2 \\ 0 & 1 & -5 & | & 21 \\ 0 & 4 & 2 & | & -4 \end{bmatrix} \xrightarrow{-4R_2+R_3} \begin{bmatrix} 1 & 3 & 0 & | & -2 \\ 0 & 1 & -5 & | & 21 \\ 0 & 0 & 22 & | & -88 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{22}R_3} \begin{bmatrix} 1 & 3 & 0 & | & -2 \\ 0 & 1 & -5 & | & 21 \\ 0 & 0 & 1 & | & -4 \end{bmatrix} \xrightarrow{5R_3+R_2} \begin{bmatrix} 1 & 3 & 0 & | & -2 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -4 \end{bmatrix} \xrightarrow{-3R_2+R_1} \begin{bmatrix} 1 & 0 & 0 & | & -5 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -4 \end{bmatrix}$$

After performing all of the necessary elementary row operations to place the augmented matrix for this system in reduced row echelon form, we have the following matrix.

$$\begin{bmatrix} 1 & 0 & 0 & | & -5 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & -4 \end{bmatrix}$$

If we now write this matrix in system form, we have the following.

$$\begin{cases} x = -5 \\ y = 1 \\ z = -4 \end{cases}$$

This is equivalent to the original system, but in a form that tells us the solution of the system. Therefore, the ordered triple $(-5,1,-4)$ solves this system of equations.

11.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When using the method of addition, the solution only needs to be checked in one of the original equations.

2. It's possible for a system of equations to have no solutions.

3. Both the addition method and the substitution method give approximate solutions.

4. The graphing method is helpful in "seeing" the geometric relationship between the lines and finding approximate solutions.

Practice

Solve each system of linear equations.

5. $\begin{cases} 2x + y = 3 \\ 4x + 2y = 7 \end{cases}$

6. $\begin{cases} y = 2x + 14 \\ x = 14 - 3y \end{cases}$

7. $\begin{cases} 4x - 2y = 8 \\ 2x - y = 4 \end{cases}$

Write an equation for the line determined by the two given points by using the formula *y* = *mx* + *b* to set up a system of equations with *m* and *b* as the unknowns.

8. $(2, 3), (1, -2)$

Applications

Each of the following applications has been modeled using a system of equations. Use the method of substitution or the method of addition to solve each system.

9. ***Baseball:*** A minor league baseball team has a game attendance of 4500 people. Tickets cost \$5 for children and \$8 for adults. The total revenue made at this game was \$26,100. How many adults and how many children attended the game?
Let x = number of adults
and y = number of children.
The system that models the problem is $\begin{cases} x + y = 4500 \\ 8x + 5y = 26{,}100 \end{cases}$

10. ***Acid Solutions:*** How many liters each of a 30% acid solution and a 40% acid solution must be used to produce 100 liters of a 36% acid solution?

Let x = amount of 30% solution
and y = amount of 40% solution.

The system that models the problem is $\begin{cases} x + y = 100 \\ 0.30x + 0.40y = 0.36(100) \end{cases}$

Writing & Thinking

11. Explain, in your own words, why the answer to a system with infinite solutions is written as an ordered pair with variables.

From here, we can see that the vertices of the feasible region may be represented by the solutions to the following systems of equations.

$$\begin{cases} y = 4000 \\ x + y \leq 19000 \end{cases}, \begin{cases} y = 0 \\ x + y = 19000 \end{cases}, \begin{cases} x = 8000 \\ y = 0 \end{cases}, \begin{cases} x = 8000 \\ y = 4000 \end{cases}$$

Solving each of these systems, we find that the vertices are (15000,4000), (19000,0), (8000,0), and (8000,4000). We know that one of these vertices must be the point that gives the maximum value. Substituting each of these ordered pairs into the objective function, $f(x,y) = 0.04x + 0.05y$, we find the following.

$$f(15000,4000) = 800$$
$$f(19000,0) = 760$$
$$f(8000,0) = 320$$
$$f(8000,4000) = 520$$

We see that income will be maximized by investing $15,000 in municipal bonds and $4000 in Treasury bills.

11.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When boundary lines are parallel, the system of linear inequalities has no solution.

2. If two half-planes overlap, that region is the union of the graphs.

3. Half-planes are the graphs of linear inequalities.

4. If the graphs of two linear inequalities have no intersection, then the system has no solution.

Practice

Solve the systems of two linear inequalities graphically.

5. $\begin{cases} y > 2 \\ x \geq -3 \end{cases}$

7. $\begin{cases} 2x - 3y \geq 0 \\ 8x - 3y < 36 \end{cases}$

6. $\begin{cases} y > 3x + 1 \\ -3x + y < -1 \end{cases}$

8. $\begin{cases} y > x - 4 \\ y < x + 2 \end{cases}$

Applications

Solve.

9. ***Fundraising:*** Robin is planning a charity ball to raise money for her favorite charity. There are two different ticket options. The VIP option includes dinner, dancing, and cocktails for $150 per ticket. The regular option includes dancing and cocktails for $75 per ticket. Robin wants to make at least $14,000 in ticket sales. The ballroom that is being used for the charity event has a maximum capacity of 150 people.

a. Write two linear inequalities to describe the situation. Let the variable x represent the number of VIP tickets sold and let the variable y represent the number of regular tickets sold.

b. Graph the two linear inequalities on the same coordinate plane.

c. Describe the solution set for the situation.

d. Can Robin reach her sales goal if she only sells tickets for the regular option? Explain why or why not.

Writing & Thinking

10. Graph the inequalities and explain how you can tell that there is no solution.

$$\begin{cases} y \leq 2x - 5 \\ y \geq 2x + 3 \end{cases}$$