Looking Ahead

Your review of rational expressions and how to simplify them will help you with identifying the vertical asymptotes, if they exist, of a given rational function.

Example Preview

Find equations for the vertical asymptotes, if any, for the following rational function.

$$f(x) = \frac{-6x^2 + 4x + 2}{-2x + 2}$$

Solution

First, we need to reduce the function by removing any common factors in the numerator and denominator.

$$\begin{aligned} f(x) &= \frac{-6x^2 + 4x + 2}{-2x + 2} \\ &= \frac{\cancel{(-2x + 2)}(3x + 1)}{\cancel{(-2x + 2)}} \\ &= 3x + 1 \end{aligned}$$

Now, the vertical asymptotes of f are at the zeros of the denominator.

Since no denominator remains after factoring the common terms, there are no vertical asymptotes for f. (Note that the domain of f excludes $x = 1$. This means the graph has a "hole" at this x-value instead of a vertical asymptote.)

4.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A simplified rational expression cannot have any common factors other than 1 and −1 in both the numerator and denominator.

2. The difference between a rational number and a rational expression is that a rational expression generally has polynomials in the numerator and/or denominator.

3. While a rational number cannot have a zero denominator, a rational expression can have a zero denominator.

4. If a denominator is $x + 5$, it is defined for all values except 5.

Practice

Reduce each expression to lowest terms. State any restrictions on the variable(s).

5. $\dfrac{9x^2y^3}{12xy^4}$

6. $\dfrac{2x-8}{16-4x}$

7. $\dfrac{xy-3y+2x-6}{y^2-4}$

8. $\dfrac{x^2+10x+24}{2x^2+x-28}$

9. $\dfrac{x}{x^2-3x}$

10. Evaluate $\dfrac{3y-4}{y^2+25}$ for $y=3$

Applications

Solve.

11. ***Event Planning:*** The cost of renting a party room with tables, chairs, and simple decorations is \$200 plus \$15 per person attending.

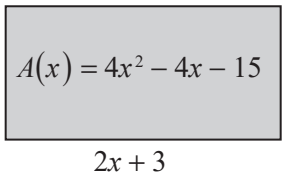
a. Write a rational expression that represents the total price per person for renting the party room, where x is the number of people attending.

b. What is the price per person to rent the party room if 10 people are attending?

c. Determine which values of the variable will make the rational expression from Part **a.** undefined.

d. Considering the context of the given problem, are there any additional restrictions on the variable? If so, explain why these restrictions are in place.

12. ***Rectangles:*** The area of a rectangle (in square feet) is represented by the polynomial function $A(x) = 4x^2 - 4x - 15$. If the length of the rectangle is $(2x + 3)$ feet, find a representation for the width.



Writing & Thinking

13. a. Define the term rational expression.

b. Give an example of a rational expression that is undefined for $x = -2$ and $x = 3$ and has a value of 0 for $x = 1$. Explain how you determined this expression.

c. Give an example of a rational expression that is undefined for $x = -5$ and never has a value of 0. Explain how you determined this expression.