From here, we can see that the vertices of the feasible region may be represented by the solutions to the following systems of equations.

$$\begin{cases} y = 4000 \\ x + y \le 19000 \end{cases}, \begin{cases} y = 0 \\ x + y = 19000 \end{cases}, \begin{cases} x = 8000 \\ y = 0 \end{cases}, \begin{cases} x = 8000 \\ y = 4000 \end{cases}$$

Solving each of these systems, we find that the vertices are (15000,4000), (19000,0), (8000,0), and (8000,4000). We know that one of these vertices must be the point that gives the maximum value. Substituting each of these ordered pairs into the objective function, $f(x,y) = 0.04x + 0.05y$, we find the following.

$$\begin{aligned} f(15000,4000) &= 800 \\ f(19000,0) &= 760 \\ f(8000,0) &= 320 \\ f(8000,4000) &= 520 \end{aligned}$$

We see that income will be maximized by investing \$15,000 in municipal bonds and \$4000 in Treasury bills.

11.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When boundary lines are parallel, the system of linear inequalities has no solution.

2. If two half-planes overlap, that region is the union of the graphs.

3. Half-planes are the graphs of linear inequalities.

4. If the graphs of two linear inequalities have no intersection, then the system has no solution.

Practice

Solve the systems of two linear inequalities graphically.

5. $\begin{cases} y > 2 \\ x \geq -3 \end{cases}$

6. $\begin{cases} y > 3x + 1 \\ -3x + y < -1 \end{cases}$

7. $\begin{cases} 2x - 3y \geq 0 \\ 8x - 3y < 36 \end{cases}$

8. $\begin{cases} y > x - 4 \\ y < x + 2 \end{cases}$

Applications

Solve.

9. ***Fundraising:*** Robin is planning a charity ball to raise money for her favorite charity. There are two different ticket options. The VIP option includes dinner, dancing, and cocktails for \$150 per ticket. The regular option includes dancing and cocktails for \$75 per ticket. Robin wants to make at least \$14,000 in ticket sales. The ballroom that is being used for the charity event has a maximum capacity of 150 people.

a. Write two linear inequalities to describe the situation. Let the variable x represent the number of VIP tickets sold and let the variable y represent the number of regular tickets sold.

b. Graph the two linear inequalities on the same coordinate plane.

c. Describe the solution set for the situation.

d. Can Robin reach her sales goal if she only sells tickets for the regular option? Explain why or why not.

Writing & Thinking

10. Graph the inequalities and explain how you can tell that there is no solution.

$$\begin{cases} y \le 2x - 5 \\ y \ge 2x + 3 \end{cases}$$