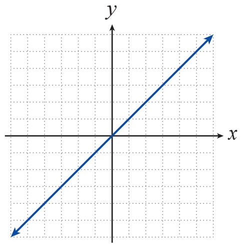
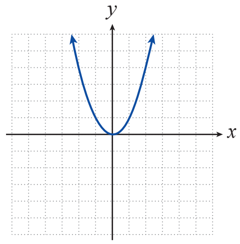
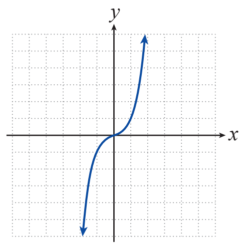
GRAPHS OF BASIC FUNCTIONS 3.4, 6.1, 6.3



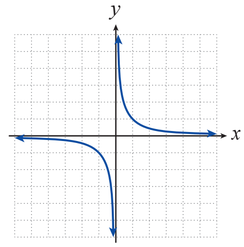
$f(x) = x$



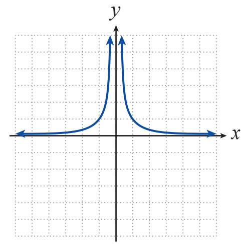
$f(x) = x^2$



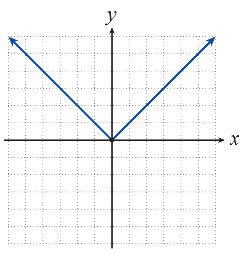
$f(x) = x^3$



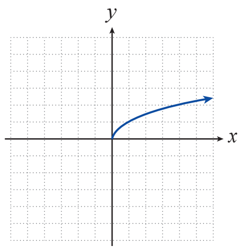
$f(x) = x^{-1} = \dfrac{1}{x}$



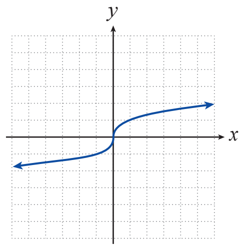
$f(x) = x^{-2} = \dfrac{1}{x^2}$



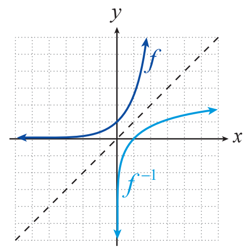
$f(x) = |x|$



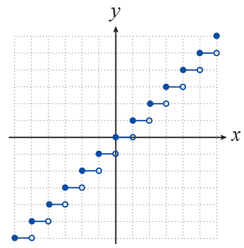
$f(x) = x^{\frac{1}{2}} = \sqrt{x}$



$f(x) = x^{\frac{1}{3}} = \sqrt[3]{x}$

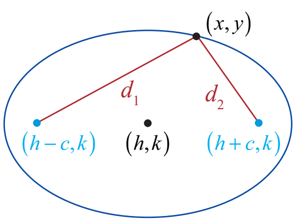


$f(x) = e^x$ and $f^{-1}(x) = \ln x$



$f(x) = [\![x]\!]$

CONIC SECTIONS 10.1, 10.2, 10.3



Ellipse

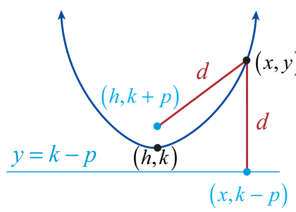
Let $a, b > 0$ with $a > b$. The standard form of the equation of an ellipse centered at (h, k) with major axis of length $2a$ and minor axis of length $2b$ is as follows.

- $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$
  (major axis is horizontal)
- $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$
  (major axis is vertical)

The foci are located on the major axis c units away from the center of the ellipse where $c^2 = a^2 - b^2$.

Eccentricity: $e = \dfrac{c}{a} = \dfrac{\sqrt{a^2 - b^2}}{a}$

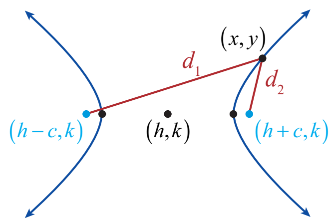
Note: If we let $a = b$, then $e = 0$ and the ellipse is a circle.



Parabola

Let p be a nonzero real number. The standard form of the equation of a parabola with vertex at (h, k) is as follows.

- $(x-h)^2 = 4p(y-k)$
  (vertically oriented)
  Focus: $(h, k+p)$
  Directrix: $y = k - p$
- $(y-k)^2 = 4p(x-h)$
  (horizontally oriented)
  Focus: $(h+p, k)$
  Directrix: $x = h - p$



Hyperbola

Let $a, b > 0$. The standard form of the equation of a hyperbola with center at (h, k) is as follows.

- $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$
  (foci are aligned horizontally)
  Asymptotes: $y - k = \pm\dfrac{b}{a}(x-h)$
- $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$
  (foci are aligned vertically)
  Asymptotes: $y - k = \pm\dfrac{a}{b}(x-h)$

The foci are located c units away from the center, where $c^2 = a^2 + b^2$.

The vertices are located a units away from the center.

GEOMETRIC FORMULAS

A = area, P = perimeter, C = circumference, SA = surface area, V = volume

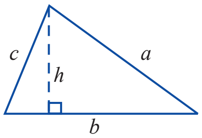
Triangle

$$A = \frac{1}{2}bh$$

Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

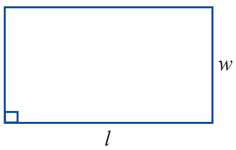
where $s = \dfrac{a+b+c}{2}$



Rectangle

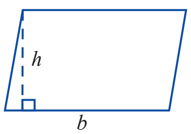
$A = lw$

$P = 2l + 2w$



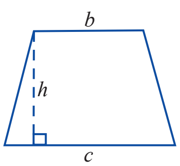
Parallelogram

$A = bh$



Trapezoid

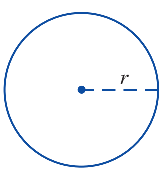
$$A = \frac{1}{2}h(b+c)$$



Circle

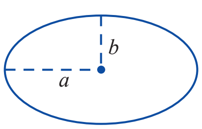
$A = \pi r^2$

$C = 2\pi r$



Ellipse

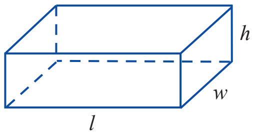
$A = \pi ab$



Rectangular Prism

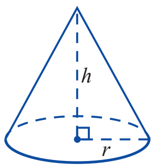
$V = lwh$

$SA = 2lh + 2wh + 2lw$



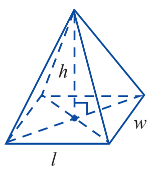
Circular Cone

$$V = \frac{1}{3}\pi r^2 h$$



Rectangular Pyramid

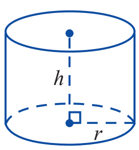
$$V = \frac{1}{3}lwh$$



Right Circular Cylinder

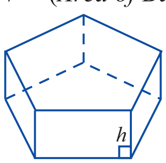
$V = \pi r^2 h$

$SA = 2\pi r^2 + 2\pi rh$



Right Cylinder

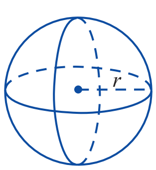
$V = (\text{Area of Base})h$



Sphere

$$V = \frac{4}{3}\pi r^3$$

$SA = 4\pi r^2$



PROPERTIES OF ABSOLUTE VALUE 1.1

For all real numbers a and b:

$|a| \geq 0$ $\qquad |-a| = |a|$

$a \leq |a|$ $\qquad |ab| = |a||b|$

$\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \; b \neq 0$

$|a+b| \leq |a| + |b|$ (the triangle inequality)

PROPERTIES OF EXPONENTS AND RADICALS 1.2

$a^n \cdot a^m = a^{n+m}$ $\qquad (a^n)^m = a^{nm}$ $\qquad (ab)^n = a^n b^n$

$\dfrac{a^n}{a^m} = a^{n-m}$ $\qquad a^{-n} = \dfrac{1}{a^n}$ $\qquad \left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$ $\qquad \sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ $\qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\qquad \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

SPECIAL PRODUCT FORMULAS 1.3

$(A-B)(A+B) = A^2 - B^2$

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

FACTORING SPECIAL BINOMIALS 1.3

$A^2 - B^2 = (A-B)(A+B)$

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

QUADRATIC FORMULA 1.8

The solutions of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

PYTHAGOREAN THEOREM 2.1

Given a right triangle with legs a and b and hypotenuse c,

$$a^2 + b^2 = c^2.$$

DISTANCE FORMULA 2.1

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

MIDPOINT FORMULA 2.1

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

STANDARD FORM OF THE EQUATION OF A CIRCLE 2.2

The standard form of the equation of a circle with radius r and center (h,k) is

$$(x-h)^2 + (y-k)^2 = r^2.$$

SLOPE OF A LINE 2.4

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Horizontal lines $y = c$ have a slope of 0.

Vertical lines $x = c$ have an undefined slope.

FORMS OF LINEAR EQUATIONS 2.4

Standard form: $ax + by = c$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

PARALLEL AND PERPENDICULAR LINES 2.5

Given a line with slope m:

slope of parallel line $= m$

slope of perpendicular line $= -\frac{1}{m}$

TRANSFORMATIONS OF FUNCTIONS 4.1

Horizontal Shifting

The graph of $g(x) = f(x - h)$ has the same shape as the graph of f, but shifted h units to the right if $h > 0$ and shifted h units to the left if $h < 0$.

Vertical Shifting

The graph of $g(x) = f(x) + k$ has the same shape as the graph of f, but shifted upward k units if $k > 0$ and downward k units if $k < 0$.

Reflecting with Respect to Axes

- The graph of the function $g(x) = -f(x)$ is the reflection of the graph of f with respect to the x-axis.
- The graph of the function $g(x) = f(-x)$ is the reflection of the graph of f with respect to the y-axis.

Stretching and Compressing

- The graph of the function $g(x) = af(x)$ is stretched vertically compared to the graph of f by a factor of a if $a > 1$.
- The graph of the function $g(x) = af(x)$ is compressed vertically compared to the graph of f by a factor of a if $0 < a < 1$.
- The graph of the function $g(x) = f(ax)$ is stretched horizontally compared to the graph of f by a factor of $\frac{1}{a}$ if $0 < a < 1$.
- The graph of the function $g(x) = f(ax)$ is compressed horizontally compared to the graph of f by a factor of $\frac{1}{a}$ if $a > 1$.

OPERATIONS WITH FUNCTIONS 4.3

Let f and g be functions.

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ when } g(x) \neq 0$$

$$(f \circ g)(x) = f(g(x))$$

RATIONAL ZERO THEOREM 5.3

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial with integer coefficients with $a_n \neq 0$, then any rational zero of f must be of the form $\frac{p}{q}$, where p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

FUNDAMENTAL THEOREM OF ALGEBRA 5.4

If p is a polynomial of degree n, with $n \geq 1$, then p has at least one zero. That is, the equation $p(x) = 0$ has at least one solution. (**Note:** The solution may be a nonreal complex number.)

COMPOUND INTEREST 6.2

An investment of P dollars, compounded n times per year at an annual interest rate of r, has a value after t years of

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}.$$

An investment compounded continuously has an accumulated value of $A(t) = Pe^{rt}$.

PROPERTIES OF LOGARITHMS 6.3, 6.4

For $a, x, y > 0$, $a \neq 1$, and any real number r:

$\log_a x = y$ and $x = a^y$ are equivalent

$\log_a 1 = 0$ $\qquad$ $\log_a a = 1$

$\log_a(a^x) = x$ $\qquad$ $a^{\log_a x} = x$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a(x^r) = r\log_a x$

CHANGE OF BASE FORMULA 6.4

For $a, b, x > 0$ and $a, b \neq 1$:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

DETERMINANT OF A 2×2 MATRIX 11.3

The determinant of a 2×2 matrix $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by the formula

$$|A| = a_{11}a_{22} - a_{21}a_{12}.$$

DETERMINANT OF AN *n*×*n* MATRIX 11.3

The minor of the element a_{ij} is the determinant of the $(n-1)\times(n-1)$ matrix formed from A by deleting the i^{th} row and the j^{th} column.

The cofactor of the element a_{ij} is $(-1)^{i+j}$ times the minor of a_{ij}. Find the determinant of an $n\times n$ matrix by expanding along a fixed row or column.

- To expand along the i^{th} row, each element of that row is multiplied by its cofactor and the n products are then added.
- To expand along the j^{th} column, each element of that column is multiplied by its cofactor and the n products are then added.

CRAMER'S RULE 11.3

A system of n linear equations in the n variables $x_1, x_2, \ldots, x_n$ can be written in the following form.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \qquad \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

The solution of the system is given by the n formulas

$$x_1 = \frac{D_{x_1}}{D}, x_2 = \frac{D_{x_2}}{D}, \ldots, x_n = \frac{D_{x_n}}{D},$$

where D is the determinant of the coefficient matrix and D_{x_i} is the determinant of the same matrix with the i^{th} column of constants replaced by the column of constants $b_1, b_2, \ldots, b_n$.

MATRIX ADDITION 11.4

$A+B =$ the matrix such that $c_{ij} = a_{ij} + b_{ij}$ (c_{ij} is the element in the i^{th} row and j^{th} column of $A+B$).

SCALAR MULTIPLICATION 11.4

$cA =$ the matrix such that the element in the i^{th} row and j^{th} column is equal to ca_{ij}.

MATRIX MULTIPLICATION 11.4

$AB =$ the matrix such that $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$. (The length of each row in A must be the same as the length of each column of B.)

PROPERTIES OF SIGMA NOTATION 12.1

For sequences $\{a_n\}$ and $\{b_n\}$ and a constant c:

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} a_i = \sum_{i=1}^{k} a_i + \sum_{i=k+1}^{n} a_i \text{ (for any } 1 \le k \le n-1)$$

SUMMATION FORMULAS 12.1

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

SEQUENCES AND SERIES

Arithmetic 12.2

Let $\{a_n\}$ be an arithmetic sequence with common difference d.

General term: $a_n = a_1 + (n-1)d$

Partial sum: $S_n = na_1 + d\left(\frac{(n-1)n}{2}\right) = \left(\frac{n}{2}\right)(a_1 + a_n)$

Geometric 12.3

Let $\{a_n\}$ be a geometric sequence with common ratio r.

General term: $a_n = a_1 r^{n-1}$

Partial sum: $S_n = \frac{a_1(1-r^n)}{1-r}$, if $r \ne 0, 1$

Infinite sum: $S = \sum_{n=0}^{\infty} a_1 r^n = \frac{a_1}{1-r}$, if $|r| < 1$

PERMUTATION FORMULA 12.5

$$_nP_k = \frac{n!}{(n-k)!}$$

COMBINATION FORMULA 12.5

$$_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

BINOMIAL THEOREM 12.5

$$(A+B)^n = \sum_{k=0}^{n} \binom{n}{k} A^k B^{n-k}$$

MULTINOMIAL COEFFICIENTS 12.5

$$\binom{n}{k_1, k_2, \ldots, k_r} = \frac{n!}{k_1! k_2! \cdots k_r!}$$

MULTINOMIAL THEOREM 12.5

$$(A_1 + A_2 + \cdots + A_r)^n = \sum_{k_1 + k_2 + \cdots + k_r = n} \binom{n}{k_1, k_2, \ldots, k_r} A_1^{k_1} A_2^{k_2} \cdots A_r^{k_r}$$

COMPLEX NUMBERS AND DE MOIVRE'S THEOREM 9.5

$z = a + bi$ $|z| = \sqrt{a^2 + b^2}$ $\tan\theta = \frac{b}{a}$

$z = |z|(\cos\theta + i\sin\theta) = |z|e^{i\theta}$

$z^n = |z|^n(\cos(n\theta) + i\sin(n\theta)) = |z|^n e^{in\theta}$

Distinct n^{th} roots of z for $k = 0, 1, \ldots, n-1$:

$$w_k = |z|^{\frac{1}{n}}\left[\cos\left(\frac{\theta + 2k\pi}{n}\right) + i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right] = |z|^{\frac{1}{n}} e^{i\left(\frac{\theta + 2k\pi}{n}\right)}$$

VECTOR OPERATIONS AND MAGNITUDE 9.6

Given two vectors $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ and a scalar a, $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$, $a\mathbf{u} = \langle au_1, au_2 \rangle$, and $\|\mathbf{u}\| = \sqrt{u_1^{\,2} + u_2^{\,2}}$.

PROPERTIES OF VECTOR OPERATIONS 9.6

For vectors **u**, **v**, and **w** and scalars a and b:

Vector Addition	Scalar Multiplication
$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$
$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$	$(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$
$\mathbf{u} + \mathbf{0} = \mathbf{u}$	$(ab)\mathbf{u} = a(b\mathbf{u}) = b(a\mathbf{u})$
$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$	$1\mathbf{u} = \mathbf{u}$, $0\mathbf{u} = \mathbf{0}$, and $a\mathbf{0} = \mathbf{0}$
	$\|a\mathbf{u}\| = \|a\|\|\mathbf{u}\|$

DOT PRODUCT 9.7

Given two vectors $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$,

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2.$$

ELEMENTARY PROPERTIES OF THE DOT PRODUCT 9.7

Given vectors **u**, **v**, and **w** and a scalar a:

$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ $\mathbf{0} \cdot \mathbf{u} = 0$

$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$ $a(\mathbf{u} \cdot \mathbf{v}) = (a\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (a\mathbf{v})$

$\mathbf{u} \cdot \mathbf{u} = \|\mathbf{u}\|^2$

THE DOT PRODUCT THEOREM 9.7

Let θ be the smaller of the two angles formed by nonzero vectors **u** and **v** (so $0 \le \theta \le \pi$). Then $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta$.

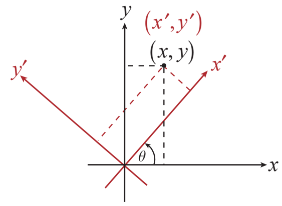
ORTHOGONAL VECTORS 9.7

Two nonzero vectors **u** and **v** are said to be orthogonal (or perpendicular) if $\mathbf{u} \cdot \mathbf{v} = 0$.

PROJECTION OF u ONTO v 9.7

Let **u** and **v** be nonzero vectors. The projection of **u** onto **v** is the vector $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$.

ROTATION OF AXES 10.4



ROTATION RELATIONS 10.4

$x = x'\cos\theta - y'\sin\theta$ $x' = x\cos\theta + y\sin\theta$

$y = x'\sin\theta + y'\cos\theta$ $y' = -x\sin\theta + y\cos\theta$

ELIMINATION OF THE *xy*-TERM 10.4

The graph of the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ in the xy-plane is the same as the graph of the equation $A'x'^2 + C'y'^2 + D'x' + E'y' + F' = 0$ in the $x'y'$-plane, where the angle of rotation θ between the two coordinate systems satisfies $\cot(2\theta) = \frac{A - C}{B}$.

CLASSIFYING CONICS 10.4

Assuming the graph of the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is a nondegenerate conic section, it is classified by its discriminant as follows:

- Ellipse if $B^2 - 4AC < 0$
- Parabola if $B^2 - 4AC = 0$
- Hyperbola if $B^2 - 4AC > 0$

POLAR EQUATIONS OF CONIC SECTIONS 10.5

A conic section consists of all points P in the plane that satisfy the equation

$$\frac{D(P, F)}{D(P, L)} = e,$$

where e is a fixed positive constant. The conic is

- an ellipse if $0 < e < 1$,
- a parabola if $e = 1$, and
- a hyperbola if $e > 1$.

Equation of Conic Section	Directrix
$r = \frac{ed}{1 + e\cos\theta}$	Vertical directrix $x = d$
$r = \frac{ed}{1 - e\cos\theta}$	Vertical directrix $x = -d$
$r = \frac{ed}{1 + e\sin\theta}$	Horizontal directrix $y = d$
$r = \frac{ed}{1 - e\sin\theta}$	Horizontal directrix $y = -d$

RECIPROCAL IDENTITIES 8.1

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

COFUNCTION IDENTITIES 8.1

$$\cos x = \sin\left(\frac{\pi}{2} - x\right) \qquad \sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$\csc x = \sec\left(\frac{\pi}{2} - x\right) \qquad \sec x = \csc\left(\frac{\pi}{2} - x\right)$$

$$\cot x = \sin\left(\frac{\pi}{2} - x\right) \qquad \tan x = \cot\left(\frac{\pi}{2} - x\right)$$

QUOTIENT IDENTITIES 8.1

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

PERIOD IDENTITIES 8.1

$$\sin(x + 2\pi) = \sin x \qquad \csc(x + 2\pi) = \csc x$$

$$\cos(x + 2\pi) = \cos x \qquad \sec(x + 2\pi) = \sec x$$

$$\tan(x + \pi) = \tan x \qquad \cot(x + \pi) = \cot x$$

EVEN/ODD IDENTITIES 8.1

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \quad \sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

PYTHAGOREAN IDENTITIES 8.1

$$\sin^2 x + \cos^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

SUM AND DIFFERENCE IDENTITIES 8.2

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

DOUBLE-ANGLE IDENTITIES 8.3

$$\sin(2u) = 2\sin u \cos u$$

$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

$$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

POWER-REDUCING IDENTITIES 8.3

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

HALF-ANGLE IDENTITIES 8.3

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}} \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

PRODUCT-TO-SUM IDENTITIES 8.3

$$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$$

$$\cos x \sin y = \frac{1}{2}[\sin(x + y) - \sin(x - y)]$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$$

SUM-TO-PRODUCT IDENTITIES 8.3

$$\sin x + \sin y = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

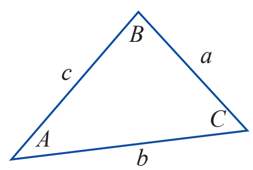
$$\cos x + \cos y = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

LAWS OF SINES AND COSINES

Law of Sines 9.1

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Law of Cosines 9.2

$$a^2 = b^2 + c^2 - 2bc\cos A$$

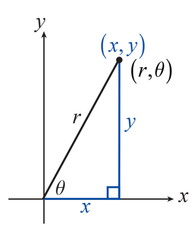
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a Triangle (Sine Formula) 9.1

$$\text{Area} = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$$

POLAR COORDINATES 9.3



$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x} \; (x \neq 0)$$

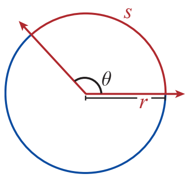
CONVERTING BETWEEN RADIAN AND DEGREE MEASURE 7.1

$$180° = \pi \text{ rad}$$

$$1° = \frac{\pi}{180} \text{ rad} \qquad \left(\frac{180}{\pi}\right)^{\circ} = 1 \text{ rad}$$

$$x° = x\left(\frac{\pi}{180}\right) \text{ rad} \qquad x \text{ rad} = x\left(\frac{180}{\pi}\right)^{\circ}$$

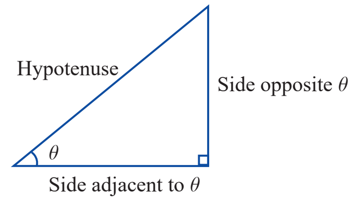
ARC LENGTH, AREA OF A SECTOR, ANGULAR SPEED, AND LINEAR SPEED 7.1



$$s = \left(\frac{\theta}{2\pi}\right)(2\pi r) = r\theta \qquad A = \left(\frac{\theta}{2\pi}\right)(\pi r^2) = \frac{r^2\theta}{2}$$

$$\omega = \frac{\theta}{t} \qquad v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$

TRIGONOMETRIC FUNCTIONS AND RIGHT TRIANGLES 7.2



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$$

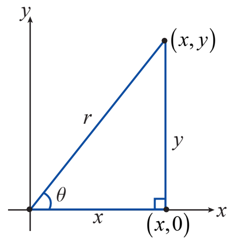
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$$

TRIGONOMETRIC FUNCTIONS OF ARBITRARY ANGLES 7.3

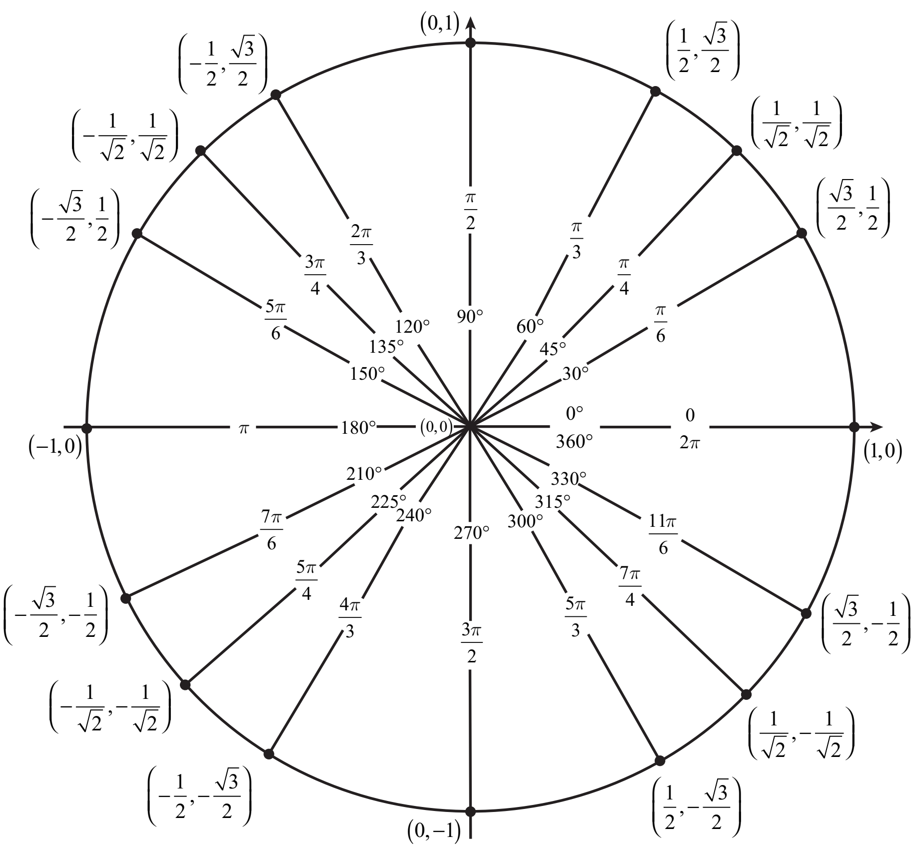
$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y} \ (\text{for } y \neq 0)$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x} \ (\text{for } x \neq 0)$$

$$\tan\theta = \frac{y}{x} \ (\text{for } x \neq 0) \qquad \cot\theta = \frac{x}{y} \ (\text{for } y \neq 0)$$

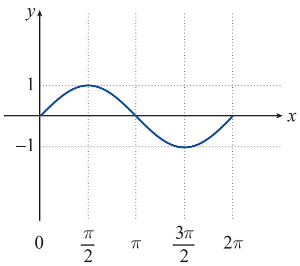


TRIGONOMETRIC FUNCTIONS AND THE UNIT CIRCLE 7.3



Each ordered pair represents $(\cos\theta, \sin\theta)$, and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

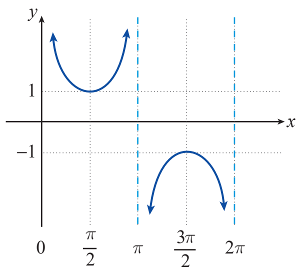
GRAPHS OF TRIGONOMETRIC FUNCTIONS 7.4, 7.5



$y = \sin x$

Domain: $(-\infty, \infty)$

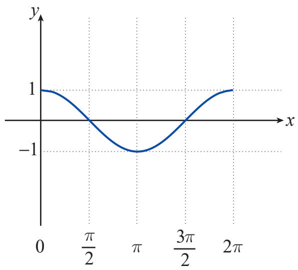
Range: $[-1, 1]$



$y = \csc x$

Domain: $\{x \mid x \neq n\pi, n \in \mathbb{Z}\}$

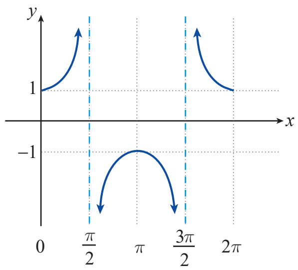
Range: $(-\infty, -1] \cup [1, \infty)$



$y = \cos x$

Domain: $(-\infty, \infty)$

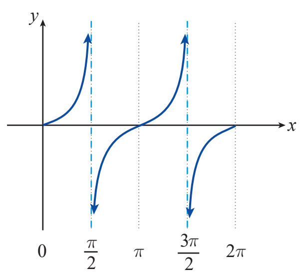
Range: $[-1, 1]$



$y = \sec x$

Domain: $\left\{x \mid x \neq (2n+1)\dfrac{\pi}{2}, n \in \mathbb{Z}\right\}$

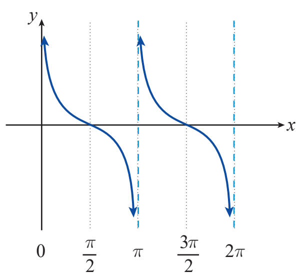
Range: $(-\infty, -1] \cup [1, \infty)$



$y = \tan x$

Domain: $\left\{x \mid x \neq (2n+1)\dfrac{\pi}{2}, n \in \mathbb{Z}\right\}$

Range: $(-\infty, \infty)$



$y = \cot x$

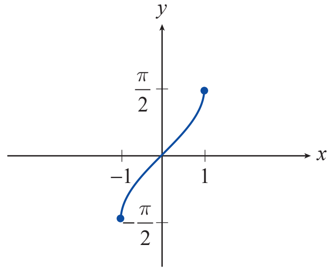
Domain: $\{x \mid x \neq n\pi, n \in \mathbb{Z}\}$

Range: $(-\infty, \infty)$

AMPLITUDE, PERIOD, AND PHASE SHIFT 7.4

Given constants a, b (such that $b > 0$), and c, the functions $f(x) = a\sin(bx - c)$ and $g(x) = a\cos(bx - c)$ have amplitude $|a|$, period $\dfrac{2\pi}{b}$, and a phase shift of $\dfrac{c}{b}$. The left endpoint of one cycle of either function is $\dfrac{c}{b}$ and the right endpoint is $\dfrac{c}{b} + \dfrac{2\pi}{b}$.

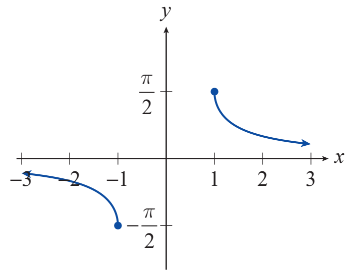
GRAPHS OF INVERSE TRIGONOMETRIC FUNCTIONS 7.6



$y = \sin^{-1} x$

Domain: $[-1, 1]$

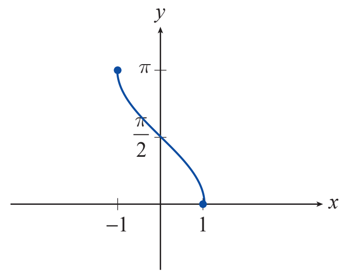
Range: $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$



$y = \csc^{-1} x$

Domain: $(-\infty, -1] \cup [1, \infty)$

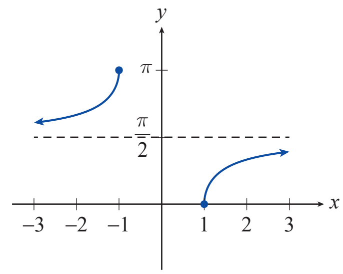
Range: $\left[-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$



$y = \cos^{-1} x$

Domain: $[-1, 1]$

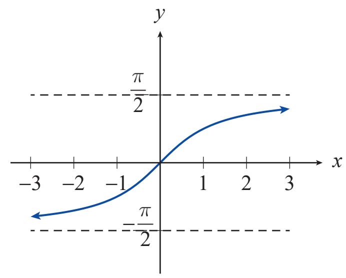
Range: $[0, \pi]$



$y = \sec^{-1} x$

Domain: $(-\infty, -1] \cup [1, \infty)$

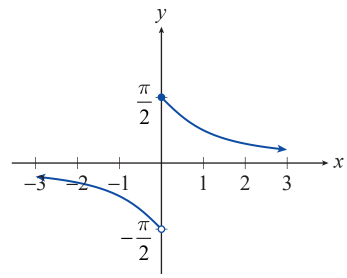
Range: $\left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$



$y = \tan^{-1} x$

Domain: $(-\infty, \infty)$

Range: $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$



$y = \cot^{-1} x$

Domain: $(-\infty, \infty)$

Range: $\left(-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$

HYPERBOLIC FUNCTIONS 9.8

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} \qquad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \coth x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$