9.7 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A rectangular array of numbers is called a/an _____ (plural _____).
2. In a matrix, entries written horizontally form a/an _____ and entries written vertically form a/an _____.
3. The _____ of a matrix is the number of rows by the number of columns.
4. A matrix that is made up of the coefficients of the variables of a system of linear equations is called a/an_____ matrix.
5. A matrix that is made up of the coefficients of the variables and the constant terms of a system of linear equations is called a/an _____ matrix.
6. Interchanging equations, multiplying an equation by a constant, and adding like terms of two equations are known as _____ operations.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so that the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A matrix that has 3 rows and 5 columns is a 5×3 matrix.
8. A matrix with the same number of rows as columns, such as a 3×3 matrix, is called a square matrix.
9. Interchanging two equations in a system of linear equations will change the solution of the system.
10. In a system of linear equations, adding like terms of one equation to another equation will not change the solution of the system.

Practice

Write the coefficient matrix and the augmented matrix for the given systems of linear equations. See Examples 1 and 2.

1. $\begin{cases} 2x+2y=13 \\ 5x-y=10 \end{cases}$
2. $\begin{cases} x+4y=-1 \\ 2x-3y=7 \end{cases}$
3. $\begin{cases} 7x-2y+7z=2 \\ -5x+3y=2 \\ 4y+11z=8 \end{cases}$
4. $\begin{cases} -8x+2y-z=6 \\ 2x+3z=-3 \\ -4x-2y+5z=13 \end{cases}$
5. $\begin{cases} 3w+x-y+2z=6 \\ w-x+2y-z=-8 \\ 2x+5y+z=2 \\ w+3x+3z=14 \end{cases}$
6. $\begin{cases} 4w+x+3y-2z=13 \\ w-2x+y-4z=-3 \\ w+x+4y+2z=12 \\ -2w+3x-y-3z=5 \end{cases}$

Write the system of linear equations represented by each of the augmented matrices. Use x, y, and z as the variables.

7. $\left[\begin{array}{cc|c} -3 & 5 & 1 \\ -1 & 3 & 2 \end{array}\right]$

8. $\left[\begin{array}{cc|c} 3 & -1 & 5 \\ -2 & 10 & 9 \end{array}\right]$

9. $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 1 \\ 2 & -3 & -2 & 0 \\ 1 & 1 & 0 & -4 \end{array}\right]$

10. $\left[\begin{array}{ccc|c} 2 & -9 & 14 & 0 \\ -3 & 0 & -8 & 5 \\ 2 & -6 & 1 & 3 \end{array}\right]$

Perform the indicated row operations on the given matrix. See Examples 1 and 2.

11. $\begin{bmatrix} 1 & 4 \\ -1 & 7 \end{bmatrix}$

a. Interchange rows 1 and 2

b. Multiply row 2 by –2

12. $\left[\begin{array}{cc|c} 3 & -2 & 8 \\ 1 & 5 & 9 \end{array}\right]$

a. Multiply row 1 by $\frac{1}{2}$

b. Add –3 times row 2 to row 1

13. $\begin{bmatrix} 1 & 3 & 7 \\ -8 & -2 & 5 \\ 4 & -1 & 6 \end{bmatrix}$

a. Interchange rows 2 and 3

b. Add –2 times row 3 to row 2

14. $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 7 \\ 0 & -2 & -1 & 9 \\ 0 & 1 & 1 & 4 \end{array}\right]$

a. Interchange rows 2 and 3

b. Using your answer from part a., add 2 times row 2 to row 3.

Use the Gaussian elimination method to solve the given system of linear equations. See Examples 3 and 4.

15. $\begin{cases} x+2y=3 \\ 2x-y=-4 \end{cases}$

16. $\begin{cases} 4x+3y=5 \\ -x-2y=0 \end{cases}$

17. $\begin{cases} -8x+2y=6 \\ x-2y=1 \end{cases}$

18. $\begin{cases} 2x+y=-2 \\ 4x+3y=-2 \end{cases}$

19. $\begin{cases} x-3y+2z=11 \\ -2x+4y+z=-3 \\ x-2y+3z=12 \end{cases}$

20. $\begin{cases} x+2y-z=6 \\ x+3y-3z=3 \\ x+y+z=6 \end{cases}$

21. $\begin{cases} x+2y+3z=4 \\ x-y-z=0 \\ 4x-3y+z=5 \end{cases}$

22. $\begin{cases} x+y-2z=-1 \\ 3x+4y-2z=0 \\ x-y+z=4 \end{cases}$

23. $\begin{cases} x-y-2z=3 \\ x+2y-z=5 \\ 2x-3y-2z=3 \end{cases}$

24. $\begin{cases} x+y+3z=2 \\ 2x-y+z=1 \\ 4x+y+7z=5 \end{cases}$

25. $\begin{cases} x-y+5z=-6 \\ x+2z=0 \\ 6x+y+3z=0 \end{cases}$

26. $\begin{cases} x-3y-z=-4 \\ 3x-2y+z=1 \\ -2x+y+2z=13 \end{cases}$

27. $\begin{cases} x-y+2z=5 \\ 2x-2y+4z=5 \\ 3x-3y+6z=8 \end{cases}$

28. $\begin{cases} 2x-y-5z=-9 \\ x-3y+2z=0 \\ 3x+2y+10z=4 \end{cases}$

29. $\begin{cases} x-5y+2z=-7 \\ x+y+4z=7 \\ 2x-y+7z=7 \end{cases}$

30. $\begin{cases} 2x-y+5z=-2 \\ 4x+y+3z=-2 \\ x+3y-z=6 \end{cases}$

31. $\begin{cases} 3x+4z=11 \\ x+y=-2 \\ 2y+z=-4 \end{cases}$

32. $\begin{cases} y+z=2 \\ x+y=5 \\ x+z=5 \end{cases}$

Use your graphing calculator to solve the systems of linear equations. See Example 5.

33. $\begin{cases} x+y=-4 \\ 2x+3y=-12 \end{cases}$

34. $\begin{cases} 2x+3y=1 \\ x-5y=-19 \end{cases}$

35. $\begin{cases} x+y+z=10 \\ 2x-y+z=10 \\ -x+2y+2z=14 \end{cases}$

36. $\begin{cases} 2x+y+2z=2 \\ x-y+4z=1 \\ 3x-y+z=-4 \end{cases}$

37. $\begin{cases} 2y-3z=-18 \\ 4x+5y=-7 \\ 6x-z=8 \end{cases}$

38. $\begin{cases} w+x+y+z=0 \\ w+x-y-z=-2 \\ -w+3x+3y-z=11 \\ y-2z=6 \end{cases}$

39. $\begin{cases} x-3y+z=0 \\ 2x+2y-z=2 \\ x+y+z=5 \end{cases}$

40. $\begin{cases} x-2y-2z=-13 \\ 2x+y-z=-5 \\ x+y+z=6 \end{cases}$

Applications

Set up a system of linear equations that represents the information and solve the system using Gaussian elimination. See Example 4.

41. The sum of three integers is 169. The first integer is twelve more than the second integer. The third integer is fifteen less than the sum of the first and second integers. What are the integers?

42. A pizzeria sells three sizes of pizzas: small, medium, and large. The pizzas sell for \$6.00, \$8.00, and \$9.50, respectively. One evening they sold 68 pizzas for a total of \$528.00. If they sold twice as many medium-sized pizzas as large-sized pizzas, how many of each size did they sell?

43. Caroline bought a pound of bacon, a dozen eggs, and a loaf of bread. The total cost was \$8.52. The eggs cost \$0.94 more than the bacon. The combined cost of the bread and eggs was \$2.34 more than the cost of the bacon. Find the cost of each item.

44. An investment firm is responsible for investing \$250,000 from an estate according to three conditions in the will of the deceased. The money is to be invested in three accounts paying 6%, 8%, and 11% interest. The amount invested in the 6% account is to be \$5000 more than the total invested in the other two accounts, and the total annual interest for the first year is to be \$19,250. How much is the firm supposed to invest in each account?

45. Maurice is looking at his options for colleges. He can only borrow \$13,125 a year for tuition. The community college in town does not offer every class he needs, and he cannot afford to attend the university full time. The university charges \$2500 per credit hour (u) and the community college charges \$625 per credit hour (c). In order to finish his degree on time, Maurice must take 12 credit hours per semester for both fall and spring semesters, but not for the summer semester.

a. Write the system of equations that represents this situation.

b. Write the system as a matrix.

c. How many hours at each school can he take?

46. You and 14 of your friends are planning a spring-break trip. There are three concerts happening in different locations all on the same night, and everyone wants to attend one concert. The group has \$617 to spend on the 15 tickets and \$195 to spend on transportation to the concerts. For concert x, tickets cost \$46 and transportation per person is \$15. For concert y, tickets cost \$35 and transportation per person is \$12. For concert z, tickets are \$40 and transportation per person is \$10.
Consider the following system of equations.

$$\begin{aligned} x + \quad y + \quad z &= 15 \\ 46x + 35y + 40z &= 617 \\ 15x + 12y + 10z &= 195 \end{aligned}$$

a. What is the dimension of the augmented matrix?

b. Set up augmented matrix.

c. Solve the system of equations using Gaussian elimination.

Writing & Thinking

47. Suppose that Gaussian elimination with a system of three linear equations in three unknowns results in the following triangular matrix. Discuss how you can use back substitution to find that the system has an infinite number of solutions and these solutions satisfy the equation $x + 5y = 6$. (**Hint:** Solve the second equation for z.)

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$