E Finding Equations of Lines Given the Slope and the *y*-Intercept

7. Find the equation of the line through the point $(0, -3)$ with a slope of $\frac{2}{3}$.

Example 7 Finding Equations Given the Slope and the *y*-Intercept

Find the equation of the line through the point $(0, -2)$ with slope $\frac{1}{2}$.

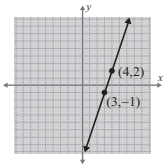
Solution

Because the x-coordinate is 0, we know that the point $(0, -2)$ is the y-intercept. So $b = -2$. The slope is $\frac{1}{2}$. So $m = \frac{1}{2}$. Substituting in slope-intercept form, $y = mx + b$, gives the result $y = \frac{1}{2}x - 2$.

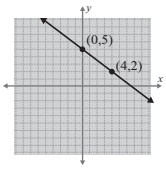
Now work margin exercise 7.

Margin Exercise Answers

1. slope = 3



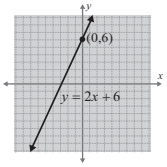
2. slope = $-\frac{3}{4}$



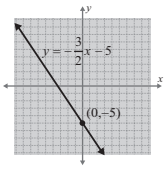
3. $y = -2$; slope is 0

4. $x = 2$; slope is undefined

5. $m = 2$; y-intercept = $(0, 6)$



6. $m = -\frac{3}{2}$; y-intercept = $(0, -5)$



7. $y = \frac{2}{3}x - 3$

8.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The slope of a line is the ratio of rise to _____.

2. Another name for slope is the rate of _____.

3. A line that rises (increases) from left to right has a/an _____ slope.

4. The slope of every vertical line is _____.

5. The slope of every horizontal line is _____.

6. In the equation $y = mx + b$, m represents the _____ and $(0, b)$ represents the _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If the y-intercept and the slope of a line are given, there is enough information to write the equation of the line.

8. When using the slope formula, the slope of a line changes if the order of the points is reversed.

9. A line that falls (decreases) from left to right has a negative slope.

10. The line that represents the equation $y = 2x + 4$ has a y-intercept of $(0, 4)$.

Practice

Find the slope of the line determined by each pair of points. See Examples 1 and 2.

1. $(2, 4); (1, -1)$

2. $(1, -2); (1, 4)$

3. $(-6, 3); (1, 2)$

4. $(-3, 7); (4, -1)$

5. $(-5, 8); (3, 8)$

6. $(-2, 3); (-2, -1)$

7. $(5, 1); (3, 0)$

8. $(0, 0); (-2, -3)$

9. $\left(\frac{3}{4}, \frac{3}{2}\right); (1, 2)$

10. $\left(4, \frac{1}{2}\right); (-1, 2)$

11. $\left(\frac{3}{2}, \frac{4}{5}\right); \left(-2, \frac{1}{10}\right)$

12. $\left(\frac{7}{2}, \frac{3}{4}\right); \left(\frac{1}{2}, -3\right)$

Determine whether each equation represents a horizontal line or vertical line and give its slope. Graph the line. See Examples 3 and 4.

13. $y = 5$

14. $y = -2$

15. $x = -3$

16. $x = 1.7$

17. $3y = -18$

18. $4x = 2.4$

19. $-3x + 21 = 0$

20. $2y + 5 = 0$

Write each equation in slope-intercept form. Find the slope and y-intercept, and then use them to draw the graph. See Examples 5 and 6.

21. $y = 2x - 1$

22. $y = 3x - 4$

23. $y = 5 - 4x$

24. $y = 4 - x$

25. $y = \frac{2}{3}x - 3$

26. $y = \frac{2}{5}x + 2$

27. $x + y = 5$

28. $x - 2y = 6$

29. $x + 5y = 10$

30. $4x + y = 0$

31. $4x + y + 3 = 0$

32. $2x + 7y + 7 = 0$

33. $2y - 8 = 0$

34. $3y - 9 = 0$

35. $2x = 3y$

36. $4x = y$

37. $3x + 9 = 0$

38. $4x + 7 = 0$

39. $5x - 6y = 18$

40. $3x + 6 = 6y$

41. $5 - 3x = 4y$

42. $5x = 11 - 2y$

43. $6x + 4y = -8$

44. $7x + 2y = 4$

45. $6y = -6 + 3x$

46. $4x = 3y - 7$

47. $5x - 2y + 5 = 0$

48. $6x + 5y = -15$

In reference to the equation $y = mx + b$, sketch the graphs of three lines for each of the two characteristics listed below.

49. $m > 0$ and $b > 0$

50. $m < 0$ and $b > 0$

51. $m > 0$ and $b < 0$

52. $m < 0$ and $b < 0$

Find an equation in slope-intercept form for the line passing through the given point with the given slope. See Example 7.

53. $(0, 3); m = -\frac{1}{2}$

54. $(0, 2); m = \frac{1}{3}$

55. $(0, -3); m = \frac{2}{5}$

56. $(0, -6); m = \frac{4}{3}$

57. $(0, -5); m = 4$

58. $(0, 9); m = -1$

59. $(0, -4); m = 1$

60. $(0, 6); m = -5$

61. $(0, -3); m = -\frac{5}{6}$

62. $(0, -1); m = -\frac{3}{2}$

The graph of a line is shown with two points labeled. Find **a.** the slope, **b.** the *y*-intercept (if there is one), and **c.** the equation of the line in slope-intercept form.

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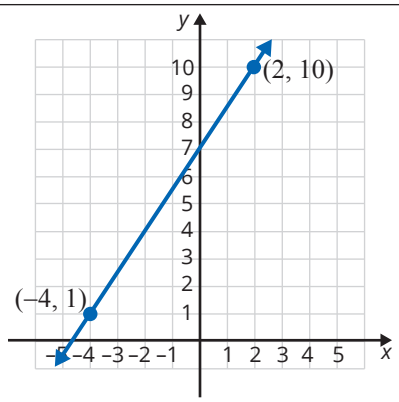

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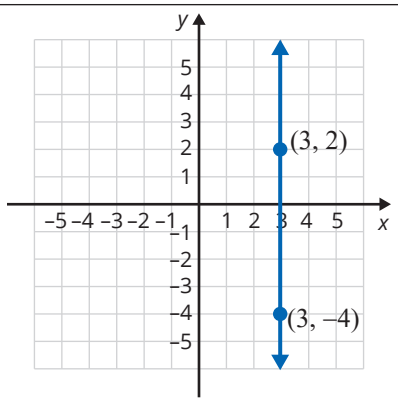

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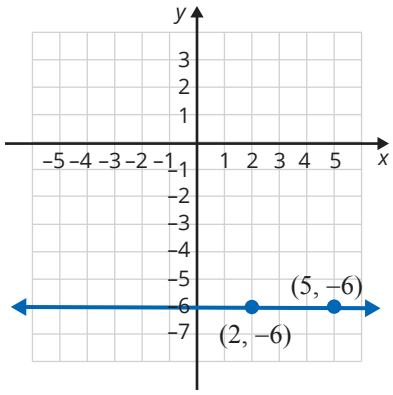

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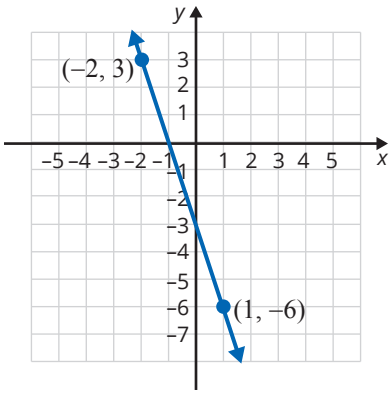

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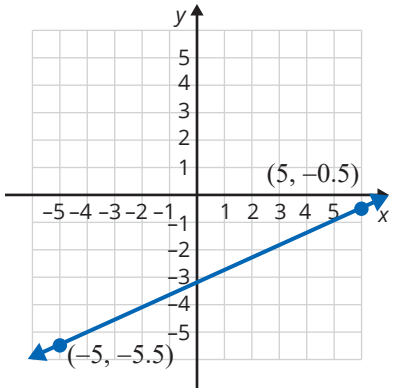

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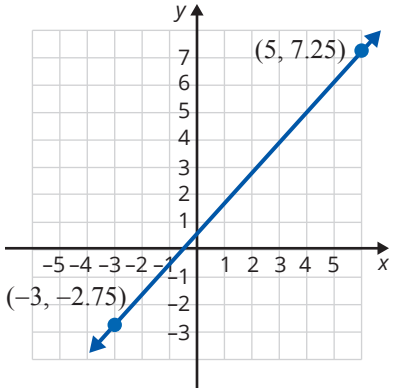

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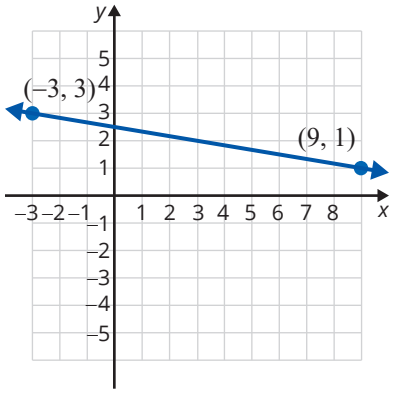

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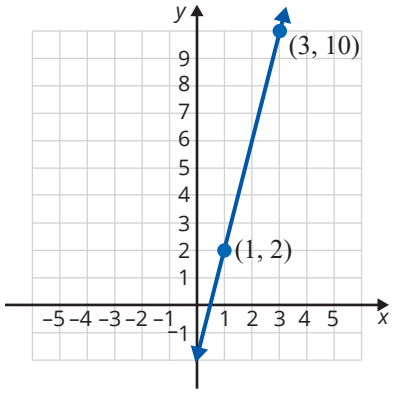


Points are said to be **collinear** if they lie on a straight line. If points are collinear, then the slope of the line through any two of them must be the same (because the line is the same line). Use this idea to determine whether the three points in each of the sets are collinear.

71. $\{(-1, 3), (0, 1), (5, -9)\}$

72. $\{(-2, -4), (0, 2), (3, 11)\}$

73. $\{(-2, 0), (0, 30), (1.5, 5.25)\}$

74. $\{(-1, -7), (1, 1), (2.5, 7)\}$

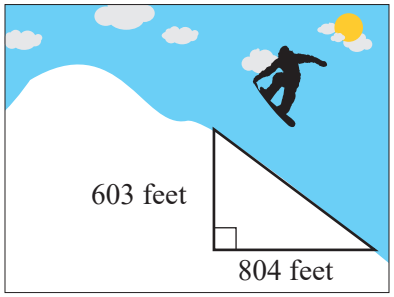
75. $\left\{\left(\frac{2}{3}, \frac{1}{2}\right), \left(0, \frac{5}{6}\right), \left(-\frac{3}{4}, \frac{29}{24}\right)\right\}$

76. $\left\{\left(\frac{3}{2}, -\frac{1}{3}\right), \left(0, \frac{1}{6}\right), \left(-\frac{1}{2}, \frac{3}{4}\right)\right\}$

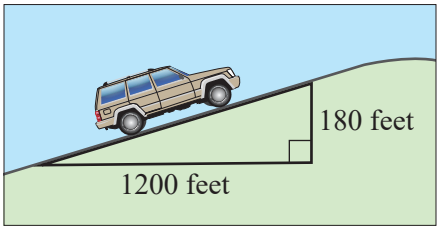
Applications

Solve.

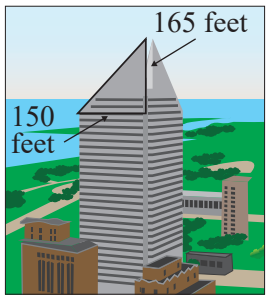
77. Find the slope of the ski slope.



78. Find the slope of the road.

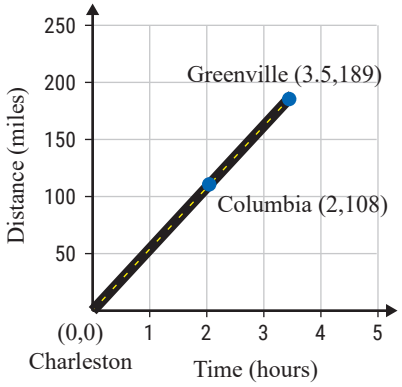


79. Find the slope of the roof of the skyscraper.

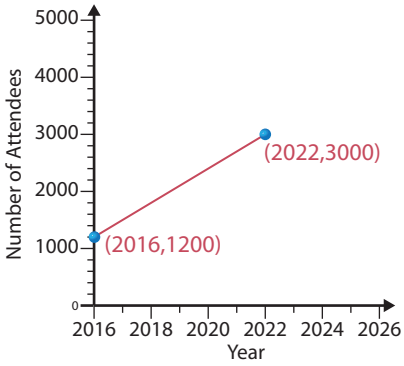


80. Find the slope of the larger sail on the sailboat.

81. A car travels from Charleston to Greenville. Its distance related to time traveled is given on the following graph. Find the average speed of the car in miles per hour from Columbia to Greenville.



82. The attendance at Smithville's Spring Festival has been increasing steadily as shown in the graph. Find the average increase in attendees per year. How many people do you predict will attend the festival in 2026?

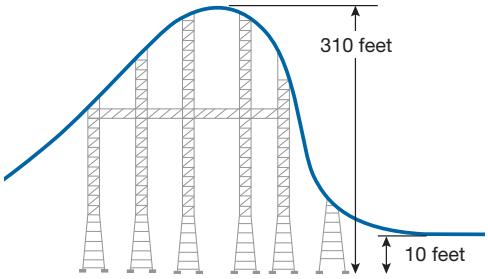


83. John bought his new car for $35,000 in the year 2022. He knows that the value of his car has depreciated linearly. If the value of the car in 2025 was $23,000, what was the annual rate of depreciation of his car? Show this information on a graph. (When graphing, use years as the x-coordinates and the corresponding values of the car as the y-coordinates.)

84. The number of people in the United States with cell phones was about 198 million in 2011 and about 232 million in 2016. If the growth in the usage of cell phones was linear, what was the approximate rate of growth per year from 2011 to 2016. Show this information on a graph. (When graphing, use years as the x-coordinates and the corresponding numbers of users as the y-coordinates.) [1]

1 Source: www.statista.com/statistics/231612/number-of-cell-phone-users-usa/

85. The Millennium Force roller coaster at Cedar Point in Sandusky, Ohio, has been voted the Best Steel Coaster by Golden Ticket eleven times since 2001. The Millennium Force is known for its steep slope on the first hill and speeds up to 93 miles per hour. The descent from the first hill starts at 310 feet above the ground and ends 10 feet above the ground. The descent runs approximately 53 feet. Find the slope of this hill to the nearest hundredth. (**Hint:** The slope is running downhill. Consider how that affects the slope.)



86. The grade, or slope, of a road is commonly given as a percentage. The grade can be determined by multiplying the slope by 100. Calculate the slope of each road or track and then determine its grade. Round each percent to the nearest hundredth if necessary.

a. A road increases in height 5 feet for every 120 feet of run.

b. The railway line with the steepest grade that does not run on a track system is the Lisbon tramway network in Portugal which has a section that increases 5 feet in height for every 37 feet of run.

c. A road on the Route des Crêtes (Route of the Ridges) in France has an elevation that increases in height 450 feet over 1500 feet.

87. Jared sells paintings at an open-air market. He starts his work day with $30 and sells each painting for $15. Jared wants to create a linear equation to model this situation where y is the amount of money Jared has at the end of the work day and x is the number of paintings sold.

a. The slope, or rate of change, is the increase in the amount of money Jared makes when he sells a painting. Determine the value of the slope and list the units for both variables.

b. The y-coordinate of the y-intercept of this equation is the amount of money Jared has before he sells any paintings. What is the y-intercept?

c. Write a linear equation in slope-intercept form to model this situation using the answers from parts a. and b.

d. Graph the equation from part c.

e. Are there any solutions to the equation which do not make sense in the context of the problem? Explain why.

f. Use the graph to determine the amount of money Jared will have after selling 4 paintings.

88. The given table shows the estimated number of internet users from 2018 through 2022. The number of users for each year is shown in millions.

a. Plot these points on a graph.

b. Connect the points with line segments.

c. Find the slope of each line segment.

d. Interpret each slope as a rate of change.

Year	Internet users (in millions)
2018	293
2019	312
2020	288
2021	299
2022	299

Source: www.statista.com/statistics/276445/number-of-internet-users-in-the-united-states

89. The following table shows the urban growth from 1850 to 2000 in New York, NY.

a. Plot these points on a graph.

b. Connect the points with line segments.

c. Find the slope of each line segment.

d. Interpret each slope as a rate of change.

Year	Population
1850	515,547
1900	3,437,202
1950	7,891,957
2000	8,008,278

Source: U.S. Census Bureau

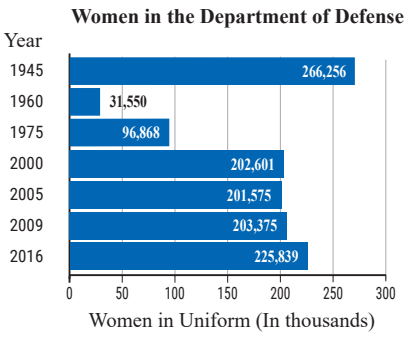
90. The following graph shows the number of female active duty military personnel over a span from 1945 to 2016. The number of women listed includes both officers and enlisted personnel from the Army, the Navy, the Marine Corps, and the Air Force.

a. Plot these points on a graph.

b. Connect the points with line segments.

c. Find the slope of each line segment.

d. Interpret each slope as a rate of change.



Source: U.S. Dept. of Defense

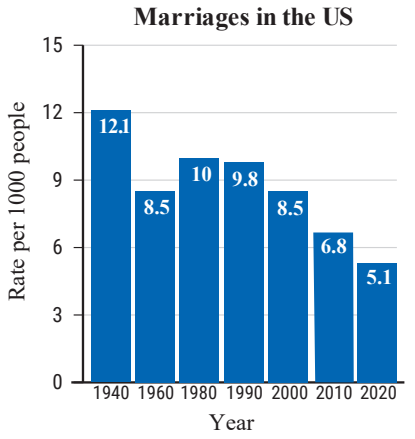
91. The following graph shows the rates of marriage per 1000 people in the US over a span from 1940 to 2020.

a. Plot these points on a graph.

b. Connect the points with line segments.

c. Find the slope of each line segment.

d. Interpret each slope as a rate of change.



Source: U.S. National Center for Health Statistics

Writing & Thinking

92. **a.** Explain in your own words why the slope of a horizontal line must be 0.

b. Explain in your own words why the slope of a vertical line must be undefined.

93. **a.** Describe the graph of the line $y = 0$.

b. Describe the graph of the line $x = 0$.

94. In the formula $y = mx + b$, explain the meaning of m and the meaning of b.

95. The slope of a road is called a **grade**. A steep grade is cause for truck drivers to have slow speed limits in mountains. What do you think that a "grade of 12%" means? Draw a picture of a right triangle that would indicate a grade of 12%.

Collaborative Learning

96. The class should be divided into teams of 2 or 3 students. Each team will need access to a digital camera, a printer, and a ruler.

a. Take pictures of 8 things with a defined slope. (**Suggestions:** A roof, a stair railing, a beach umbrella, a crooked tree, etc. Be creative!)

b. Print each picture.

c. Use a ruler to draw a coordinate system on top of each picture. You will probably want to use increments of in. or cm, depending on the size of your picture.

d. Identify the line in each picture whose slope you are calculating and then use the coordinate systems you created to identify the coordinates of two points on each line.

e. Use the points you just found to calculate the slope of the line in each picture.

f. Share your findings with the class.