10. Determine whether the equation $5x + 9 = -13 + 3x$ is a conditional equation, an identity, or a contradiction.

Example 10 Determining Types of Equations

Determine whether the equation $-2(x-7)+x=14-x$ is a conditional equation, an identity, or a contradiction.

Solution

$-2(x-7)+x=14-x$	Write the equation.
$-2x+14+x=14-x$	Use the distributive property.
$14-x=14-x$	Combine like terms.
$14-x-14=14-x-14$	Add -14 to both sides.
$-x=-x$	Simplify.
$-x+x=-x+x$	Add x to both sides.
$0=0$	Simplify.

The last equation is always true. Therefore, the original equation is an identity, and has an infinite number of solutions. Every real number is a solution.

Now work margin exercise 10.

Completion Example Answers

7.

$4(x+3)=2(3x-1)+6$	Write the equation.
$4x+\underline{12}=6x-\underline{2}+6$	Use the distributive property.
$4x+12=6x+\underline{4}$	Combine like terms.
$4x+12-\underline{12}=6x+4-\underline{12}$	Subtract $\underline{12}$ from both sides.
$4x=6x-\underline{8}$	Simplify.
$4x-\underline{6x}=6x-8-\underline{6x}$	Subtract $\underline{6x}$ from both sides.
$\underline{-2}x=-8$	Simplify.
$\frac{-2x}{\underline{-2}}=\frac{-8}{\underline{-2}}$	Divide both sides by $\underline{-2}$.
$x=\underline{4}$	Simplify.

Margin Exercise Answers

1. $x=-6$ **2.** $x=-7$ **3.** $y=2.1$ **4.** $x=-5$ **5.** $y=-4$ **6.** $x=-6$ **7.** $x=2$ **8.** contradiction **9.** identity **10.** conditional

7.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A/An _____ is an equation that has an infinite number of solutions.

2. If an equation has a finite number of solutions, it is a/an _____ equation.

3. Every linear equation is a/an _____ equation.

4. If a linear equation simplifies to a statement that is never true, then the original equation is called a/an _____.

5. The solution set of an identity can be written as all _____ numbers or _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. Every linear equation has exactly one solution.

7. If a linear equation simplifies to a statement that is always true, then the original equation is called an identity.

8. If an equation has no solution, it is called an identity.

9. The most general form of a linear equation is $ax + b = cx + d$.

Practice

Solve each equation. See Examples 1 through 7.

1. $3x+2=x-8$

2. $5x+1=2x-5$

3. $4n-3=n+6$

4. $6y+3=y-7$

5. $3y+18=7y-6$

6. $2y+5=8y+10$

7. $3x+11=8x-4$

8. $9x+3=5x-9$

9. $14n=3n$

10. $1.6x=0.8x$

11. $6y-2.1=y-2.1$

12. $13x+5=2x+5$

13. $2(z+1)=3z+3$

14. $6x-3=3(x+2)$

15. $16y+23y-3=16y-2y+2$

16. $5x-2x+4=3x+x-1$

17. $0.25+3x+6.5=0.75x$

18. $0.9y+3=0.4y+1.5$

19. $6.5+1.2x=0.5-0.3x$

20. $x-0.1x+0.8=0.2x+0.1$

21. $\frac{2}{3}x+1=\frac{1}{3}x-6$

22. $\frac{4}{5}n+2=\frac{2}{5}n-4$

23. $\frac{y}{5}+\frac{3}{4}=\frac{y}{2}+\frac{3}{4}$

24. $\frac{5n}{6}+\frac{1}{9}=\frac{3n}{2}+\frac{1}{9}$

25. $\frac{3}{8}\left(y-\frac{1}{2}\right)=\frac{1}{8}\left(y+\frac{1}{2}\right)$

26. $\frac{1}{2}\left(\frac{x}{2}+1\right)=\frac{1}{3}\left(\frac{x}{2}-1\right)$

27. $\frac{2x}{3}+\frac{x}{3}=-\frac{3}{4}+\frac{x}{2}$

28. $\frac{3}{4}x+\frac{1}{5}x=\frac{1}{2}x-\frac{3}{10}$

29. $x+\frac{2}{3}x-2x=\frac{x}{6}-\frac{1}{8}$

30. $3x+\frac{1}{2}x-\frac{2}{5}x=\frac{x}{10}+\frac{7}{20}$

31. $3(1+9x)=6(2-4x)$

32. $4(5-x)=8(3x+10)$

33. $3(4x-1)=4(2x-3)+8$

34. $7(2x-1)=5(x+6)-13$

35. $5-3(2x+1)=4(x-5)+6$

36. $-2(y+5)-4=6(y-2)+2$

37. $8+4(2x-3)=5-(x+3)$

38. $8(3x+5)-9=9(x-2)+14$

39. $4.7-0.3x=0.5x-0.1$

40. $5.8-0.1x=0.2x-0.2$

41. $0.2(x+3)=0.1(x-5)$

42. $0.4(x+3)=0.3(x-6)$

43. $\frac{1}{2}(4-8x)=\frac{1}{3}(4x+7)-3$

44. $3+\frac{1}{4}(x-4)=\frac{2}{5}(2+3x)$

45. $0.6x-22.9=1.5x-18.4$

46. $0.1y+3.8=5.72-0.3y$

47. $0.12n+0.25n-5.895=4.3n$

48. $0.15n+32n-21.0005=10.5n$

49. $0.7(x+14.1)=0.3(x+32.9)$

50. $0.8(x-6.21)=0.2(x-24.84)$

Determine whether each equation is a conditional equation, an identity, or a contradiction. See Examples 8 through 10.

51. $2(3x-1)+5=3$

52. $-2x+13=-2(x-7)$

53. $5x+13=-2(x-7)+3$

54. $3x+9=-3(x-3)+6x$

55. $7(x-1)=-3(3-x)+4x$

56. $3(x-2)+4x=6(x-1)+x$

57. $5(x+1)=3(x+1)+2(x+1)$

58. $8x-20+x=-3(5-2x)+3(x-4)$

59. $2x+3x=5.2(3-x)$

60. $5.2x+3.4x=0.2(x-0.42)$

Use a calculator to help solve each equation.

61. $0.17x - 23.0138 = 1.35x + 36.234$

62. $48.512 - 1.63x = 2.58x + 87.63553$

63. $0.32(x+14.1)=2.47x+2.21795$

64. $1.6(9.3+2x)=0.2(3x+133.94)$

Applications

Solve.

65. Caitlyn and Steve are planning their wedding reception and must decide between two catering halls. The first site, A Wedding Space, rents for $800 for one day and charges $50 per person for dinner. The second venue, A Wedding Place, costs $1000 to rent for one day and charges $40 per person for the same dinner. Solve the equation $800 + 50x = 1000 + 40x$ to determine how many guests they can invite so that the cost they pay will be the same at both wedding catering halls.

66. The value of a new car depreciates at a rate of about $250 per month. Suppose a car originally costs $30,000. The car was bought with a $1000 down payment and a loan with 0% financing for 60 months with payments of $200 a month. Solve the equation $30{,}000 - 250t = 29{,}000 - 200t$ to determine how many months it will take for the value of the vehicle to equal the amount owed on the loan?

67. Heidrick has won a chance to "Eat the Elite" at the Zombie Dash 7-mile race. He will receive a 2.25-mile head start. The elite runners run at a pace of 12.6 miles per hour and Heidrick runs at a pace of 6 miles per hour. Solve the equation $12.6t = 6t + 2.25$ to determine how many hours it will take the elite runners catch up to Heidrick? Enter your answer in hours rounded to the nearest hundredth.

68. A company has two packaging options for shipping quantities of a certain inventory item. Option A uses 20 boxes and there are 5 items unpacked. Option B requires more filler and uses 23 boxes, where each box holds 2 fewer items than Option A and there are only 3 items unpacked. This situation can be represented by $20x + 5 = 23(x - 2) + 3$, where x is the number of items that can fit in the box used for Option A.

a. What does $20x + 5$ represent in the equation?

b. What does $x - 2$ represent?

c. Solve the equation for x.

d. Check the solution.

e. What does the answer from part c. mean? Write a complete sentence.

69. Two advertisement flyers have the same area. The first flyer has a length of 12 inches and a width of x inches. The second flyer has a length of 4 inches and a width that is 10 inches more than x. This situation can be represented by $12x = 4(10 + x)$, where x is the width of the first flyer.

a. What does $12x$ represent in the equation?

b. What does $10 + x$ represent?

c. Solve the equation for x.

d. Check the solution.

e. What does the answer from part c. mean? Write a complete sentence.

70. The manager of a café wants to list a price for the weekly featured combo that includes tax. He wants to sell a medium house-blend coffee with a pastry for a total of \$5.45. He doesn't know which pastry to sell with the coffee to avoid losing money on the combo. The medium coffee costs \$2.75 and the tax is 9%. He uses the equation $1.09(2.75 + x) = 5.45$ to determine the price of the pastry, which is represented by the variable x.

a. What does the sum $2.75 + x$ represent?

b. Solve the equation for x.

c. Which of the following pastries would you choose to be a part of the combo? Explain why you made your choice.

cherry pie for \$2.50, coffee cake for \$2.25, bagel for \$2.00

71. A farmer is putting a shed on his property. He has two designs. One uses wood and would cost \$2 per square foot plus an extra \$8400 in materials. The other design is metal and would cost \$4 per square foot plus an additional \$8800. Both sheds are the same size, and the wood shed costs $\frac{3}{4}$ what the metal shed costs. Solve the equation $2x + 8400 = \frac{3}{4}(4x + 8800)$ to determine how many square feet the shed will be.

72. Two rival shoe companies want to rent the same building for their office and shipping space. Schulster's Shoes would use 1000 square feet for offices, 600 for shipping, and another 6 square feet for every packaged box of shoes. Shoes, Shoes, Shoes! would use 750 square feet for offices, 400 for shipping, and 9 square feet per packaged box of shoes. If only one company may occupy the building and the maximum amount of inventory kept by both companies is the same, solve the equation, $1000 + 600 + 6x = 750 + 400 + 9x$ to determine the maximum number of boxes of shoes each company plans to have.

73. An ice cream shop is having a special "Ice Cream Sunday" event in which they are giving away giant mixed sundaes of 3 scoops of vanilla ice cream and 2 scoops of chocolate. If they have 24 gallons of chocolate and 36 gallons of vanilla to start with, solve the equation, $36-\frac{1}{20}(3x)=24-\frac{1}{20}(2x)$ to determine how many sundaes they will have made when they run out of ice cream. (For this problem, we assume a gallon equals 20 scoops.)

74. A guitarist and a drummer are getting ready for a gig. The length of the gig will depend on how much material they have prepared. For every hour of the show, the guitarist must practice for 5 days and the drummer for 3 days. Since the guitarist already knows some of the songs, he saves 3 days of practice time. The drummer hurts his hand and loses 3 days of practice. If they plan to start and finish practicing at the same time, solve the equation, $5x-3=3x+3$ to determine how long the show will be.

Writing & Thinking

75. Answer each question.

a. Simplify the expression $3(x+5)+2(x-7)$.

b. Solve the equation $3(x+5)+2(x-7)=31$.

c. How are the methods you used to answer parts a. and b. similar? How are they different?

76. Write an equation to represent each situation, using x to represent Ryan's current age. Determine whether each equation is a conditional equation, an identity, or a contradiction, and explain why that makes sense for the situation represented.

a. In 6 years, Ryan will be 20 years old.

b. In 6 years, Ryan will be 8 years older than he is now.

c. In 6 years, Ryan will be 3 years older than he will be 3 years from now.