6. Solve the equation:

$$\frac{4x}{5}-\frac{2x}{3}-\frac{1}{2}=0$$

Example 6 Solving Equations with Fractions

Solve the equation: $\frac{x}{2}+\frac{3x}{4}-\frac{5}{3}=0$

Solution

$\frac{x}{2}+\frac{3x}{4}-\frac{5}{3}=0$	Write the equation.
$12\left(\frac{x}{2}+\frac{3x}{4}-\frac{5}{3}\right)=12(0)$	Multiply both sides by 12, the LCM of 2, 4, and 3.
$12\left(\frac{x}{2}\right)+12\left(\frac{3x}{4}\right)-12\left(\frac{5}{3}\right)=12(0)$	Apply the distributive property.
$6x+9x-20=0$	Simplify.
$15x-20=0$	Simplify.
$15x-20+20=0+20$	Add 20 to both sides.
$15x=20$	Simplify.
$\frac{15x}{15}=\frac{20}{15}$	Divide both sides by 15.
$x=\frac{4}{3}$ or $1\frac{1}{3}$	Simplify.

Checking will confirm that $\frac{4}{3}\left(\text{or } 1\frac{1}{3}\right)$ is the solution.

Now work margin exercise 6.

As illustrated in Example 6, the solution can be in the form of an improper fraction or a mixed number. Either form is correct and is acceptable mathematically. In general, improper fractions are preferred in algebra and the mixed number form is more appropriate if the solution indicates a measurement.

Margin Exercise Answers

1. $y=-40$ **2.** $x=\frac{9}{2}$ or $4\frac{1}{2}$ **3.** $x=\frac{9}{10}$ **4.** $x=\frac{6}{7}$ **5.** $n=-\frac{8}{13}$ **6.** $x=\frac{15}{4}$ or $3\frac{3}{4}$

3.9 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The addition principle states that adding an expression to both sides of an equation doesn't change the __________ of the equation.

2. Once you solve an equation, you should check the solution by plugging the solution in for the __________ in the original equation.

3. An equation of the form $ax+b=c$ can be solved by applying the addition principle and the __________ principle.

4. An equation where a variable has a fractional coefficient and all the constants are on the other side of the equation can be solved by multiplying both sides of the equation by the __________ of the coefficient.

5. Equations containing fractions can be solved by first multiplying both sides of the equation by the __________ of all the denominators.

6. In the expression $\frac{5}{6}x,$ the operation being performed is __________.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The equations $x + 3 = 4$ and $16(x+3)=16(4)$ have the same solutions.

8. The goal of solving equations is to have all of the constants on one side of the equation and the variable on the other side of the equation with a coefficient of 0.

9. The equation $\frac{3}{4}x=\frac{8}{15}$ can be solved by multiplying both sides of the equation by $\frac{15}{8}$.

10. The first step to solve $\frac{1}{3}x+5=\frac{1}{7}$ is to multiply both sides of the equation by 28.

Practice

Give a brief explanation of what is happening in each step of the solution process. See Examples 1through 6.

1.
$$4x-12=-10 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$
$$4x-12+12=-10+12 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$
$$4x=2 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$
$$\frac{4x}{4}=\frac{2}{4} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$
$$x=\frac{1}{2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

2.
$$7x+25=19 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$
$$7x+25-25=19-25 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$
$$7x=-6 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$
$$\frac{7x}{7}=\frac{-6}{7} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$
$$x=-\frac{6}{7} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

3.

$$\frac{4}{3}y-\frac{2}{3}=7 \qquad \underline{\hspace{6cm}}$$

$$3\left(\frac{4}{3}y\right)-3\left(\frac{2}{3}\right)=3(7) \qquad \underline{\hspace{6cm}}$$

$$4y-2=21 \qquad \underline{\hspace{6cm}}$$

$$4y-2+2=21+2 \qquad \underline{\hspace{6cm}}$$

$$4y=23 \qquad \underline{\hspace{6cm}}$$

$$\frac{4y}{4}=\frac{23}{4} \qquad \underline{\hspace{6cm}}$$

$$y=\frac{23}{4}\left(\text{or } y=5\frac{3}{4}\right) \qquad \underline{\hspace{6cm}}$$

4.

$$\frac{1}{2}x+\frac{1}{5}=3 \qquad \underline{\hspace{6cm}}$$

$$10\left(\frac{1}{2}x\right)+10\left(\frac{1}{5}\right)=10(3) \qquad \underline{\hspace{6cm}}$$

$$5x+2=30 \qquad \underline{\hspace{6cm}}$$

$$5x+2-2=30-2 \qquad \underline{\hspace{6cm}}$$

$$5x=28 \qquad \underline{\hspace{6cm}}$$

$$\frac{5x}{5}=\frac{28}{5} \qquad \underline{\hspace{6cm}}$$

$$x=\frac{28}{5}\left(\text{or } x=5\frac{3}{5}\right) \qquad \underline{\hspace{6cm}}$$

Solve each equation. See Examples 1 through 6.

5. $16x+23x-5=8$

6. $15m-4m+3=21$

7. $x-5-4x=-18$

8. $x-4-6x=24$

9. $5(n-2)=-24$

10. $4(y-1)=-6$

11. $2(n+1)=-3$

12. $3(x+2)=-10$

13. $2(x+9)+10=3$

14. $4(x-3)-7=0$

15. $5x+2(6-x)=10$

16. $3y+2(y+1)=4$

17. $\frac{3}{4}x=15$

18. $\frac{5}{8}x=40$

19. $\frac{7}{10}y=-28$

20. $\frac{2}{3}y=-30$

21. $\frac{4}{5}x=-\frac{2}{3}$

22. $\frac{5}{7}x=-\frac{5}{8}$

23. $-\frac{1}{9}n=\frac{3}{4}$

24. $-\frac{2}{5}n=\frac{1}{3}$

25. $\frac{7}{8}x=56$

26. $-\frac{2}{9}x=-18$

27. $\frac{3}{4}x+2=17$

28. $\frac{2}{3}y-5=21$

29. $\frac{1}{5}x+10=-32$

30. $\frac{1}{2}y-3=-23$

31. $\frac{1}{2}x-\frac{2}{3}=11$

32. $\frac{2}{3}y-\frac{1}{5}=-2$

33. $\frac{y}{4}+\frac{2}{3}=-\frac{1}{6}$

34. $\frac{x}{5}+\frac{1}{6}=-\frac{1}{10}$

35. $\frac{3}{5}y-4=\frac{1}{5}$

36. $\frac{n}{3}-6=\frac{2}{3}$

37. $\frac{5}{8}y-\frac{1}{4}=\frac{1}{3}$

38. $\frac{1}{7}x+\frac{1}{3}=\frac{1}{12}$

39. $\frac{n}{5}-\frac{1}{5}=\frac{1}{5}$

40. $\frac{x}{15}+\frac{2}{15}=-\frac{2}{15}$

41. $\frac{3}{4}=\frac{1}{5}x-\frac{5}{8}$

42. $\frac{1}{2}=\frac{1}{3}x+\frac{4}{15}$

43. $\frac{7}{8}=\frac{3}{4}x-\frac{3}{8}$

44. $-\frac{2}{25}=\frac{1}{5}x-\frac{3}{5}$

45. $\frac{1}{10}=\frac{4}{5}x+\frac{3}{10}$

46. $-\frac{5}{6}=\frac{2}{3}n+\frac{1}{6}$

47. $\frac{3x}{5}+\frac{2x}{5}=-\frac{1}{8}$

48. $\frac{x}{7}+\frac{6x}{7}=\frac{3}{4}$

49. $\frac{5}{8}x-\frac{3}{5}x=-\frac{1}{10}$

50. $\frac{5}{6}n-\frac{1}{15}n=-\frac{2}{3}$

51. $\frac{y}{7}+\frac{y}{28}=\frac{3}{4}$

52. $\frac{5y}{6}-\frac{y}{3}=\frac{5}{12}$

Applications

Solve.

53. Margot lost $8\frac{3}{4}$ pounds in 5 weeks. This can be modeled by the equation $5x=8\frac{3}{4}$, where x is the average number of pounds Margot lost per week. Solve the equation for x to determine the average number of pounds Margot lost per week.

54. Thomas Brown won the election for class president by earning 96 votes, which was $\frac{2}{3}$ of the total votes. This situation can be modeled by the equation $\frac{2}{3}x=96$, where x is the number of people who voted. Solve the equation for x to determine the number of students who voted in the election for class president.

55. One serving of Ritz Bits Peanut Butter Sandwiches has $2\frac{1}{2}$ g of saturated fat. This represents $\frac{5}{22}$ of the total fat content. This situation can be modeled by the equation $\frac{5}{22}x=2\frac{1}{2}$, where x represents the total number of grams of fat in the serving. Solve the equation for x to determine the total number of grams of fat are in one serving of Ritz Bits Peanut Butter Sandwiches.

56. A newly wed couple is writing thank-you cards to the guests who attended their wedding. The groom has written $\frac{1}{4}$ of the thank-you cards and the bride has written $\frac{1}{3}$ of the thank-you cards. So far they have completed 77 thank-you cards. This situation can be modeled by the equation $\frac{1}{4}x+\frac{1}{3}x=77$, where x is the total number of thank-you cards they need to write. Solve the equation for x to determine the total number of thank-you cards that the couple will write.

57. At the beginning of the work day, Sven had $\frac{1}{6}$ of his project completed. At the end of the work day, he had $\frac{1}{5}$ of his project completed. This situation can be modeled by $\frac{1}{6}+x=\frac{1}{5}$, where x is the fraction of the project Sven completed during the day. Solve the equation for x to determine the fraction of the project that was completed during the day.

58. Kristen watched $\frac{1}{4}$ of an hour of a documentary. She has $\frac{2}{3}$ of an hour left to watch. This can be modeled by the equation $x-\frac{1}{4}=\frac{2}{3}$, where x represents the total length of the documentary in hours. Solve the equation for x to determine the length of the documentary.

59. A baker purchased $4\frac{3}{4}$ pounds of apples. After making a few pies, he is left with $1\frac{1}{2}$ pounds of apples. This situation can be modeled by the equation $4\frac{3}{4} - x = 1\frac{1}{2}$, where x is the weight of apples used in the pies. Solve the equation for x to determine the number of pounds of apples the baker used to make the pies.

60. Silas received a bag of chocolate-covered peanuts for his birthday. His older brother ate $\frac{1}{5}$ of them and his younger sister ate $\frac{2}{9}$ of them. He discovers that he only has 26 left. This situation can be modeled by the equation $x - \left(\frac{1}{5}x + \frac{2}{9}x\right) = 26$, where x represents the original number of chocolate-covered peanuts. Solve this equation to find the number of chocolate-covered peanuts that were in the bag to begin with.

61. Caleb has been saving to buy a new video game. He has saved \$40, which is $\frac{5}{8}$ of the total cost. This situation can be modeled by the equation $\frac{5}{8}x = 40$, where x is the total cost of the game. Solve this equation to find out how much the game costs.

Writing & Thinking

62. In your own words, explain why multiplying an equation containing fractions by the LCD will result in an equation with only integer coefficients.

63. Below is a student's work while solving the equation $\frac{2}{3} = \frac{5}{8}x - \frac{1}{6}$. Next to each step, write the operation that was performed to reach that equation from the previous one.

$$\frac{2}{3} = \frac{5}{8}x - \frac{1}{6} \qquad \underline{\hspace{6cm}}$$

$$24\left(\frac{2}{3}\right) = 24\left(\frac{5}{8}x - \frac{1}{6}\right) \qquad \underline{\hspace{6cm}}$$

$$16 = 15x - \frac{1}{6} \qquad \underline{\hspace{6cm}}$$

$$16 + \frac{1}{6} = 15x - \frac{1}{6} + \frac{1}{6} \qquad \underline{\hspace{6cm}}$$

$$\frac{97}{6} = 15x \qquad \underline{\hspace{6cm}}$$

$$\frac{1}{15}\left(\frac{97}{6}\right) = \frac{1}{15}(15x) \qquad \underline{\hspace{6cm}}$$

$$\frac{97}{90} = x \qquad \underline{\hspace{6cm}}$$

Did the student perform the work correctly? If not, identify any mistakes the student made and give the correct answer.