Example 9 Application: Solving Equations of the Form $ax + b = c$

Last night's low temperature was reported to be 24 °F. The weather report said the temperature has steadily risen 2 degrees per hour since the lowest temperature of the day and it is currently 34 °F. This situation can be modeled by the equation $24 + 2x = 34$, where x is the number of hours since the lowest temperature was recorded. Solve the equation for x to determine how many hours have passed since the lowest temperature was recorded.

Solution

$24 + 2x = 34$	Write the equation.
$24 + 2x - 24 = 34 - 24$	Add -24 to both sides.
$2x = 10$	Simplify.
$\frac{2x}{2} = \frac{10}{2}$	Divide both sides by 2.
$x = 5$	Simplify.

5 hours have passed since the lowest temperature was recorded.

Now work margin exercise 9.

9. After class, Adrian works as an auto mechanic performing 15 oil changes a week. His boss plans to give him a pay raise after he's changed the oil on 360 cars. At the end of this week he will have completed 105 oil changes. This situation can be modeled by the equation $105 + 15x = 360$, where x is the number of weeks Adrian still needs to work before getting the raise. Solve the equation for x to determine how many weeks he still needs to work before receiving a raise.

Completion Example Answers

8. Apply the distributive property. Combine like terms. Add 15 to both sides. Simplify. Divide both sides by **2.** Simplify.

Margin Exercise Answers

1. a. Substituting -10 for x gives $(-10) - 3 = -13$, which is true. **b.** Substituting -2 for x gives $2(-2) + 8 = 4$, which is true. **2.** $x = -7$ **3.** $n = -8$ **4.** $y = -1$ **5.** $y = 4$ **6.** $x = -2$ **7.** $x = -6$ **8.** $x = 16$ **9.** 17 weeks

2.7 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A/An _____ to an equation is a number that gives a true statement when substituted for the variable.

2. A fundamental fact of algebra is that every equation of the form $ax + b = c$ (where $a \neq 0$) has exactly _____ solution(s).

3. The subtraction of a number can be thought of as the _____ of the opposite of that number.

4. The first step to solving an equation of the form $ax + b = c$ is to apply the _____ property whenever necessary to remove parentheses.

5. Checking an equation can be done by substituting the solution found into the _____ _____ to see if the resulting statement is true.

6. The first step that should be taken when solving the equation $14x + 3 - 8x = 9$ is to _____ like terms.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. In the addition and division principles, A, B, and C can be represented by positive or negative numbers.

8. The object in solving an equation is to isolate the constant with coefficient 1 on one side of the equation.

9. When solving an equation of the form $ax + b = c$, if a variable has a constant coefficient other than 1, use the addition principle to add the opposite of the coefficient to both sides.

10. When checking the solution to an equation with more than one occurrence of the variable, you can choose which instance of the variable to substitute the solution found into.

Practice

In each exercise, an equation and a list of possible solutions is given. Substitute each number into the equation until you find the solution of the equation. See Example 1.

1. $x + 4 = -8$; Possible solutions: $-10, -12, -14, -16, -18$

2. $x + 3 = -5$; Possible solutions: $-4, -5, -6, -7, -8, -9$

3. $-32 = y - 7$; Possible solutions: $-20, -21, -22, -23, -24, -25$

4. $-13 = y - 12$; Possible solutions: $0, -1, -2, -3, -4, -5$

5. $7x = -105$; Possible solutions: $0, -5, -10, -15, -20$

6. $-72 = 8n$; Possible solutions: $-6, -7, -8, -9, -10$

7. $-13n = 39$; Possible solutions: $0, -1, -2, -3, -4, -5$

8. $-14x = 56$; Possible solutions: $0, -1, -2, -3, -4, -5$

Solve each equation. Combine like terms whenever necessary. See Examples 2 through 8.

9. $x + 13 = 25$

10. $x + 7 = 33$

11. $-22 = x + 10$

12. $-15 = x + 31$

13. $y - 15 = -35$

14. $y - 12 = -40$

15. $x + 27 = 15$

16. $x + 8 = 3$

17. $y + 9 = 0$

18. $y - 18 = 0$

19. $-12 = x - 3$

20. $-83 = y + 13$

21. $-8x = -32$

22. $-9x = -72$

23. $-84 = 4n$

24. $-88 = 11n$

25. $3y - 2y = -11$

26. $6y - 5y = 8$

27. $4x - x = -12$

28. $5x - x = -24$

29. $30 = 11x - 5x$

30. $16 = 12y - 4y$

31. $3x - 2x + 6 = -28 + 8$

32. $4x - 3x + 3 = 16 + 30$

33. $7x + 3x = 30 + 10$

34. $6x + 2x = 14 + 26$

35. $10n - 3n - 6n + 1 = -41$

36. $6x - 2x - 3x + 3 = -24$

37. $6y + y - 3y = -20 - 4$

38. $-5n - 4n + n = 34 - 2$

39. $4x - 10x = -35 + 1 - 2$

40. $-2n - 7n = 10 - 11 + 100$

41. $2x + 3 = 13$

42. $4x + 5 = 21$

43. $3x - 1 = 20$

44. $5x - 4 = 21$

45. $23 = 7n - 5$

46. $45 = 8n - 3$

47. $5x + 15 = 40$

48. $8x + 12 = 44$

49. $15 = 6x - 9$

50. $14 = 5x - 21$

51. $23 = 7x + 2$

52. $32 = 7x + 4$

53. $10x - 2 = 78$

54. $11x + 7 = 40$

55. $6x + 15 = 63$

56. $7x + 26 = 61$

57. $61 = 11x - 5$

58. $74 = 9x - 7$

59. $5y - 2 - 4y = -6$

60. $7n - 6 - 6n = -13$

61. $5 = 7x + 14 - 10x$

62. $3 = 13y + 15 - 7y$

63. $15 + 19 = 4x - 2$

64. $8 + 23 = 5y - 4$

65. $23 - 30 = 4y + 1$

66. $18 - 25 = 3y + 2$

67. $4n + 7n + 5 = 18 + 9$

68. $3n + 5n + 12 = 25 + 19$

69. $4(x+2) - 5 = 19$

70. $3(x+4) - 7 = 17$

71. $5 = 2(y-1) + 1$

72. $23 = 5(y-2) + 8$

73. $35 = 3(x+5) + 2x$

74. $33 = 4(n-3) + 5n$

75. $6(n+1) - 4n = -10$

76. $5(x-2) - 2x = -19$

Applications

Solve.

77. At the beginning of his diet, Jim weighed 256 pounds. After one month, Jim weighed 238 pounds. This situation can be modeled by the equation $256 + w = 228$, where w is the change in Jim's weight during the month. Solve the equation for w to determine the change in Jim's weight.

78. The temperature decreased by 27 degrees during a 9-hour period. This situation can be modeled by $9t = -27$, where t represents the average change in temperature per hour. Solve the equation for t to determine the average change in temperature per hour.

79. A kitchen has 71 pounds of fresh produce in its inventory. After two days, the kitchen had 45 pounds of fresh produce. This situation can be modeled by the equation $2x + 71 = 45$, where x is the change in the amount of fresh produce per day. Solve the equation for x to determine the change in the amount of fresh produce per day.

80. A public pool is being drained for cleaning and maintenance. At noon, the water depth was 59 inches. At 3 p.m., the depth was 26 inches. This situation can be modeled by the equation $59 + 3x = 26$, where x is change of water depth in inches per hour. Solve the equation for x to determine the change in depth per hour.

81. During a snowstorm, the snow fell at a steady rate for 4 hours. Before the snowstorm started, there were 3 inches of snow on the ground. After the snowstorm, there were 11 inches of snow on the ground. This situation can be modeled by the equation $11 = 4s + 3$, where s represents the snowfall in inches per hour. Solve the equation for s to determine the amount of snow that fell per hour.

82. Tristen participated in a basketball shootout to raise money for charity. His sponsors could choose to donate a certain amount of money for every basket he made or donate any amount before the shootout. Before the shootout, Tristen had received \$25 in donations. He made 68 baskets at the shootout, and was able to donate a total of \$297 to charity. This situation can be modeled by the equation $68x + 25 = 297$, where x represents the amount of money his sponsors donated per basket. Solve this equation for x to find the amount of money that his sponsors donated for each basket Tristen made.

83. At 6 p.m. the temperature was 73 °F. Seven hours later, the temperature was 59 °F. This situation can be modeled by the equation $7t + 73 = 59$, where t is the change in temperature per hour. Solve the equation for t to determine the change in temperature per hour.

84. At the beginning of a craft show, a jeweler started with –\$75 of profit. The jeweler earned the same amount of profit for each item sold. At the end of the craft show, the jeweler sold 65 pieces and made \$510 in profit. This situation can be modeled by the equation $65x - 75 = 510$, where x is the amount of profit per piece of jewelry sold. Solve the equation for x to determine the amount of profit per piece of jewelry.

Writing & Thinking

Give a brief explanation of what is happening in each step of the solution process.

85. $3x - x = -10$ _____

$2x = -10$ _____

$\frac{2x}{2} = \frac{-10}{2}$ _____

$x = -5$ _____

86. $71y - 62y = -36$ _____

$9y = -36$ _____

$\frac{9y}{9} = \frac{-36}{9}$ _____

$y = -4$ _____

87. Explain in your own words why the addition and subtraction principles can be stated as one principle of addition.

88. Explain what the goal in solving an equation is, and how you achieve that goal.