Example 7 Verifying Absolute Value Inequalities

Determine whether each statement is true or false. Rewrite any false statement so that it is true. (**Note:** There may be more than one correct new statement.)

a. $|-12| \geq 12$ **b.** $|-20| < |-21|$

Solution

a. True, since $|-12| = 12$ and $12 \geq 12$. (Remember that the symbol $\geq$ is read "greater than or equal to" so that "equal to" is valid with this symbol.)

b. True, since $|-20| = 20$, $|-21| = 21$, and $20 < 21$.

Now work margin exercise 7.

7. Determine whether each statement is true or false. Rewrite any false statement so that it is true. (**Note:** There may be more than one correct new statement.)

a. $|-5| \leq 5$

b. $|-38| > |-39|$

Example 8 Solving Absolute Value Equations

If $|x| = 7$, what are the possible values for x?

Solution

Since $|-7| = 7$ and $|7| = 7$, $x = -7$ or $x = 7$.

Now work margin exercise 8.

8. If $|z| = 3$, what are the possible values for z?

Example 9 Solving Absolute Value Equations

If $|a| = -2$, what are the possible values for a?

Solution

There are no values of a for which $|a| = -2$. The absolute value can never be negative. There is **no solution**.

Now work margin exercise 9.

9. If $|y| = -19$, what are the possible values for y?

Margin Exercise Answers

1. a. 10 **b.** 8 **c.** −17 **2.** 

3. a. False; $6 \leq 7$ **b.** True **c.** True **d.** False; $2 \geq -2$ **4. a.** 1 **b.** 1 **5.** 4 **6. a.** 2 **b.** −13 **c.** −7 **7. a.** True **b.** False; $|-38| < |-39|$ **8.** $z = -3, 3$ **9.** No solution

2.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The set of numbers that includes the whole numbers and their opposites is the set of _____.

2. To aid in understanding numbers and their relation to each other, a/an _____ _____ can be used to show that information as a "picture."

3. The number _____ is neither positive nor negative.

4. The symbols < and > are known as _____ symbols.

5. A number's distance from 0 is the number's _____ _____.

6. The _____ of a number is the point that corresponds to the number on a number line.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If –8 lies to the right of a number on a number line, then –8 is less than that number.

8. All whole numbers have an opposite number.

9. All whole numbers are also integers.

10. The absolute value of a positive number is a positive number.

Practice

Find the opposite of each integer. See Example 1.

1. –3
2. –7
3. 0
4. –0
5. +2
6. +6

Graph each set of integers on a number line. See Example 2.

7. $\{0, 1, 2\}$
8. $\{0, 2, 4\}$
9. $\{-3, -1, 1\}$
10. $\{-3, -2, 0\}$
11. $\{-5, 0, 5\}$
12. $\{-4, 3, 4\}$
13. $\{1, 2, 5, 6\}$
14. $\{1, 3, 4, 6\}$
15. $\{-10, -9, -8, -7\}$
16. $\{-5, -4, -2, -1\}$
17. $\{-3, 1, -2, 0\}$
18. $\{2, -3, 0, -1\}$
19. $\{-3, -1, 0, 1, 3\}$
20. $\{-3, -2, 0, 1, 2\}$
21. $\{-5, -4, -3, -2, 0, 1\}$
22. $\{-2, -1, 0, 2, 3, 4\}$

Graph each set of numbers on a number line.

23. All whole numbers less than 4
24. All negative integers greater than –4
25. All whole numbers less than 0
26. All natural numbers less than or equal to –1

Fill in each blank with the appropriate symbol that will make the statement true: <, >, or =. See Example 3.

27. $4 \_\_\_ 6$

28. $3 \_\_\_ 0$

29. $7 \_\_\_ -1$

30. $10 \_\_\_ -10$

31. $-3 \_\_\_ 1$

32. $4 \_\_\_ -8$

33. $-8 \_\_\_ 0$

34. $-1 \_\_\_ 0$

35. $-2 \_\_\_ -4$

36. $-7 \_\_\_ -6$

37. $-20 \_\_\_ -19$

38. $-67 \_\_\_ -50$

Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.) See Example 3.

39. $0 = -0$

40. $-22 < -16$

41. $-17 \leq 17$

42. $-9 < -10$

43. $-2 < 0$

44. $-5 > 5$

Simplify. See Examples 4 through 6.

45. $|-4|$

46. $|-11|$

47. $|5|$

48. $|-1|$

49. $|0|$

50. $|1|$

51. $|-42|$

52. $|23|$

53. $-|20|$

54. $-|19|$

55. $-(-13)$

56. $-(-21)$

57. $-|-12|$

58. $-|-8|$

List the possible values for x for each statement. See Examples 8 and 9.

59. $|x| = 5$

60. $|x| = 8$

61. $|x| = 2$

62. $|x| = 23$

63. $|x| = 0$

64. $|x| = 105$

65. $|x| = -6$

66. $|x| = -1$

Choose the response that correctly completes each sentence. Assume that the variables represent integers. In each problem give two examples that illustrate your reasoning.

67. $|a|$ is (never, sometimes, always) equal to a.

68. $|x|$ is (never, sometimes, always) equal to $-x$.

69. $|y|$ is (never, sometimes, always) equal to a positive integer.

70. $|x|$ is (never, sometimes, always) greater than x.

Applications

Represent each quantity with a signed integer.

71. The Alvin is a manned deep-ocean research submersible that has explored the wreck of the Titanic. The operating depth of the Alvin is 4500 meters below sea level.

72. The Mariana trench is the deepest known location on the Earth's ocean floor. The deepest known part of the Mariana Trench is approximately 11 kilometers below sea level.

73. Mount Everest is considered to be the highest mountain on Earth. Its peak reaches to a height of approximately 8844 meters.

74. The lowest temperature ever recorded was at the Vostok Station on Antarctica. On July 21, 1983, the temperature was approximately 128 degrees Fahrenheit below zero.

75. Cynthia believes that the ideal daytime temperature for growing tomato plants is 77 °F, and the plants will be okay as long as the temperature does not vary more than 7 °F from that ideal temperature during the day.

a. How low can the daytime temperature be to accommodate the tomato plant growth?

b. How high can the daytime temperature be to accommodate the tomato plant growth?

76. The following table shows the elevation of six California cities. (Sea level is defined to have 0 feet of elevation.)

City	Elevation (in feet)
Alameda	50
Death Valley	–282
El Centro	–39
Fresno	296
Salton City	–125
Windsor	118

a. Which of these cities has an elevation farthest away from sea level?

b. Which of these cities has an elevation closest to sea level?

Writing and Thinking

77. Give one example each of the use of a positive number, a negative number, and the number zero (outside of a class).

78. Explain, in your own words, how an expression such as $-y$ might represent a positive number.

79. Compare and contrast absolute value with opposites.