

**Example 5 Using the Change-of-Base Formula**

Use the change-of-base formula to find the value of  $x$  (accurate to the nearest ten-thousandth) in the equation  $5^x = 16$ .

**Solution**

Because the base is 5, we can take  $\log_5$  of both sides.

(This method is not necessary, but it does show how the change-of-base formula can be used.)

$$\begin{aligned} 5^x &= 16 \\ \log_5 5^x &= \log_5 16 \\ x &= \log_5 16 \\ x &= \frac{\ln 16}{\ln 5} && \text{Change-of-base formula} \\ x &\approx \frac{2.7726}{1.6094} \approx 1.7228 \end{aligned}$$

5. Use the change-of-base formula to find the value of  $x$ .

$$8^x = 25$$

**Now work margin exercise 5.****Margin Exercise Answers**

1. a.  $x = 1$  or  $x = 3$    b.  $x = -1$  or  $x = 2$    2. a.  $x = \frac{\log 3}{\log 4} \approx 0.7925$    b.  $x = \frac{\log 20}{\log 5} \approx 1.8614$

3. a.  $x = 50$    b.  $x = -1$    4.  $\frac{\ln 4}{\ln 2} = 2$    5.  $\frac{\ln 25}{\ln 8} \approx 1.5480$

## 15.7 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

- When solving exponential equations, if the bases are not the same, the equations are solved by taking the \_\_\_\_\_ of both sides.
- Logarithms are defined only for \_\_\_\_\_ numbers, so each answer should be checked in the original equation.
- For any positive real number  $b$ ,  $b^0$  is equal to \_\_\_\_\_.
- For any positive real number  $b$  and any real numbers  $x$  and  $y$ ,  $(b^x)^y$  is equivalent to \_\_\_\_\_.
- For any positive real number  $b$  and any value of  $x$ ,  $b^{-x}$  is equivalent to \_\_\_\_\_.
- For  $b > 0$  and  $b \neq 1$ , if  $\log_b x = \log_b y$ , then  $x =$  \_\_\_\_\_.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The change of base formula is  $\log_b x = \frac{\log_a b}{\log_a x}$ , for  $a, b, x > 0$  and  $b \neq 1$ .
8. If the terms in an exponential equation all have the same base, there is no need to use logarithms to solve the equation.
9. Exponential equations with different bases can be solved by taking either the log of both sides or the ln of both sides.
10. If  $5^x = 5^y$ , then  $x$  is equal to  $y$ .

## Practice

Use the properties of exponents and logarithms to solve each of the equations. If necessary, use a calculator and round answers to the nearest ten-thousandth. See Examples 1 through 3.

1.  $2^4 \cdot 2^7 = 2^x$
2.  $3^7 \cdot 3^{-2} = 3^x$
3.  $(3^5)^2 = 3^{x+1}$
4.  $(2^x)^3 = 2^{x+4}$
5.  $\frac{10^4 \cdot 10^{\frac{1}{2}}}{10^x} = 10$
6.  $(10^2)^x = \frac{10 \cdot 10^{\frac{2}{3}}}{10^{\frac{1}{2}}}$
7.  $2^{5x} = 4^3$
8.  $7^{3x} = 49^4$
9.  $(25)^x = 5^3 \cdot 5^4$
10.  $10^x \cdot 10^8 = 100^3$
11.  $8^{x+3} = 2^{x-1}$
12.  $100^{2x+1} = 1000^{x-2}$
13.  $27^x = 3 \cdot 9^{x-2}$
14.  $16^x = 2 \cdot 8^{2x+3}$
15.  $2^{3x+5} = 2^{x^2+1}$
16.  $10^{x^2+x} = 10^{x+9}$
17.  $10^{2x^2+3} = 10^{x+6}$
18.  $3^{x^2+5x} = 3^{2x-2}$
19.  $(3^{x+1})^x = (3^{x+3})^2$
20.  $(10^x)^{x+3} = (10^{x+2})^{-2}$
21.  $3^{x+4} = 9$
22.  $2^{5x-8} = 4$
23.  $4^{x^2-x} = \left(\frac{1}{2}\right)^{5x}$
24.  $25^{x^2+x} = \left(\frac{1}{5}\right)^{3x}$
25.  $5^{2x-x^2} = \frac{1}{125}$
26.  $10^{x^2-2x} = 1000$
27.  $10^{3x} = 140$
28.  $10^{2x} = 97$
29.  $10^{0.32x} = 253$
30.  $10^{-0.48x} = 88.6$
31.  $4 \cdot 10^{-0.94x} = 126.2$
32.  $3 \cdot 10^{-2.1x} = 83.5$
33.  $e^{3x} = 2.1$
34.  $e^{4t} = 184$

35.  $e^{-0.5x} = 47$
36.  $e^{-0.006t} = 50.3$
37.  $3e^{-0.12t} = 3.6$
38.  $5e^{2.4t} = 44$
39.  $2^x = 10$
40.  $3^{x-2} = 100$
41.  $5^{2x} = \frac{1}{100}$
42.  $7^{3x} = \frac{1}{10}$
43.  $5^{1-x} = 1$
44.  $12^{5x+2} = 1$
45.  $4^{2x+5} = 0.01$
46.  $4^{2-3x} = 0.1$
47.  $14^{3x-1} = 10^3$
48.  $12^{2x+7} = 10^4$
49.  $7^x = 9$
50.  $2^x = 20$
51.  $3^{3x} = 23$
52.  $5^{2x} = 23$
53.  $6^{2x-1} = 14.8$
54.  $4^{7-3x} = 26.3$
55.  $5 \log x = 7$
56.  $3 \log x = 13.2$
57.  $4 \log x - 6 = 0$
58.  $2 \log x - 15 = 0$
59.  $4 \log x^{\frac{1}{2}} + 8 = 0$
60.  $\frac{2}{3} \log x^{\frac{2}{3}} + 9 = 0$
61.  $5 \ln x - 8 = 0$
62.  $2 \ln x + 3 = 0$
63.  $\ln x^2 + 2.2 = 0$
64.  $\ln x^2 - 41.6 = 0$
65.  $\log x + \log 2x = \log 18$
66.  $\log(x+4) + \log(x-4) = \log 9$
67.  $\log x^2 - \log x = 2$
68.  $\log x + \log x^2 = 3$
69.  $\ln(x-3) + \ln x = \ln 18$
70.  $\ln(x+5) + \ln(x-1) = \ln 16$
71.  $\log(x-15) = 2 - \log x$
72.  $\log(2x-17) = 2 - \log x$
73.  $\log(3x-5) + \log(x-1) = 1$
74.  $\log(2x-3) + \log(x+3) = 3$
75.  $\log(x-3) - \log(x+1) = 1$
76.  $\ln(x+1) + \ln(x-1) = 0$
77.  $\log(x^2-9) - \log(x-3) = -2$
78.  $\ln(x^2+4x-5) - \ln(x+5) = -2$
79.  $\log(x^2-4x-5) - \log(x+1) = 2$
80.  $\log(x^2-x-12) - \log(x-4) = -2$
81.  $\ln(x^2-4) = 3 + \ln(x+2)$
82.  $\ln(x^2+2x-3) = 1 + \ln(x-1)$
83.  $\log \sqrt[3]{x^2+2x+20} = \frac{2}{3}$
84.  $\log \sqrt{x^2-24} = \frac{3}{2}$

Use the change-of-base formula to evaluate each of the expressions or solve the equations. Round answers to the nearest ten-thousandth. See Examples 4 and 5.

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85.  $\log_3 12$

86.  $\log_4 36$

87.  $\log_5 1.68$

88.  $\log_{11} 39.6$

89.  $\log_8 0.271$

90.  $\log_7 0.849$

91.  $\log_{15} 739$

92.  $\log_2 14.2$

93.  $\log_{20} 0.0257$

94.  $\log_9 2.384$

95.  $2^x = 5$

96.  $3^{2x} = 10$

97.  $9^{2x-1} = 100$

98.  $5^{x-1} = 30$

99.  $4^{3-x} = 20$

100.  $6^{4-3x} = 25$

## Writing & Thinking

101. Solve the following equation for  $x$  two different ways:  $a^{2x-1} = 1$ .

102. Rewrite each of the following expressions as products.

a.  $5^{x+2}$

b.  $3^{x-2}$

103. Explain, in your own words, why  $7 \cdot 7^x \neq 49^x$  when  $x \neq 1$ . Show each of the expressions  $7 \cdot 7^x$  and  $49^x$  as a single exponential expression with base 7.