OR

$$\begin{aligned}\log_b \sqrt{x} + \log_b \sqrt[3]{x} &= \log_b x^{\frac{1}{2}} + \log_b x^{\frac{1}{3}} \\ &= \log_b \left(x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}\right) && \text{Product rule} \\ &= \log_b x^{\left(\frac{1}{2}+\frac{1}{3}\right)} \\ &= \log_b x^{\frac{5}{6}}\end{aligned}$$

Now work margin exercise 5.

Common Misunderstandings about Logarithms

There is no logarithmic property for the logarithm of a sum or a difference.

$\log_b (x+y)$	Cannot be simplified
$\log_b (x-y)$	Cannot be simplified

Also,

$\log_b (xy) \neq \log_b x \cdot \log_b y$	The log of a product does not equal the product of the logs.
$\log_b \frac{x}{y} \neq \frac{\log_b x}{\log_b y}$	The log of a quotient does not equal the quotient of the logs.

CAUTION

Margin Exercise Answers

1. a. 4 **b.** 1.6021 **c.** 1.7782 **2. a.** −0.4771 **b.** −2 **c.** 0.3802 **3. a.** 0.2386 **b.** 1.4314 **c.** 1.5563

4. a. $\log_b 3 + 2\log_b x$ **b.** $3\log_b x + \log_b y - \log_b z$ **c.** $-2\log_b m - 2\log_b n$ **d.** $\frac{1}{2}\log_b 2 + \frac{1}{2}\log_b a$

5. a. $\log_b \left(\frac{x^3}{y^4}\right)$ **b.** $\log_d \left(\frac{3}{4x}\right)$ **c.** $\log_a \left(y^2 - 4\right)$ **d.** $\log_b y^{\frac{3}{4}}$

15.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Because logarithms are _____, their properties are similar to those of _____.
2. The logarithm of a product is equal to the _____ of the logarithms of the factors.
3. The logarithm of a quotient is equal to the _____ _____ the logarithm of the numerator and the logarithm of the denominator.
4. The logarithm of a number raised to a power is equal to the _____ of the exponent and the logarithm of the number.

5. There is no logarithmic property for the logarithm of a/an _____ or a/an _____.

6. When dealing with logarithms of the form $\log_b x$, we assume that $b >$ _____ and $b \neq$ _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The properties of exponents are used to prove the properties of logarithms.

8. The power rule for logarithms states that the exponent r must be a positive integer.

9. The log of a product does not equal the product of the logs.

10. The expression $\log_5 \frac{4}{3}$ is equivalent to $\frac{\log_5 4}{\log_5 3}$.

Practice

Use the following logarithms (accurate to 4 decimal places) in Exercises 1 and 2. See Example 1.

$\log_{10} 2 \approx 0.3010$ $\log_{10} 3 \approx 0.4771$
$\log_{10} 5 \approx 0.6990$ $\log_{10} 6 \approx 0.7782$

1. Find the values of the following expressions.

a. $10^{0.3010}$ **c.** $10^{0.6990}$

b. $10^{0.4771}$ **d.** $10^{0.7782}$

2. Find the values of the following expressions.

a. $10^{0.3010 + 0.7782}$ **c.** $10^{0.4771 + 0.6990}$

b. $10^{0.4771 + 0.7782}$ **d.** $10^{0.6990 - 0.3010}$

Use your knowledge of logarithms and exponents to find the value of each expression.

3. $\log_2 32$

4. $\log_3 9$

5. $\log_4 \frac{1}{16}$

6. $\log_5 \frac{1}{125}$

7. $\log_3 \sqrt{3}$

8. $\log_2 \sqrt{8}$

9. $5^{\log_5 10}$

10. $3^{\log_3 17}$

11. $6^{\log_6 \sqrt{3}}$

12. $5^{\log_5 5}$

Use the properties of logarithms to expand each expression as much as possible. See Example 4.

13. $\log_b 5x^4$

14. $\log_b 3x^2 y$

15. $\log_b 2x^{-3} y$

16. $\log_5 xy^2 z^{-1}$

17. $\log_6 \dfrac{2x}{y^3}$

18. $\log_3 \dfrac{xy}{4z}$

19. $\log_b \dfrac{x^2}{yz}$

20. $\log_3 \dfrac{xy^2}{z^2}$

21. $\log_5 (xy)^{-2}$

22. $\log_b (x^2 y)^4$

23. $\log_6 \sqrt[3]{xy^2}$

24. $\log_5 \sqrt{2x^3 y}$

25. $\log_3 \sqrt{\dfrac{xy}{z}}$

26. $\log_6 \sqrt[3]{\dfrac{x^2}{y}}$

27. $\log_5 21x^2 y^{\frac{2}{3}}$

28. $\log_b 15x^{-\frac{1}{2}} y^{\frac{1}{3}}$

29. $\log_6 \dfrac{x}{\sqrt{x^3 y^5}}$

30. $\log_3 \dfrac{1}{\sqrt{x^4 y}}$

31. $\log_b \left(\dfrac{x^3 y^2}{z}\right)^{-3}$

32. $\log_4 \left(\dfrac{x^{-\frac{1}{2}} y}{z^2}\right)^{-2}$

Use the properties of logarithms to write each expression as a single logarithm of a single expression. See Example 5.

33. $2\log_b 3 + \log_b x - \log_b 5$

34. $\dfrac{1}{2}\log_b 25 + \log_b 3 - \log_b x$

35. $\log_2 7 + \log_2 9 + 2\log_2 x$

36. $\log_5 4 + \log_5 6 + \log_5 y$

37. $2\log_b x + \log_b y$

38. $\log_2 x + 3\log_2 y$

39. $3\log_5 y - \dfrac{1}{2}\log_5 x$

40. $3\log_{10} x - 2\log_{10} y$

41. $\dfrac{1}{2}(\log_5 x - \log_5 y)$

42. $\dfrac{1}{3}(\log_{10} x - 2\log_{10} y)$

43. $\log_2 x - \log_2 y + \log_2 z$

44. $\log_b x - 2\log_b y - 2\log_b z$

45. $\log_b x + 2\log_b y - \dfrac{1}{2}\log_b z$

46. $-\dfrac{2}{3}\log_2 x - \dfrac{1}{3}\log_2 y + \dfrac{2}{3}\log_2 z$

47. $2\log_5 x + \log_5 (2x+1)$

48. $\log_b (3x+1) + 2\log_b x$

49. $\log_2 (x-1) + \log_2 (x+3)$

50. $\log_{10} (x+3) + \log_{10} (x-3)$

51. $\log_b (x^2 - 2x - 3) - \log_b (x-3)$

52. $\log_2 (x-4) - \log_2 (x^2 - 2x - 8)$

53. $\log_{10} (x+6) - \log_{10} (2x^2 + 9x - 18)$

54. $\log_5 (3x^2 + 5x - 2) - \log_5 (3x-1)$

Writing & Thinking

55. Prove the quotient rule for logarithms: For $b > 0$, $b \neq 1$, and $x, y > 0$,

$$\log_b \frac{x}{y} = \log_b x - \log_b y.$$

56. Prove the following property of logarithms: For $b > 0$, $b \neq 1$, and $x > 0$, $\log_b b^x = x$.