- For the logarithmic function $y = \log_b x \ (\text{or } x = b^y)$,
  the domain is all $x > 0$, and (The graph is to the right of the y-axis.)
  the range is all real y.
  There is a vertical asymptote at $x = 0$.

Margin Exercise Answers

1. a. $\log_4 4 = 1$ **b.** $\log_4 64 = 3$ **c.** $\log_4\left(\frac{1}{4}\right) = -1$ **d.** $3^2 = 9$ **e.** $3^4 = 81$ **f.** $3^{-1} = \frac{1}{3}$ **2. a.** 0 **b.** 1 **c.** 30 **d.** 6 **e.** −3 **3.** 4 **4.** $\frac{5}{2}$

15.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The function $x = b^y$ is equivalent to $y =$ _____.
2. The line $y = 0$ is the _____ asymptote of $y = b^x$.
3. The inverse of an exponential function is a/an _____ function.
4. Regardless of the base, the logarithm of 1 is _____.
5. The graph of a logarithmic function can be found by _____ the corresponding exponential function across the line $y = x$.
6. The points on the graph of the inverse function can be found by _____ the coordinates of the ordered pairs.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so that the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Exponential functions of the form $y = b^x$ are one-to-one functions and have inverses.
8. The exponent of an exponential function is the base of its inverse logarithmic function.
9. Exponents are logarithms.
10. The logarithm of the base is always 1.

Practice

Express each equation in logarithmic form. See Example 1.

1. $7^2 = 49$
2. $3^3 = 27$
3. $5^{-2} = \frac{1}{25}$
4. $2^{-5} = \frac{1}{32}$
5. $1 = \pi^0$
6. $6^0 = 1$
7. $10^2 = 100$
8. $10^1 = 10$
9. $10^k = 23$

10. $4^k = 11.6$

11. $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

12. $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

Express each equation in exponential form. See Example 1.

13. $\log_3 9 = 2$

14. $\log_5 125 = 3$

15. $\log_9 3 = \frac{1}{2}$

16. $\log_b 4 = \frac{2}{3}$

17. $\log_7 \frac{1}{7} = -1$

18. $\log_{1/2} 8 = -3$

19. $\log_{10} N = 1.74$

20. $\log_2 42.3 = x$

21. $\log_b 18 = 4$

22. $\log_b 39 = 10$

23. $\log_n y^2 = x$

24. $\log_b a = x^2$

Use the four basic properties of logarithms to evaluate each expression. See Example 2.

25. $\log_3 81$

26. $7^{\log_7 15}$

27. $\log_7 1$

28. $5^{\log_5 25}$

29. $\log_4 \frac{1}{64}$

30. $\log_{12} 12$

Solve by first changing each equation to exponential form. See Examples 3 and 4.

31. $\log_4 x = 2$

32. $\log_3 x = 4$

33. $\log_{14} 196 = x$

34. $\log_{25} 125 = x$

35. $\log_5 \frac{1}{125} = x$

36. $\log_3 \frac{1}{9} = x$

37. $\log_{36} x = -\frac{1}{2}$

38. $\log_{81} x = -\frac{3}{4}$

39. $\log_x 32 = 5$

40. $\log_x 121 = 2$

41. $\log_8 x = \frac{5}{3}$

42. $\log_{16} x = \frac{3}{4}$

43. $\log_8 8^{3.7} = x$

44. $\log_{10} 10^{1.52} = x$

45. $\log_5 5^{\log_5 25} = x$

46. $\log_4 4^{\log_2 8} = x$

Graph each function and its inverse on the same set of axes. Label two points on each graph.

47. $f(x) = 6^x$

48. $f(x) = 2^x$

49. $y = \left(\frac{2}{3}\right)^x$

50. $y = \left(\frac{1}{4}\right)^x$

51. $f(x) = \log_4 x$

52. $f(x) = \log_5 x$

53. $y = \log_{1/2} x$

54. $y = \log_{1/3} x$

55. $y = \log_8 x$

56. $y = \log_7 x$

57. Consider the function $y = c\left(3^x\right)$ where c is a constant greater than zero. List the following:

a. The domain of the function.

b. The range of the function.

c. Any asymptotes of the graph of the function.

d. Give c two different values and sketch the graphs of both functions.

58. Consider the function $y = c\left(3^{-x}\right)$ where c is a constant greater than zero. List the following:

a. The domain of the function.

b. The range of the function.

c. Any asymptotes of the graph of the function.

d. Give c two different values and sketch the graphs of both functions.

Writing & Thinking

59. Discuss, in your own words, the symmetrical relationship of the graphs of the two functions $y = 10^x$ and $y = \log_{10} x$.

60. Discuss, in your own words, the symmetrical relationship of the graphs of the two logarithmic functions $y = \log_{10} x$ and $y = -\log_{10} x$.