15.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. To evaluate a function at an algebraic expression, replace the ______ with the expression everywhere the ______ appears.

2. Given two functions $f(x)$ and $g(x)$ a new function $f(g(x))$, called the composition of f and g, is found by substituting the ______ for $g(x)$ into the place of x in the ______ f.

3. In general, $f(g(x))$ ______ $g(f(x))$.

4. The domain of $f \circ g$ consists of those values of x in the ______ of g for which $g(x)$ is in the ______ of f.

5. Functions that have only one x-value for each y-value in the range are said to be ________ functions.

6. In general, every ______ function has a/an ______ function.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The vertical line test is used to determine whether a graph represents a vertical line.

8. In a one-to-one function, each x-value corresponds to exactly one y-value.

9. The horizontal line test is used to determine whether a graph of a function is one-to-one.

10. The notation $f^{-1}(x)$ means $\frac{1}{f(x)}$.

Practice

Find the indicated function values for each function given. See Example 1.

1. $f(x) = 8x - 5$
   a. $f(r)$
   b. $f(3a - 1)$

2. $r(x) = 4x - 6$
   a. $r(g - 5)$
   b. $r(h^2 + 8)$

3. $g(y) = 5y^2 + 4$
   a. $g(x - 2)$
   b. $g(3n^2)$

4. $h(y) = y^4 + 8$
   a. $h(3p)$
   b. $h(2s^2)$

5. $f(c) = 3c^2 + 6c - 9$
   a. $f(n - 2)$
   b. $f(4y^3)$

6. $b(t) = t^2 - 2t + 7$
   a. $b(5k)$
   b. $b(x + 1)$

Find the following function compositions.

7. $f(x)=3x+5,\ g(x)=\frac{x+4}{2}$ Find **a.** $f(g(2))$ and **b.** $g(f(2))$.

8. $f(x)=\frac{1}{4}x+1,\ g(x)=6x-7$ Find **a.** $f(g(4))$ and **b.** $g(f(4))$.

9. $f(x)=x^2,\ g(x)=2x+3$ Find **a.** $(f\circ g)(-5)$ and **b.** $(g\circ f)(-1)$.

10. $f(x)=x^2+1,\ g(x)=x-6$ Find **a.** $(f\circ g)(3)$ and **b.** $(g\circ f)(-2)$.

Form the compositions $f(g(x))$ and $g(f(x))$ for each pair of functions. See Examples 2 through 4.

11. $f(x)=\sqrt{x}, \quad g(x)=x^2$

12. $f(x)=\frac{1}{x}, \quad g(x)=\frac{1}{x}$

13. $f(x)=\sqrt{x}, \quad g(x)=x-2$

14. $f(x)=\sqrt{x}, \quad g(x)=x^2-9$

15. $f(x)=x-1, \quad g(x)=\frac{1}{x^2}$

16. $f(x)=\frac{1}{x^2}, \quad g(x)=x^2+1$

17. $f(x)=x^3+x+1, \quad g(x)=x+1$

18. $f(x)=x^3, \quad g(x)=2x-1$

19. $f(x)=\frac{1}{\sqrt{x}}, \quad g(x)=x^2$

20. $f(x)=\frac{1}{\sqrt{x}}, \quad g(x)=x^2-4$

21. $f(x)=\frac{1}{x}, \quad g(x)=x^2+7x-8$

22. $f(x)=\frac{1}{x+1}, \quad g(x)=x^2+x-3$

23. $f(x)=x^{3n}, \quad g(x)=2x-6$

24. $f(x)=x^{\frac{1}{3}}, \quad g(x)=4x+7$

25. $f(x)=x^3, \quad g(x)=\sqrt{x-8}$

26. $f(x)=x^3+1, \quad g(x)=\frac{1}{x}$

Solve.

27. For the functions $f(x)=6x-3$ and $g(x)=\frac{1}{3}x+3$, find:

a. $f(g(3))$ **b.** $g(f(0))$

c. Does it appear that *f* and *g* are inverses of each other? Explain.

28. For the functions $h(x)=-2x+4$ and $g(x)=\frac{4-x}{2}$, find:

a. $h(g(6))$ **b.** $g(h(-4))$

c. Does it appear that *h* and *g* are inverses of each other? Explain.

29. Given $f(x)=\frac{1}{2x+1}$ and $g(x)=-\frac{1}{x}$, find:

a. $g(f(4))$ **b.** $f(g(2))$

c. Explain the different results from parts a. and b.

30. Given $f(x)=\sqrt{x-9}$ and $g(x)=x-9$, find:

a. $g(f(109))$ **b.** $f(g(9))$

c. Explain the different results from parts a. and b.

Show that the given one-to-one functions are inverses of each other. Then graph both functions on the same set of axes and show the line $y = x$ as a dotted line on each graph. (You may use a calculator as an aid in finding the graphs.) See Examples 6 and 7.

31. $f(x) = 3x + 1$ and $g(x) = \dfrac{x-1}{3}$

32. $f(x) = -2x + 3$ and $g(x) = \dfrac{3-x}{2}$

33. $f(x) = \sqrt[3]{x-1}$ and $g(x) = x^3 + 1$

34. $f(x) = x^3 - 4$ and $g(x) = \sqrt[3]{x+4}$

35. $f(x) = x^2$ for $x \geq 0$ and $g(x) = \sqrt{x}$

36. $f(x) = \sqrt{x+3}$ and $g(x) = x^2 - 3$ for $x \geq 0$

37. $f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$

38. $f(x) = \sqrt[5]{x+6}$ and $g(x) = x^5 - 6$

39. $f(x) = \dfrac{2}{x}$ and $g(x) = \dfrac{2}{x}$

40. $f(x) = \dfrac{3}{x}$ and $g(x) = \dfrac{3}{x}$

Find the inverse of the given function. Then graph both functions on the same set of axes and show the line $y = x$ as a dotted line on the graph. See Examples 9 and 10.

41. $f(x) = 2x - 3$

42. $f(x) = 2x - 5$

43. $g(x) = x$

44. $g(x) = 1 - 4x$

45. $f(x) = 5x + 1$

46. $g(x) = -3x + 1$

47. $g(x) = \dfrac{2}{3}x + 2$

48. $f(x) = -\dfrac{1}{2}x - 3$

49. $f(x) = -x - 2$

50. $f(x) = -2x + 4$

51. $f(x) = x^2 + 1,\ x \geq 0$

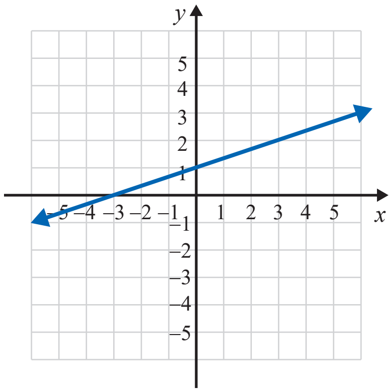
52. $f(x) = x^2 - 1,\ x \geq 0$

53. $f(x) = -\sqrt{x},\ x \geq 0$

54. $f(x) = -\sqrt{x-2},\ x \geq 2$

Using the horizontal line test, determine which of the graphs are graphs of one-to-one functions. If the graph represents a one-to-one function, graph its inverse by reflecting the graph of the function across the line $y = x$. (**Hint:** If a function is one-to-one, label a few points on the graph and use the fact that the x- and y-coordinates are interchanged on the graph of the inverse.) See Example 5.

55.



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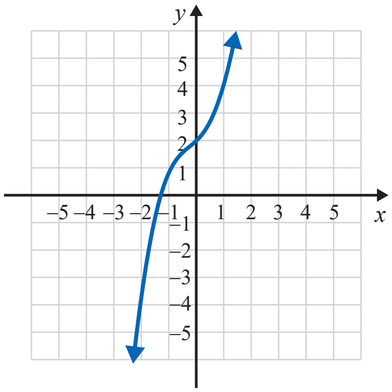

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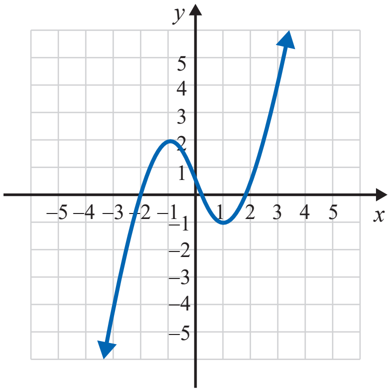

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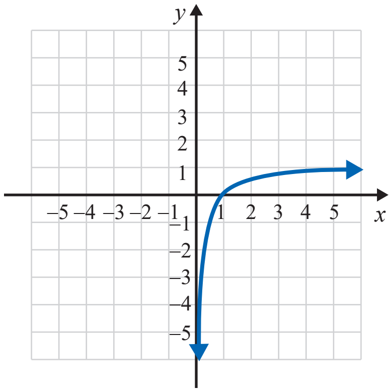

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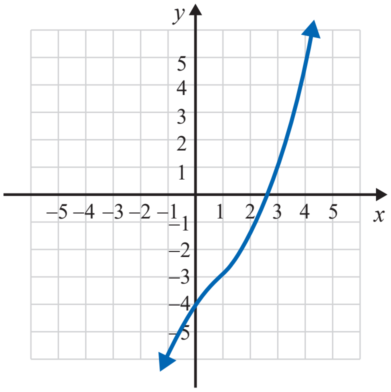

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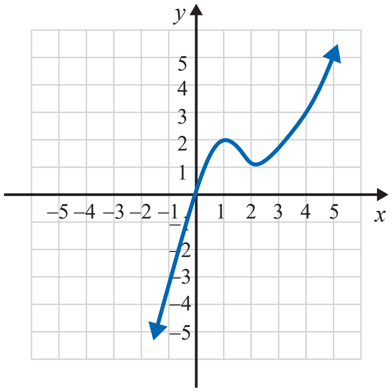

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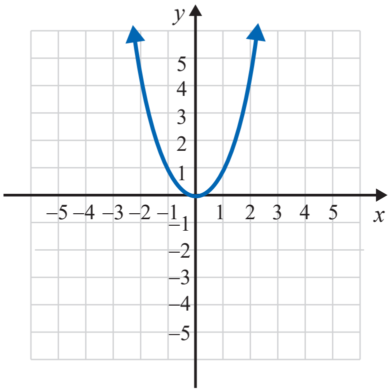

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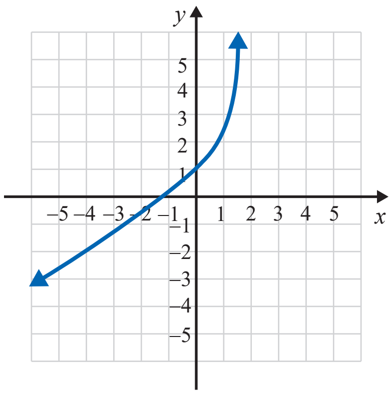

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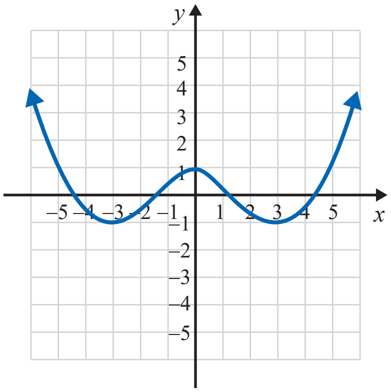

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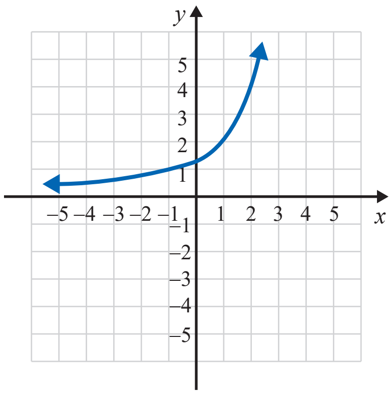

Use a graphing calculator to graph each of the functions and determine which of the functions are one-to-one by inspecting the graph and using the horizontal line test.

65. $f(x) = 2x + 3$

66. $f(x) = 7 - 4x$

67. $g(x) = x^2 - 2$

68. $g(x) = 9 - x^2$

69. $f(x) = 4 - x^3$

70. $f(x) = x^3 + 2$

71. $f(x) = \frac{4}{x}$

72. $g(x) = \frac{1}{x}$

73. $g(x) = \sqrt{x-3}$

74. $f(x) = \sqrt{x+5}$

75. $f(x) = |x+1|$

76. $f(x) = |x-5|$

Find the inverse of the given function. Then use a graphing calculator to graph both the function and its inverse. Set the `WINDOW` so that it is "square."

77. $f(x) = x^3$

78. $f(x) = (x+1)^3$

79. $f(x) = \frac{1}{x-3}$

80. $f(x) = \frac{1}{x}$

81. $f(x) = x^2, \ x \geq 0$

82. $f(x) = x^2 + 2, \ x \geq 0$

83. $g(x) = x^3 + 2$

84. $g(x) = 6 - x^3$

85. $f(x) = \sqrt{x+5}, \ x \geq -5$

86. $g(x) = \sqrt{x-3}, \ x \geq 3$

87. $f(x) = -x^2 + 1, \ x \geq 0$

88. $g(x) = -x^2 - 2, \ x \geq 0$

Writing & Thinking

89. Explain in your own words why the domains of the two composite functions $f(g(x))$ and $g(f(x))$ might not be the same. Give an example of two functions that illustrate this possibility.

90. Explain briefly why a function must be one-to-one to have an inverse.