15.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Operating algebraically with functions, as well as understanding and finding the ______ and ______ of functions, relies heavily on function notation.

2. Logarithms are ______.

3. If two or more functions have the same ______, then we can perform the operations of addition, subtraction, multiplication, and division with these functions.

4. In the case of finding the quotient of functions, no denominator can be ______.

5. When operating with functions, the operations are performed with the ______ for each value of ______ in the common domain.

6. In general, graphing the sum of two functions will involve a/an ______ number of points.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. One way to find the sum of two functions is to find the algebraic sum of the two expressions.

8. The function $(f+g)(x)$ means the same as $f(x)+g(x)$.

9. If functions do not have the same domain, any algebraic sums, differences, products, and quotients are restricted to portions of the range that are in common.

10. If two functions have graphs that consist of line segments, the sum of the two functions will produce a graph that is a continuous line.

Practice

For the following pairs of functions find, **a.** $(f+g)(x)$, **b.** $(f-g)(x)$, **c.** $(f\cdot g)(x)$, and **d.** $\left(\frac{f}{g}\right)(x)$. See Examples 1 and 2.

1. $f(x)=x+2,\ g(x)=x-5$
2. $f(x)=2x,\ g(x)=x+4$
3. $f(x)=x^2,\ g(x)=3x-4$
4. $f(x)=x-3,\ g(x)=x^2+1$
5. $f(x)=x^2-9,\ g(x)=x-3$
6. $f(x)=x^2-25,\ g(x)=x+5$
7. $f(x)=2x^2+x,\ g(x)=x^2+2$
8. $f(x)=x^3+6x,\ g(x)=x^2+6$
9. $f(x)=x^2+4x+1, g(x)=x^2-4x+1$
10. $f(x)=x^3-x^2,\ g(x)=6-x^2$

Let $f(x)=x^2+4$ and $g(x)=-x+3$. Find the values of the indicated expressions. See Examples 1 and 2.

11. $f(2)+g(2)$

12. $f(2)\cdot g(2)$

13. $g(a)-f(a)$

14. $\dfrac{g(a)}{f(a)}$

15. $(f+g)(-4)$

16. $(f-g)(0.5)$

17. $\left(\dfrac{f}{g}\right)(-2)$

18. $(f\cdot g)(-3)$

19. $(g-f)(-6)$

20. $\left(\dfrac{g}{f}\right)(-1)$

Find the indicated functions and state their domains in interval notation. See Example 3.

21. If $f(x)=\sqrt{2x-6}$ and $g(x)=x+4$, find $(f+g)(x)$.

22. If $f(x)=x^2-2x+1$ and $g(x)=x-1$, find $\left(\dfrac{f}{g}\right)(x)$.

23. Find $f(x)\cdot g(x)$ given that $f(x)=3x+2$ and $g(x)=x-7$.

24. Find $f(x)-g(x)$ given that $f(x)=x^2$ and $g(x)=x^2-2$.

25. For $f(x)=x-5$ and $g(x)=\sqrt{x+3}$, find $\dfrac{f(x)}{g(x)}$.

26. For $f(x)=2x-8$ and $g(x)=\sqrt{2-x}$, find $f(x)\cdot g(x)$.

27. If $f(x)=-\sqrt{x-3}$ and $g(x)=3x$, find $(f\cdot g)(x)$.

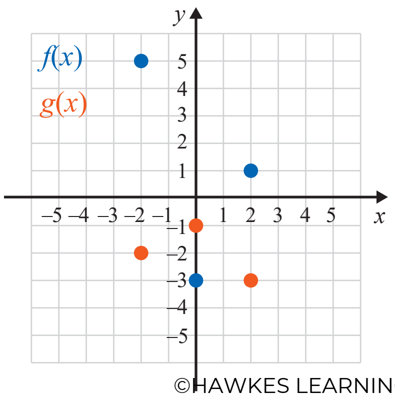
28. If $f(x)=-\sqrt{4-x}$ and $g(x)=5-x$, find $(g-f)(x)$.

29. If $f(x)=\sqrt[3]{x+3}$ and $g(x)=\sqrt{5+x}$, find $f(x)+g(x)$.

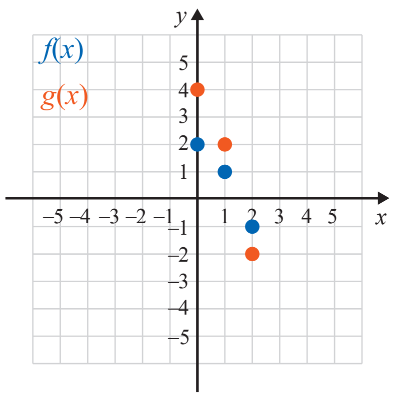
30. If $f(x)=\sqrt{x-1}$ and $g(x)=\sqrt[3]{2x+1}$, find $f(x)-g(x)$.

For the following pairs of functions, graph **a.** the sum $(f+g)$ and **b.** the difference $(f-g)$ on two different graphs.

31. $f=\{(-2,5),(0,-3),(2,1)\}$
$g=\{(-2,-2),(0,-1),(2,-3)\}$

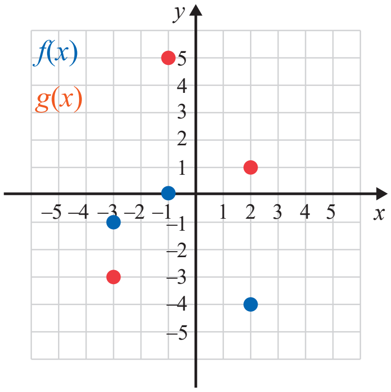


32. $f=\{(0,2),(1,1),(2,-1)\}$
$g=\{(0,4),(1,2),(2,-2)\}$



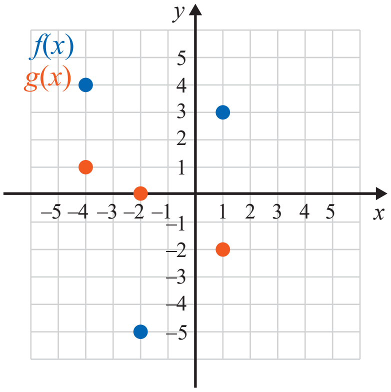
33. $f=\{(-3,-1),(-1,0),(2,-4)\}$

$g=\{(-3,-3),(-1,5),(2,1)\}$



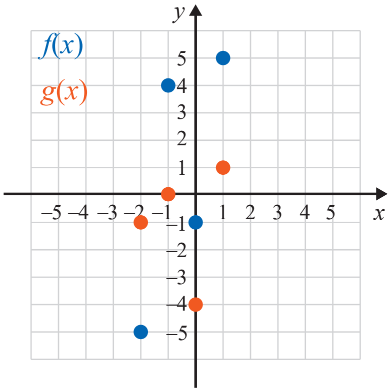
34. $f=\{(-4,4),(-2,-5),(1,3)\}$

$g=\{(-4,1),(-2,0),(1,-2)\}$



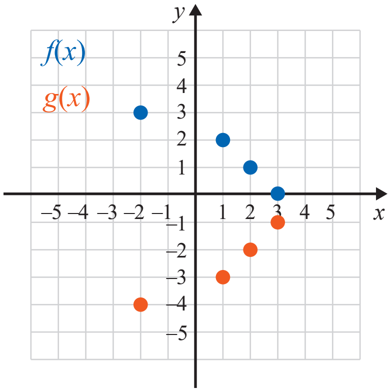
35. $f=\{(-2,-5),(-1,4),(0,-1),(1,5)\}$

$g=\{(-2,-1),(-1,0),(0,-4),(1,1)\}$



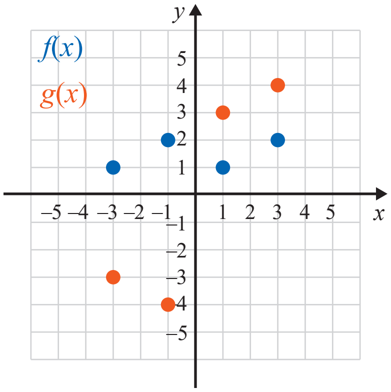
36. $f=\{(-2,3),(1,2),(2,1),(3,0)\}$

$g=\{(-2,-4),(1,-3),(2,-2),(3,-1)\}$



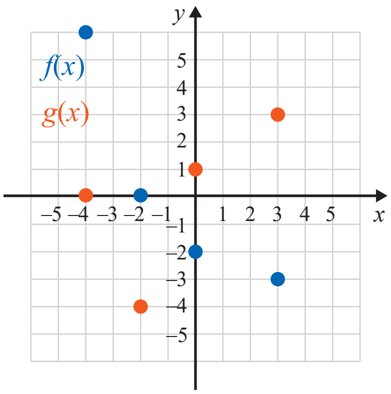
37. $f=\{(-3,1),(-1,2),(1,1),(3,2)\}$

$g=\{(-3,-3),(-1,-4),(1,3),(3,4)\}$

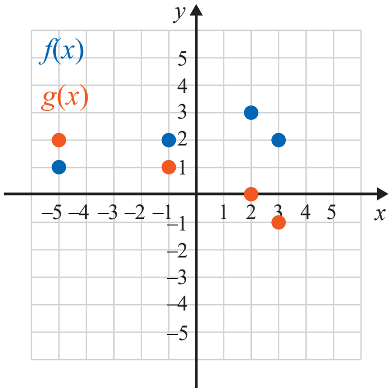


38. $f=\{(-4,6),(-2,0),(0,-2),(3,-3)\}$

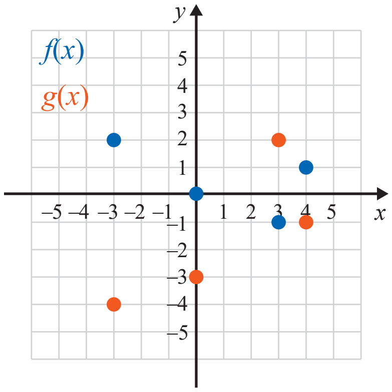
$g=\{(-4,0),(-2,-4),(0,1),(3,3)\}$



39. $f = \{(-5, 1), (-1, 2), (2, 3), (3, 2)\}$
$g = \{(-5, 2), (-1, 1), (2, 0), (3, -1)\}$



40. $f = \{(-3, 2), (0, 0), (3, -1), (4, 1)\}$
$g = \{(-3, -4), (0, -3), (3, 2), (4, -1)\}$



Graph each pair of functions and the sum of these functions on the same set of axes.

41. $f(x) = x^2$ and $g(x) = -1$

42. $f(x) = x^2$ and $g(x) = 2$

43. $f(x) = x + 1$ and $g(x) = 2x$

44. $f(x) = x + 5$ and $g(x) = x - 5$

45. $f(x) = x + 4$ and $g(x) = -x$

46. $f(x) = 2 - x$ and $g(x) = x$

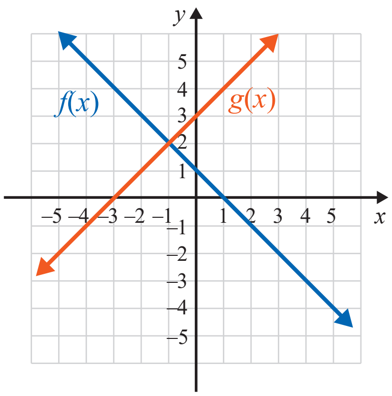
47. $f(x) = x + 1$ and $g(x) = x^2 - 1$

48. $f(x) = x^2 + 2$ and $g(x) = x^2 - 2$

49. $f(x) = \sqrt{x - 6}$ and $g(x) = 2$

50. $f(x) = \sqrt{3 - x}$ and $g(x) = -1$

Use the graph shown here to find the values indicated.



51. $(f + g)(-2)$

52. $(f - g)(2)$

53. $(f \cdot g)(3)$

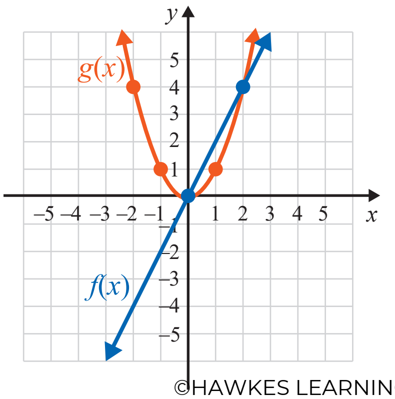
54. $(g - f)(0)$

55. $\left(\frac{f}{g}\right)(4)$

56. $(g \cdot f)(4)$

Graph the sum of each function.

57.



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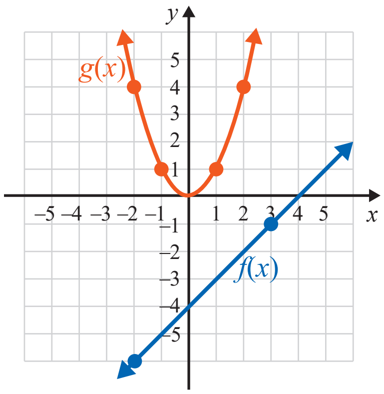

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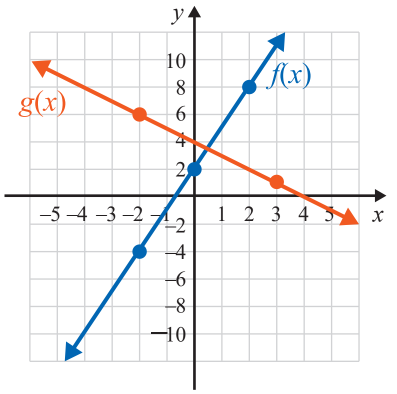

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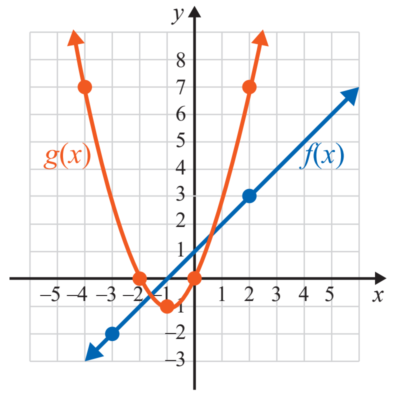

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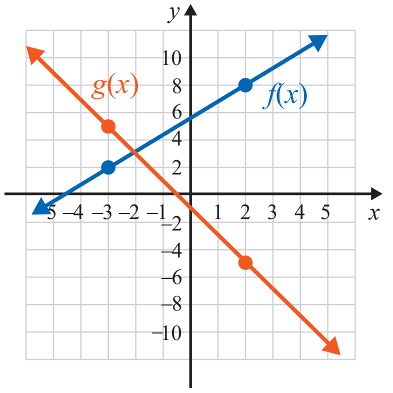

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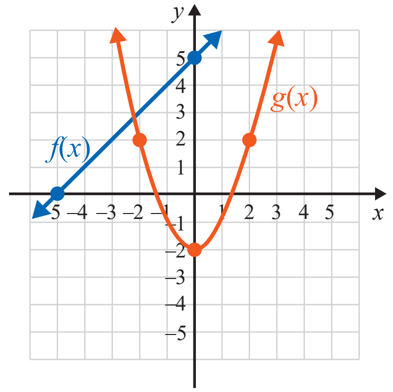


Use a graphing calculator to graph each pair of functions and the sum of these functions on the same set of axes.

63. $f(x)=x^2$ and $h(x)=2x+1$

64. $g(x)=x^2+x$ and $h(x)=3x+4$

65. $f(x)=\sqrt{x+4}$ and $g(x)=-2$

66. $f(x)=-\sqrt{x-1}$ and $g(x)=3$

67. $f(x)=\sqrt[3]{x+5}$ and $h(x)=2x$

68. $h(x)=\sqrt[3]{x-1}$ and $g(x)=x-1$

69. $g(x)=7-x^2$ and $h(x)=x^2-3$

70. $f(x)=x^2+5$ and $g(x)=4-x^2$

Writing & Thinking

71. Explain why, in general, $(f-g)(x)\neq(g-f)(x)$ if $f(x)\neq g(x)$.

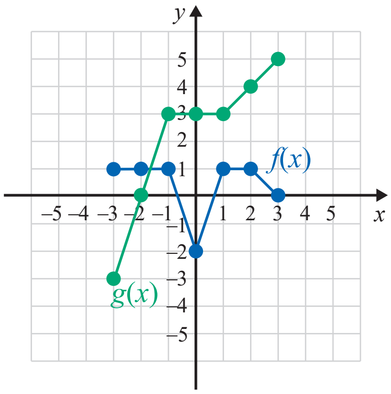
72. Given the two functions *f* and *g*,

$$f=\{(-2,0),(-1,1),(0,4),(2,4),(3,5),(4,1)\}$$
$$g=\{(-2,3),(-1,4),(0,1),(2,-1),(3,2),(4,6)\},$$

find and graph the following.

a. $f-g$ **b.** $f\cdot g$ **c.** $\frac{f}{g}$

73. Use the graphs of the two functions f and g shown in the graph.



a. Sketch the graph of $f - g$.

b. Sketch the graph of $f \cdot g$.

c. Is $\dfrac{f}{g}$ defined on the entire interval [–3, 3]? Briefly explain your reasoning.