Solution

$$b^2 - 4ac = 0$$
$$(8)^2 - 4(1)(c) = 0$$
$$64 - 4c = 0$$
$$-4c = -64$$
$$c = 16$$

Check

$$x^2 + 8x + (16) = 0$$
$$x^2 + 8x + 16 = 0$$
$$(x+4)^2 = 0$$
$$x = -4$$

There is only one real solution. Thus, -4 is a double root.

Now work margin exercise 7.

8. Determine the value(s) for a such that $ax^2 - 8x + 1 = 0$ will have two nonreal solutions.

Example 8 Understanding the Discriminant

Determine the value(s) for a such that $ax^2 - 8x + 4 = 0$ will have two nonreal solutions. (**Hint:** Set the discriminant less than 0 and solve for a.)

Solution

$$b^2 - 4ac < 0$$
$$(-8)^2 - 4(a)(4) < 0$$
$$64 - 16a < 0$$
$$-16a < -64$$
$$a > 4$$

Thus, if a is any real number greater than 4, the discriminant will be negative and the equation will have two nonreal solutions.

Now work margin exercise 8.

Margin Exercise Answers

1. $\frac{7 \pm \sqrt{33}}{2}$ **2.** $\frac{3 \pm i\sqrt{31}}{10}$ **3.** $\pm 3i$ **4.** $x = \frac{3 \pm \sqrt{13}}{2}$ **5.** $x = 0, \frac{5 \pm \sqrt{17}}{2}$ **6. a.** 253, there are two real solutions. **b.** 0, there is one real solution, a double root. **c.** -36, there are two nonreal solutions. **7.** $c = 9$ **8.** $a > 16$

14.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The general form of a quadratic equation is ____________.
2. In the quadratic formula, the expression $b^2 - 4ac$ is called the ____________.
3. The quadratic formula is $x =$ ______________.
4. When using the quadratic formula, the value of a cannot be ______.
5. To develop the quadratic formula, the general quadratic equation is solved by ______ the square.
6. In the case where the discriminant is zero, there is one real solution, also called a/an ______ root.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The quadratic formula will always work when solving quadratic equations.

8. If the discriminant is a perfect square, the quadratic equation is factorable.

9. When using the quadratic formula, if the discriminant is greater than zero, there are infinite solutions.

10. If the discriminant is less than zero, there is no real solution.

Practice

Find the discriminant and determine the nature of the solutions of each quadratic equation. See Examples 6 through 8.

1. $x^2 + 6x - 8 = 0$

2. $x^2 + 3x + 1 = 0$

3. $x^2 - 8x + 16 = 0$

4. $x^2 + 3x + 5 = 0$

5. $4x^2 + 2x + 3 = 0$

6. $3x^2 - x + 2 = 0$

7. $5x^2 + 8x + 3 = 0$

8. $4x^2 + 12x + 9 = 0$

9. $100x^2 - 49 = 0$

10. $9x^2 + 121 = 0$

11. $3x^2 + x + 1 = 0$

12. $5x^2 - 3x - 2 = 0$

Solve each of the quadratic equations using the quadratic formula See Examples 1 through 4.

13. $x^2 + 4x - 4 = 0$

14. $x^2 - 6x - 1 = 0$

15. $9x^2 + 12x + 4 = 0$

16. $4x^2 - 20x + 25 = 0$

17. $x^2 - 2x + 7 = 0$

18. $x^2 - 2x + 3 = 0$

19. $2x^2 + 5x - 3 = 0$

20. $3x^2 - 7x + 4 = 0$

21. $4x^2 + 6x + 1 = 0$

22. $2x^2 - 3x - 1 = 0$

23. $4x^2 + 6x + 3 = 0$

24. $x^2 - 5x + 7 = 0$

Solve the given equations using any of the techniques discussed for solving quadratic equations: factoring, completing the square, or using the quadratic formula.

25. $x^2 + 3x - 5 = 0$

26. $x^2 - 7x - 3 = 0$

27. $x^2 + 4x + 3 = 0$

28. $x^2 + 14x + 49 = 0$

29. $x^2 + 8 = 0$

30. $x^2 - 7 = 0$

31. $x^2 - 5x + 2 = 0$

32. $3x^2 + 2x - 2 = 0$

33. $16x^2 + 8x = -1$

34. $6x^2 = 5x + 1$

35. $3x^2 - 4 = 0$

36. $4x^2 + 9 = 0$

37. $9x^2 - 12x + 4 = 0$

38. $9x^2 - 6x + 1 = 0$

39. $2x^2 = -8x - 9$

40. $3x^2 = 6x - 4$

41. $5x^2 + 5 = 7x$

42. $4x^2 - 5x = -3$

43. $6x^2 + 2x = 20$

44. $10x^2 + 30 = -35x$

45. $3x^2 = 18x - 33$

46. $2x^2 = 16x - 36$

47. $x^2 + 4x = x - 2x^2$

48. $3x^2 + 4x = 0$

49. $x^3 - 9x^2 + 4x = 0$

50. $x^3 - 8x^2 = 3x^2 + 3x$

51. $x^3 + 3x^2 + x = 0$

52. $4x^3 + 10x^2 - 3x = 0$

53. $(2x+1)(x+3) = 2x+6$

54. $(x+5)(x-1) = -3$

55. $(3x-1)(x-2) = x+5$

56. $(x+4)(x-2) = -4$

First multiply each side of the equation by the LCD to get integer coefficients and then solve the resulting equation. See Example 3.

57. $3x^2 - 4x + \frac{1}{3} = 0$

58. $\frac{3}{4}x^2 - 2x + \frac{1}{8} = 0$

59. $2x^2 - \frac{2}{3}x + \frac{2}{9} = 0$

60. $2x^2 + 3x + \frac{5}{4} = 0$

61. $\frac{1}{2}x^2 - x + \frac{3}{4} = 0$

62. $\frac{2}{3}x^2 - \frac{1}{3}x + \frac{1}{2} = 0$

63. $\frac{1}{4}x^2 + \frac{7}{8}x + \frac{1}{2} = 0$

64. $\frac{5}{12}x^2 - \frac{1}{2}x - \frac{1}{4} = 0$

65. Determine the value(s) for c such that $x^2 - 8x + c = 0$ will have two real solutions.

66. Determine the value(s) for c such that $x^2 + 5x + c = 0$ will have two real solutions.

67. Determine the value(s) for c such that $x^2 + 9x + c = 0$ will have one real solution.

68. Determine the value(s) for c such that $x^2 - 7x + c = 0$ will have one real solution.

69. Determine the value(s) for a such that $ax^2 - 6x + 3 = 0$ will have two nonreal solutions.

70. Determine the value(s) for a such that $ax^2 + 4x - 2 = 0$ will have two nonreal solutions.

71. Determine the value(s) for a such that $ax^2 + x - 9 = 0$ will have two real solutions.

72. Determine the value(s) for a such that $ax^2 + 6x + 3 = 0$ will have two real solutions.

73. Determine the value(s) for a such that $ax^2 + 7x + 12 = 0$ will have one real solution.

74. Determine the value(s) for a such that $ax^2 - 2x + 8 = 0$ will have one real solution.

75. Determine the value(s) for c such that $3x^2 + 4x + c = 0$ will have two nonreal solutions.

76. Determine the value(s) for c such that $2x^2 + 3x + c = 0$ will have two nonreal solutions.

Solve the quadratic equations using the quadratic formula and your calculator. Write the solutions accurate to the ten-thousandth.

77. $0.02x^2 - 1.26x + 3.14 = 0$

78. $0.5x^2 + 0.07x - 5.6 = 0$

79. $\sqrt{2}x^2 - \sqrt{3}x - \sqrt{5} = 0$

80. $x^2 - 2\sqrt{10}x + 10 = 0$

81. $0.3x^2 + \sqrt{2}x + 0.72 = 0$

82. $\sqrt[3]{4}x^2 - \sqrt[4]{2}x - \sqrt{11} = 0$

83. $x^2 + 2\sqrt{15} - 15 = 0$

84. $0.05x^2 - \sqrt{30} = 0$

Applications

Solve.

85. An orange is thrown down from the top of a building that is 300 feet tall with an initial velocity of 6 feet per second. The distance of the object from the ground can be calculated using the equation $d = 300 - 6t - 16t^2$, where t is the time in seconds after the orange is thrown.

a. On a balcony, a cup is sitting on a table located 100 feet from the ground. If the orange is thrown with the right aim to fall into the cup, how long will the orange fall? Round to the nearest hundredth. (**Hint:** The distance is 100 feet.)

b. If the orange misses the cup and falls to the ground, how long will it take for the orange to splatter on the sidewalk? (**Hint:** What is the height of the orange when it hits the ground?)

c. Approximately how much longer would it take for the orange to fall to the sidewalk than it would for the orange to fall into the cup?

86. Merida is practicing archery with her recurve bow. Her target is the top of a 3-foot tall bale of hay that is 400 feet away. She aims at a 45° angle and shoots the arrow with an initial velocity of 140 feet per second. The height of the arrow can be described by $h = 99t - 16t^2 + 5$, where 99 is the vertical velocity of the arrow, h is the height of the arrow, and t is the time in seconds that passes after the arrow leaves the bow.

a. Solve the equation $3 = 99t - 16t^2 + 5$ to determine the time in seconds when the height of the arrow will be 3 feet. Round your answer to the nearest hundredth.

b. When shot at a 45° angle, the horizontal velocity of the arrow is also 99 feet per second. Use this velocity to determine how long will it take the arrow to reach the bale of hay? Round your answer to the nearest hundredth. (**Hint:** Use the $d = rt$ formula.)

c. Did Merida hit the target, undershoot the target, or overshoot the target? (**Hint:** Compare the answers from part a. and part b.)

Writing & Thinking

87. Find an equation of the form $Ax^4 + Bx^2 + C = 0$ that has the four roots ±2 and ±3. Explain how you arrived at this equation.

88. The surface area of a right circular cylinder can be found using the following formula: $S = 2\pi r^2 + 2\pi rh$, where r is the radius of the cylinder and h is the height. Estimate the radius of a circular cylinder of height 30 cm and surface area 300 cm^2. Explain how you used your knowledge of quadratic equations.