Common Error

"Divide out" only common factors.

Wrong Solution

$$\frac{4x+8}{8}$$

8 is not a common factor.

$$\frac{x^2-9}{x-3}$$

3 and x are not common factors.

Correct Solution

$$\frac{4x+8}{8}=\frac{4(x+2)}{8}=\frac{(x+2)}{2}$$

4 is a common factor.

$$\frac{x^2-9}{x-3}=\frac{(x+3)(x-3)}{(x-3)}=x+3$$

$x-3$ is a common factor.

CAUTION

Margin Exercise Answers

1. a. $x \neq \frac{1}{5}$ **b.** $x \neq 3, 4$ **c.** no restrictions **2. a.** $-\frac{4}{11}$ **b.** $-\frac{1}{8}$

3. a. $\frac{2}{5}; x \neq 3$ **b.** $\frac{x+4}{x+5}; x \neq -5, 5$ **c.** $-1; x \neq 5$

12.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The technical name for a fraction with integers in the numerator and denominator is _____ number.
2. To simplify a rational expression, divide out any common _____ from the numerators and denominators.
3. When a numerator and denominator are multiplied by the same number, this is an example of the _____ principle of rational expressions.
4. Values that make an expression undefined cannot be used and are called _____ on the variable.
5. The rules for rational expressions are the same as those for _____ in arithmetic.
6. The denominator of a rational expression can never equal _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A simplified rational expression cannot have any common factors other than 1 and –1 in both the numerator and denominator.

8. The difference between a rational number and a rational expression is that a rational expression generally has polynomials in the numerator and/or denominator.

9. While a rational number cannot have a zero denominator, a rational expression can have a zero denominator.

10. If a denominator is $x + 5$, it is defined for all values except 5.

Practice

Reduce each expression to lowest terms. State any restrictions on the variable(s). See Examples 1 and 3.

1. $\dfrac{9x^2y^3}{12xy^4}$

2. $\dfrac{18xy^4}{27x^2y}$

3. $\dfrac{20x^5}{30x^2y^3}$

4. $\dfrac{15y^4}{20x^3y^2}$

5. $\dfrac{x}{x^2-3x}$

6. $\dfrac{3x}{x^2+5x}$

7. $\dfrac{7x-14}{x-2}$

8. $\dfrac{4-2x}{2x-4}$

9. $\dfrac{9-3x}{4x-12}$

10. $\dfrac{2x-8}{16-4x}$

11. $\dfrac{6x^2+4x}{3xy+2y}$

12. $\dfrac{1+3y}{4x+12xy}$

13. $\dfrac{x^2+6x}{x^2+5x-6}$

14. $\dfrac{x^2-y^2}{3x^2+3xy}$

15. $\dfrac{x^2-4x-21}{x^2-9}$

16. $\dfrac{x^2-11x+18}{x^2-4}$

17. $\dfrac{xy-3y+2x-6}{y^2-4}$

18. $\dfrac{3x^2+14x-24}{18-9x-2x^2}$

19. $\dfrac{x^2+5x-14}{5x-2y+xy-10}$

20. $\dfrac{x^2+10x+24}{2x^2+x-28}$

Evaluate each rational expression for the given value of the variable. See Example 2.

21. $\dfrac{x-3}{3x^2};\ x=5$

22. $\dfrac{2x+1}{3x-2};\ x=1$

23. $\dfrac{5x^2}{x^2-4};\ x=-3$

24. $\dfrac{3y-4}{y^2+25};\ y=3$

25. $\dfrac{2x^2+5x}{x^2-1};\ x=0$

26. $\dfrac{n^3}{n^2-5n+6};\ n=-1$

27. $\dfrac{2m-7}{m^2+8m+12};\ m=2$

28. $\dfrac{-x+3}{x-3};\ x=-10$

29. $-\dfrac{15-x}{x-15};\ x=1000$

30. $-\dfrac{16+x}{x^2-16};\ x=20$

Applications

Solve.

31. The cost of renting a party room with tables, chairs, and simple decorations is $200 plus $15 per person attending.

a. Write a rational expression that represents the total price per person for renting the party room, where x is the number of people attending.

b. What is the price per person to rent the party room if 10 people are attending?

c. Determine which values of the variable will make the rational expression from part a. undefined.

d. Considering the context of the given problem, are there any additional restrictions on the variable? If so, explain why these restrictions are in place.

32. Amelia wants to join the Fit4Life gym, a local gym that offers a variety of different fitness classes. At Fit4Life gym, it costs $85 for a lifetime membership, and then you pay $8 per class. They are currently running a special where you get the first 10 classes for free.

a. Write a rational expression that represents the average price per class where x is the number of classes Amelia takes.

b. What is the price per class after taking 100 classes?

c. Determine which values of the variable will make the rational expression from part a. undefined.

d. Considering the context of the given problem, are there any additional restrictions on the variable? If so, explain why these restrictions are in place.

33. Columbus High School plans to buy scientific calculators for use in their Earth Science classes. If they buy the calculators in bulk from Math Supplies Plus, the cost per calculator depends on how many calculators are purchased. The cost per calculator c is determined by the equation $c = \dfrac{4(x+60)}{x+20}$, where x is the number of calculators purchased.

a. Find the cost per calculator if only 1 calculator is purchased.

b. Find the cost per calculator if 50 calculators are purchased.

c. Find the cost per calculator if 100 calculators are purchased.

d. What trend do you notice about the cost per calculator as the number of calculators purchased increases?

e. For what values of the variable is the price per calculator function undefined?

f. Considering the context of the equation, are there any additional restrictions on the variable? If so, explain why these restrictions are in place.

34. An annuity is a type of savings account that you put money into after equal periods of time to reach a goal amount. Annuities are a type of investment that is generally used to meet long-term savings goals such as college funds or retirement funds. The future value of an annuity is determined by the equation

$$FV = P\left[\frac{(1+r)^n - 1}{r}\right],$$

where FV is the future value of the annuity, P is the size of the periodic payment, r is the interest rate, and n is the number of payments or times the interest is compounded.

a. Determine the future value of an annuity if the monthly payment is \$100, the interest rate is 3%, and the payments are made for 60 months. Round your answer to the nearest cent.

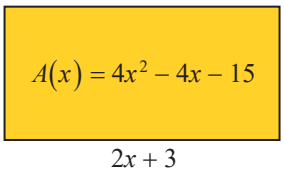
b. Determine the future value of an annuity if the monthly payment is \$200, the interest rate is 3%, and the payments are made for 60 months. Round your answer to the nearest cent.

c. What was the total amount of money paid into the annuity from part a.?

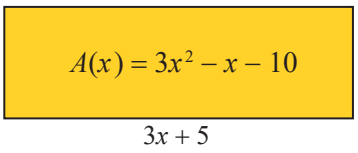
d. What was the total amount of money paid into the annuity from part b.?

e. The regular payment in part b. is double the regular payment in part a. Is the future value from part b. double the future value from part a.? Why do you think this is?

35. The area of a rectangle (in square feet) is represented by the polynomial function $A(x) = 4x^2 - 4x - 15$. If the length of the rectangle is $(2x+3)$ feet, find a representation for the width.



36. The area of a rectangle (in square feet) is represented by the polynomial function $A(x) = 3x^2 - x - 10$. If the length of the rectangle is $(3x+5)$ feet, find a representation for the width.



Writing & Thinking

37. **a.** Define the term rational expression.

b. Give an example of a rational expression that is undefined for $x = -2$ and $x = 3$ and has a value of 0 for $x = 1$. Explain how you determined this expression.

c. Give an example of a rational expression that is undefined for $x = -5$ and never has a value of 0. Explain how you determined this expression.

38. Write the opposite of each of the following expressions.

a. $3 - x$

b. $2x - 7$

c. $x + 5$

d. $-3x - 2$