

**Solution**

$$\begin{aligned} \text{a. } x^3 - 8 &= x^3 - 2^3 \\ &= (x-2)(x^2 + 2 \cdot x + 2^2) \\ &= (x-2)(x^2 + 2x + 4) \end{aligned}$$

**Note:** Remember that the second polynomial is not a perfect square trinomial and cannot be factored.

$$\begin{aligned} \text{b. } x^6 + 64y^3 &= (x^2)^3 + (4y)^3 \\ &= (x^2 + 4y) \left[ (x^2)^2 - 4y \cdot x^2 + (4y)^2 \right] \\ &= (x^2 + 4y)(x^4 - 4x^2y + 16y^2) \end{aligned}$$

- c. Factor out the GCF first. Then factor the **difference of two cubes**.

$$\begin{aligned} 16y^{12} - 250 &= 2(8y^{12} - 125) \\ &= 2 \left[ (2y^4)^3 - 5^3 \right] \\ &= 2(2y^4 - 5) \left[ (2y^4)^2 + (5)(2y^4) + 5^2 \right] \\ &= 2(2y^4 - 5)(4y^8 + 10y^4 + 25) \end{aligned}$$

**Now work margin exercise 4.****Margin Exercise Answers**

1. a.  $7a(x-7)(x+7)$  b.  $(y^3+10)(y^3-10)$  2. a. Not factorable b.  $5(9x^2+4)$   
 3. a.  $(z+20)^2$  b.  $(y-7)^2$  c.  $3z(x-3y)^2$  d.  $(y+4-z)(y+4+z)$   
 4. a.  $(y-3)(y^2+3y+9)$  b.  $(2y-x^2)(4y^2+2x^2y+x^4)$  c.  $6(2x^4-5)(4x^8+10x^4+25)$

## 11.4 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete each sentence using information found in this section.

- Factoring a perfect square trinomial gives a square \_\_\_\_\_.
- In a perfect square trinomial, both the first and last terms must be perfect \_\_\_\_\_.
- If the first term of a perfect square trinomial is  $x^2$ , and the last term is of the form  $a^2$ , then the middle term must be of the form \_\_\_\_\_ or \_\_\_\_\_.
- The formula for factoring the difference of cubes is  $x^3 - a^3 =$  \_\_\_\_\_.
- The formula for factoring the sum of two cubes is  $x^3 + a^3 =$  \_\_\_\_\_.
- The first 6 perfect cubes are 1, 8, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The expression  $x^2 + 20x + 100$  is a perfect square trinomial.
8. When factoring polynomials, always look for a common monomial factor first.
9. The sum of two squares,  $(x^2 + a^2)$ , is factorable.
10. Sixty-four is a perfect square and a perfect cube.

## Practice

Completely factor each of the given polynomials. If a polynomial cannot be factored, write "not factorable." See Examples 1 through 4.

- |                       |                              |                       |
|-----------------------|------------------------------|-----------------------|
| 1. $x^2 - 25$         | 22. $9y^2 + 12y + 4$         | 43. $4x^3 - 32$       |
| 2. $y^2 - 121$        | 23. $16x^2 - 40x + 25$       | 44. $64x^3 + 27y^3$   |
| 3. $81 - y^2$         | 24. $9x^2 - 12x + 4$         | 45. $54x^3 - 2y^3$    |
| 4. $25 - z^2$         | 25. $4x^3 - 64x$             | 46. $3x^4 + 375xy^3$  |
| 5. $2x^2 - 128$       | 26. $50x^3 - 8x$             | 47. $x^3y + y^4$      |
| 6. $3x^2 - 147$       | 27. $2x^3y + 32x^2y + 128xy$ | 48. $x^4y^3 - x$      |
| 7. $4x^4 - 64$        | 28. $3x^2y - 30xy + 75y$     | 49. $x^2y^2 - x^2y^5$ |
| 8. $5x^4 - 125$       | 29. $y^2 + 6y + 9$           | 50. $2x^2 - 16x^2y^3$ |
| 9. $y^2 + 100$        | 30. $y^2 + 4y + 4$           | 51. $24x^4y + 81xy^4$ |
| 10. $4x^2 + 49$       | 31. $x^2 - 20x + 100$        | 52. $x^6 - 64y^3$     |
| 11. $y^2 - 16y + 64$  | 32. $25x^2 - 10x + 1$        | 53. $x^6 - y^9$       |
| 12. $z^2 + 18z + 81$  | 33. $x^4 + 10x^2y + 25y^2$   | 54. $64x^2 + 1$       |
| 13. $-4x^2 + 100$     | 34. $16x^4 + 8x^2y + y^2$    | 55. $27x^3 + y^6$     |
| 14. $-12x^4 + 3$      | 35. $x^3 - 125$              | 56. $x^3 + 64z^3$     |
| 15. $9x^2 - 25$       | 36. $x^3 - 64$               | 57. $8x^3 + y^3$      |
| 16. $4x^2 - 49$       | 37. $y^3 + 216$              | 58. $x^3 + 125y^3$    |
| 17. $y^2 - 10y + 25$  | 38. $y^3 + 1$                | 59. $8y^3 - 8$        |
| 18. $x^2 + 12x + 36$  | 39. $x^3 + 27y^3$            | 60. $36x^3 + 36$      |
| 19. $4x^2 - 4x + 1$   | 40. $8x^3 + 1$               | 61. $9x^2 - y^2$      |
| 20. $49x^2 - 14x + 1$ | 41. $x^2 + 64y^2$            | 62. $x^2 - 4y^2$      |
| 21. $25x^2 + 30x + 9$ | 42. $3x^3 + 81$              | 63. $x^4 - 16y^4$     |



74. Compound interest is interest earned on interest. If a principal  $P$  is invested and compounded annually (once a year) at a rate of  $r$ , then the amount,  $A_1$  accumulated in one year is  $A_1 = P + Pr$ .

In factored form, we have  $A_1 = P + Pr = P(1+r)$ .

At the end of the second year the amount accumulated is  $A_2 = (P + Pr) + (P + Pr)r$ .

- a. Write the expression for  $A_2$  in factored form similar to that for  $A_1$ .
- b. Write an expression for the amount accumulated in three years,  $A_3$ , in factored form.
- c. Write an expression for  $A_n$  the amount accumulated in  $n$  years.
- d. Use the formula you developed in part c. and your calculator to find the amount accumulated if \$10,000 is invested at 6% and compounded annually for 20 years.

75. You may have heard of (or studied) the following rules for division of an integer by 3 and 9:

1. An integer is divisible by 3 if the sum of its digits is divisible by 3.
2. An integer is divisible by 9 if the sum of its digits is divisible by 9.

The proofs of both **1.** and **2.** can be started as follows.

Let  $abc$  represent a three-digit integer.

$$\begin{aligned} \text{Then } abc &= 100a + 10b + c \\ &= (99+1)a + (9+1)b + c \\ &= (\text{now you finish the proofs}) \end{aligned}$$

Use the pattern just shown and prove both **1.** and **2.** for a four-digit integer.