Example 4 Using Long Division (Terms Missing)

Simplify $\dfrac{x^4+9x^2-3x+5}{x^2-x+2}$ using the division algorithm.

Solution

Note that 0 is written as a placeholder for any missing powers of the variable. In this way, like terms are easily aligned vertically.

$$\begin{array}{r}
x^2+\ \ x+8 \\
x^2-x+2\overline{\big)x^4+0x^3+9x^2-3x+5} \\
\underline{-\left(x^4-\ x^3+2x^2\right)} \\
x^3+7x^2-3x \\
\underline{-\left(x^3-\ x^2+2x\right)} \\
8x^2-5x+5 \\
\underline{-\left(8x^2-8x+16\right)} \\
3x-11
\end{array}$$

Note that the remainder is of smaller degree than the divisor.

Thus, the quotient is x^2+x+8 and the remainder is $3x-11$.

In the form $Q+\dfrac{R}{D}$, we can write $x^2+x+8+\dfrac{3x-11}{x^2-x+2}$.

Now work margin exercise 4.

4. Simplify $\dfrac{7x^3+4x-9}{x-2}$ using the division algorithm.

Margin Exercise Answers

1. a. $6x^4-2x^3+3$ **b.** $\dfrac{5y^3}{2}-3y^2-4$ **2.** $3x^2-x-1-\dfrac{7}{6x+4}$ **3.** $2x^2-3x+9$

4. $7x^2+14x+32+\dfrac{55}{x-2}$

10.8 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Fractions in which the numerator and denominator are polynomials are called __________ expressions.

2. When polynomials are divided, if the remainder is 0, then both the divisor and quotient are ________ of the dividend.

3. To divide a polynomial by a monomial, divide each term in the __________ by the monomial denominator and then _______ each fraction.

4. The division algorithm with polynomials tells us that for $\frac{P}{D} = Q + \frac{R}{D}$, where P is the ________, D is the ________, Q is the quotient, and R is the remainder.

5. When dividing polynomials, the denominator (or divisor) can never equal ______.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. When dividing polynomials, any remainder must be of smaller degree than the divisor.

7. The first step in the division algorithm is to align the polynomials in ascending order.

8. To aid in organization and clarity when dividing polynomials, it is best to fill in any missing powers with ones.

9. The process followed when dividing two polynomials is called the division algorithm with polynomials.

Practice

Express each quotient as a sum (or difference) of fractions and simplify, if possible. See Example 1.

1. $\frac{8y^3 - 16y^2 + 24y}{8y}$

2. $\frac{18x^4 + 24x^3 + 36x^2}{6x^2}$

3. $\frac{34x^5 - 51x^4 + 17x^3}{17x^3}$

4. $\frac{14y^4 + 28y^3 + 12y^2}{2y^2}$

5. $\frac{110x^4 - 121x^3 + 11x^2}{11x}$

6. $\frac{15x^7 + 30x^6 - 45x^3}{15x^3}$

7. $\frac{-56x^4 + 98x^3 - 35x^2}{14x^2}$

8. $\frac{108x^6 - 72x^5 + 63x^4}{18x^4}$

9. $\frac{16y^6 - 56y^5 - 120y^4 + 64y^3}{16y^3}$

10. $\frac{20y^5 - 14y^4 + 21y^3 + 42y^2}{4y^2}$

Divide by using the division algorithm. Write the answers in the form $Q + \frac{R}{D}$, where the degree of R is less than the degree of D. See Examples 2 through 4.

11. $\frac{x^2 - 2x - 20}{x + 4}$

12. $\frac{x^2 + 9x - 5}{x - 1}$

13. $\frac{6x^2 - 11x - 3}{2x - 1}$

14. $\frac{10x^2 + 16x + 5}{5x + 3}$

15. $\frac{21x^2 + 25x - 3}{7x - 1}$

16. $\frac{15x^2 - 14x - 11}{3x - 4}$

17. $\dfrac{x^2-12x+27}{x-3}$

18. $\dfrac{x^2-12x+35}{x-5}$

19. $\dfrac{x^3-9x^2+8x-3}{x-8}$

20. $\dfrac{x^3-6x^2+8x-5}{x-2}$

21. $\dfrac{4x^3+2x^2-3x+1}{x+2}$

22. $\dfrac{3x^3+6x^2+8x-5}{x+1}$

23. $\dfrac{x^3+6x+3}{x-7}$

24. $\dfrac{2x^3+3x-2}{x-1}$

25. $\dfrac{2x^3-5x^2+6}{x+2}$

26. $\dfrac{4x^3-x^2+13}{x-1}$

27. $\dfrac{21x^3+41x^2+13x+5}{3x+5}$

28. $\dfrac{6x^3-7x^2+14x-8}{3x-2}$

29. $\dfrac{2x^3+7x^2+10x-6}{2x+3}$

30. $\dfrac{6x^3-4x^2+5x-7}{x-2}$

31. $\dfrac{x^3-x^2-10x-10}{x-4}$

32. $\dfrac{2x^3-3x^2+7x+4}{2x-1}$

33. $\dfrac{10x^3+11x^2-12x+9}{5x+3}$

34. $\dfrac{6x^3+19x^2-3x-7}{6x+1}$

35. $\dfrac{2x^3-7x+2}{x+4}$

36. $\dfrac{2x^3+4x^2-9}{x+3}$

37. $\dfrac{9x^3-19x+9}{3x-2}$

38. $\dfrac{4x^3-8x^2-9x}{2x-3}$

39. $\dfrac{6x^3+11x^2+25}{2x+5}$

40. $\dfrac{16x^3+7x+12}{4x+3}$

41. $\dfrac{x^4-3x^3+2x^2-x+2}{x-3}$

42. $\dfrac{x^4+x^3-4x^2+x-3}{x+6}$

43. $\dfrac{x^4+2x^2-3x+5}{x-2}$

44. $\dfrac{3x^4+2x^3-2x^2-1}{x+1}$

45. $\dfrac{x^4-x^2+3}{x-\frac{1}{2}}$

46. $\dfrac{x^3+2x^2+1}{x-\frac{2}{3}}$

47. $\dfrac{3x^3+5x^2+7x+9}{x^2+2}$

48. $\dfrac{2x^4+2x^3+3x^2+6x-1}{2x^2+3}$

49. $\dfrac{x^4+x^3-4x+1}{x^2+4}$

50. $\dfrac{2x^4+x^3-8x^2+3x-2}{x^2-5}$

51. $\dfrac{6x^3+5x^2-8x+3}{3x^2-2x-1}$

52. $\dfrac{x^3-9x^2+20x-38}{x^2-3x+5}$

53. $\dfrac{3x^4-7x^3+5x^2+x-2}{x^2+x+1}$

54. $\dfrac{2x^4-x^3-10x^2-3x-1}{x^2-3x+1}$

55. $\dfrac{x^4+3x-7}{x^2+2x-3}$

56. $\dfrac{3x^4-4x^2+3}{x^2+x-1}$

57. $\dfrac{x^3-27}{x-3}$

58. $\dfrac{x^3+125}{x+5}$

59. $\dfrac{x^6-1}{x+1}$

60. $\dfrac{x^6-1}{x-1}$

61. $\dfrac{x^5+1}{x-1}$

62. $\dfrac{x^6+1}{x+1}$

63. $\dfrac{x^5-x^3+x}{x+\frac{1}{2}}$

64. $\dfrac{x^4-2x^3+4}{x+\frac{4}{5}}$

Applications

Solve.

65. A moving company uses a box that has a volume of $x^3-2x^2-13x-10$ cubic inches.

a. If the height of the box is $x+2$, what is the area of the base of the box?

b. If the height of the box is $x+1$, what is the area of the base of the box?

66. A rectangular garden requires a volume of top soil modeled by the equation $200x^3+350x^2+150x$, where x is the depth in inches of top soil needed.

a. Determine an expression for the area of the garden.
(**Hint:** Volume = length · width · height and Area = length · width.)

b. If the width of the garden is $10x+10$ inches, use the expression from part a. to find an expression for the length of the garden.

c. If the depth of soil needed is 3 inches, find the volume of top soil that needs to be purchased for the garden.

d. Determine how many cubic feet of top soil is needed by dividing the answer from part c. by 1728 in.^{3}. Round your answer to the nearest tenth.
(**Note:** 1 cubic foot = 12 in. · 12 in. · 12 in. = 1728 in.^{3})

e. If the top soil comes in bags that contain 0.75 cubic feet of soil, how many bags will need to be purchased?

f. If the cost of the top soil is \$2.10 per bag (including tax), what will be the total cost of the top soil needed for the garden?

Writing & Thinking

67. Suppose that a polynomial is divided by $(3x-2)$ and the answer is given as

$$x^2+2x+4+\frac{20}{3x-2}.$$

What was the original polynomial? Explain how you arrived at this conclusion.

68. Suppose that a polynomial is divided by $(x+5)$ and the answer is given as

$$x^2-3x+2-\frac{6}{x+5}.$$

What was the original polynomial? Explain how you arrived at this conclusion.

69. Given that $P(x)=2x^3-8x^2+10x+15$.

a. Find $P(2)$ then divide $P(x)$ by $x-2$.

b. Find $P(-1)$ then divide $P(x)$ by $x+1$.

c. Find $P(4)$ then divide $P(x)$ by $x-4$.

Do you see any pattern in the values of $P(a)$ for $x=a$ and the remainders you found in the division process?