A complete summary of the rules for exponents includes the following eight rules.

Summary of the Rules for Exponents

The following rules are true for any nonzero real numbers a and b and integers m and n.

1. The exponent 1: $a = a^1$
2. The exponent 0: $a^0 = 1$
3. The product rule: $a^m \cdot a^n = a^{m+n}$
4. The quotient rule: $\dfrac{a^m}{a^n} = a^{m-n}$
5. Negative exponents: $a^{-n} = \dfrac{1}{a^n}$.
6. Power rule: $\left(a^m\right)^n = a^{mn}$.
7. Power of a product: $(ab)^n = a^n b^n$.
8. Power of a quotient: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$.

PROPERTIES

Margin Exercise Answers

1. a. x^{15} **b.** $\dfrac{1}{x^{12}}$ **c.** $\dfrac{1}{y^{15}}$ **d.** $\dfrac{1}{3^6}$ or $\dfrac{1}{9^3}$ or $\dfrac{1}{729}$ **2. a.** $16x^2$ **b.** x^7y^7 **c.** $81a^2b^2$ **d.** $\dfrac{1}{a^3b^3}$ **e.** $\dfrac{x^6}{y^8}$ **3. a.** $\dfrac{x^7}{y^7}$ **b.** $\dfrac{25}{36}$ **c.** $\dfrac{27}{a^3}$ **d.** $\dfrac{x^3}{216}$ **4. a.** $\dfrac{-27x^3}{y^9}$ **b.** $\dfrac{16b^2}{a^2}$ **5.** $\dfrac{y^{15}}{x^{30}}$ **6.** $\dfrac{64}{225x^{18}y^2}$

10.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Moving any term from the numerator to the denominator, or vice versa, changes the sign of the corresponding _____.
2. A power of a quotient (in fraction form) is found by raising both the _____ and the _____ to that power.
3. To find the value of a power raised to a power, _____ the exponents and _____ the base.
4. A power of a product can be found by _____ each factor to that power.
5. In an expression such as $-x^2$, we know that -1 is understood to be the _____ of x^2.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. Taking the reciprocal of a fraction changes the sign of any exponent in the fraction.

7. For an exponent to refer to -7 as the base, -7 must be in parentheses.

8. When simplifying an expression with exponents, the rules for exponents must be used in a specific order or the answer will vary.

9. The expression -8^2 simplifies to -64.

Practice

Use the rules for exponents to simplify each of the expressions. Assume that all variables represent nonzero real numbers. See Examples 1 through 6.

1. -3^4

2. -5^2

3. -2^4

4. -20^2

5. $(-10)^6$

6. $(-4)^6$

7. $\left(a^3\right)^2$

8. $\left(b^2\right)^{-4}$

9. $\left(x^{-5}\right)^2$

10. $\left(x^{-2}\right)^{-3}$

11. $\left(2^4\right)^{-2}$

12. $\left(2^{-3}\right)^{-2}$

13. $(3y)^2$

14. $(ab)^4$

15. $(-4xy)^2$

16. $\left(3x^{-2}\right)^2$

17. $(xy)^{-6}$

18. $\left(a^3b^{-2}\right)^3$

19. $\left(6x^3\right)^2$

20. $\left(-3x^4\right)^2$

21. $4\left(-3x^2\right)^3$

22. $7\left(2y^{-2}\right)^4$

23. $5\left(x^2y^{-1}\right)$

24. $-3\left(7xy^2\right)^0$

25. $-2\left(3x^5y^{-2}\right)^{-3}$

26. $-4\left(5x^{-3}y\right)^{-1}$

27. $\left(\frac{a}{b}\right)^4$

28. $\left(\frac{x}{2}\right)^3$

29. $\left(\frac{2}{3}\right)^2$

30. $\left(\frac{a}{4}\right)^3$

31. $\left(\frac{x}{y}\right)^6$

32. $\left(\frac{2}{5}\right)^2$

33. $\left(\frac{3x}{y}\right)^3$

34. $\left(\frac{-4x}{y^2}\right)^2$

35. $\left(\frac{6m^3}{n^5}\right)^0$

36. $\left(\frac{3x^2}{y^3}\right)^2$

37. $\left(\frac{-2x^2}{y^{-2}}\right)^2$

38. $\left(\frac{2x}{y^5}\right)^{-2}$

39. $\left(\frac{x}{y}\right)^{-2}$

40. $\left(\frac{2a}{b}\right)^{-1}$

41. $\left(\frac{3x}{y^{-2}}\right)^{-1}$

42. $\left(\frac{4a^2}{b^{-3}}\right)^{-3}$

43. $\left(\frac{-3}{xy^2}\right)^{-3}$

44. $\left(\frac{5xy^3}{y}\right)^2$

45. $\left(\frac{m^2n^3}{mn}\right)^2$

46. $\left(\frac{2ab^3}{b^2}\right)^4$

47. $\left(\frac{-7^2x^2y}{y^3}\right)^{-1}$

48. $\left(\frac{2ab^4}{b^2}\right)^{-3}$

49. $\left(\frac{5x^3y}{y^2}\right)^2$

50. $\left(\frac{2x^2y}{y^3}\right)^{-4}$

51. $\left(\frac{x^3y^{-1}}{y^2}\right)^2$

52. $\left(\frac{2a^2b^{-1}}{b^2}\right)^3$

53. $\left(\frac{6y^5}{x^2y^{-2}}\right)^2$

54. $\left(\frac{3x^4}{x^{-2}y^{-4}}\right)^3$

55. $\frac{\left(7x^{-2}y\right)^2}{\left(xy^{-1}\right)^2}$

56. $\frac{\left(-5x^3y^4\right)^2}{\left(3x^{-3}y\right)^2}$

57. $\frac{\left(3x^2y^{-1}\right)^{-2}}{\left(6x^{-1}y\right)^{-3}}$

58. $\frac{\left(2x^{-3}\right)^{-3}}{\left(5y^{-2}\right)^{-2}}$

59. $\frac{\left(4x^{-2}\right)\left(6x^5\right)}{\left(9y\right)\left(2y^{-1}\right)}$

60. $\frac{\left(5x^2\right)\left(3x^{-1}\right)^2}{\left(25y^3\right)\left(6y^{-2}\right)}$

61. $\left(\frac{3xy^3}{4x^2y^{-3}}\right)^{-1}\left(\frac{2x^3y^{-1}}{9x^{-3}y^{-1}}\right)^2$

62. $\left(\frac{5a^4b^{-2}}{6a^{-4}b^3}\right)^{-2}\left(\frac{5a^3b^4}{2^{-2}a^{-2}b^{-2}}\right)^3$

63. $\left(\frac{6x^{-4}yz^{-2}}{4^{-1}x^{-4}y^3z^{-2}}\right)^{-1}\left(\frac{2^{-2}xyz^{-3}}{12x^2y^2z^{-1}}\right)^{-2}$

64. $\left(\frac{3^{-5}a^5b^3c^{-1}}{3^{-2}abc}\right)^{-2}\left(\frac{7^{-1}a^{-4}bc^2}{7^{-2}a^{-3}bc^{-2}}\right)^{-2}$

Use a graphing calculator to evaluate each expression. Round quotients to the nearest ten-thousandth, if necessary.

65. $\left(2.1^2\right)^2$

66. $\left(1.4^{-2}\right)^5$

67. $\left(3.8x\right)^4$

68. $\left(5.2x^2\right)^3$

69. $\left(\frac{8.1}{1.7}\right)^2$

70. $\left(\frac{2.3}{4.5}\right)^3$