

Chapter 13 Project

Rationally Increasing Precision in Population Problems

An activity to demonstrate the use of rational exponents in real life.

Sometimes we need to find the value of an exponential expression where the exponent is not an integer. This often happens when dealing with exponential growth. In the essay “Observations Concerning the Increase of Mankind, Peopling of Countries, etc.,” written in 1751, Benjamin Franklin projected that the human population in the thirteen US colonies was doubling in size every twenty-five years. For example, if one year the population was 300,000 people, then to estimate the number of people 75 years later, calculate $300,000 \cdot 2^3 = 2,400,000$. Note that this works because $\frac{75}{25} = 3$, meaning the population would double 3 times in 75 years. Also notice that since $2 \cdot 2 \cdot 2 = 2^3 = 8$, we multiply 300,000 by 8 to get the final population.

If the timespan we want to estimate the future population for is not a multiple of 25, we can still calculate this value using rational exponents (with as much precision as we like). This investigation will suggest which types of numbers can be exponents, a topic which will be expanded upon in Chapter 15.

In the following investigation, do not round the exponents. Round the answers to have 10 digits, if necessary.

- Calculate 2 to each of the following powers.
 - 3
 - 3.1
 - 3.14
 - 3.141
 - 3.1415
 - 3.14159
 - 3.141592
 - 3.1415926
 - 3.14159265
- The sequence of exponents in Problem 1 is approaching which special number? (**Hint:** See Section 6.6.)
- In Problem 1, are the calculated powers of 2 increasing, decreasing, or is there no discernible pattern? If increasing (or decreasing), are they increasing (or decreasing) toward a particular value?
- Consider raising the value 2 to the power of the special number found in Problem 2.
 - If it is possible, state the result and compare it to the results of Problem 1. If this is not possible, explain why.
 - How does this relate to your answer in Problem 3? If there is no relation, explain why.
- Returning to Franklin’s population prediction, consider if someone wanted to know the population 77.5 years later, instead of 75 years later.
 - Determine the decimal form of the exponent $x = \frac{77.5}{25}$.
 - Substitute the value of x found in part a. into the population equation and simplify: $300,000 \cdot 2^x$.
 - Explain what x stands for. Interpret the answer to part b. In this case, does it make sense to round or not? If rounding does make sense, what place would you round to and what is the result?
 - If the population starts at 300,000, how many years have passed if $300,000 \cdot 2^{3.14}$ provides an estimate of the population size?