R.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. When an expression has an exponent of 3, the base is said to be _____.
2. Exponents are used to represent repeated _____.
3. In 2^4 the "2" is called the _____ and the "4" is called the _____.
4. 10 squared is equal to _____.
5. If any nonzero number has an exponent of 0, the value is always _____.
6. If there are multiple grouping symbols to be simplified, begin with the _____ group.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Nine squared is equal to eighteen.
8. $2^7 = 128$
9. 7^0 is undefined.
10. According to the order of operations, multiplication is always performed before division.

Practice

Rewrite each product by using exponents. See Example 1.

1. $11 \cdot 11 \cdot 11$
2. $13 \cdot 13 \cdot 13$
3. $7 \cdot 7 \cdot 7 \cdot 7$
4. $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$
5. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
6. $2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$
7. $5 \cdot 5 \cdot 5 \cdot 7 \cdot 7$
8. $3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7$
9. $2 \cdot 3 \cdot 3 \cdot 11 \cdot 11$
10. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 11 \cdot 11 \cdot 13 \cdot 13$

For each exponential expression **a.** identify the base, **b.** identify the exponent, and **c.** evaluate the exponential expression. See Example 2.

11. 4^2
12. 6^2
13. 2^3
14. 3^3
15. 1^6
16. 1^5

17. 5^3

18. 4^3

19. 2^4

20. 2^6

21. 9^2

22. 11^2

23. 7^2

24. 7^3

25. 3^5

26. 4^5

27. 30^2

28. 40^2

29. 20^3

30. 15^2

31. 1^{57}

32. 1^{99}

33. 4^0

34. 19^0

35. 13^1

36. 24^1

37. 22^0

38. 99^0

Determine which numbers, if any, in each set of counting numbers are prime. See Example 3.

39. $\{13, 15, 17, 21\}$

40. $\{11, 19, 23, 51\}$

41. $\{2, 4, 6, 8, 10, 12, 14\}$

42. $\{7, 16, 25, 36, 47, 49\}$

Find two factors of each number (other than 1 and the number itself) to determine that the number is composite. (Answers will vary.) See Example 4.

43. 72

44. 63

45. 68

46. 39

47. 502

48. 417

49. 170

50. 99

51. 444

52. 230

Find the prime factorization of each of the numbers. If a number is prime, write "prime." See Examples 5 and 6.

53. 52

54. 60

55. 616

56. 460

57. 308

58. 155

59. 79

60. 43

61. 289

62. 361

63. 125

64. 343

65. 400

66. 500

67. 120

68. 196

69. 231

70. 675

71. 1692

72. 1716

73. List the first ten **multiples** of each number.

a. 5

b. 6

c. 10

d. 15

74. From the lists you made in Exercise 73, find the least common multiple for each of the following sets of numbers.

a. $\{5, 6\}$

b. $\{6, 10\}$

c. $\{5, 10, 15\}$

d. $\{6, 10, 15\}$

Find the LCM of each of the following sets of counting numbers. See Examples 7 and 8.

75. $\{3, 5, 7\}$

76. $\{2, 7, 11\}$

77. $\{8, 10\}$

78. $\{9, 12\}$

79. $\{2, 3, 11\}$

80. $\{3, 5, 13\}$

81. $\{4, 14, 35\}$

82. $\{10, 12, 20\}$

83. $\{50, 75\}$

84. $\{30, 70\}$

85. $\{20, 90\}$

86. $\{50, 80\}$

87. $\{28, 98\}$

88. $\{45, 75\}$

89. $\{10, 15, 35\}$

90. $\{6, 24, 30\}$

91. $\{15, 45, 90\}$

92. $\{14, 28, 56\}$

93. $\{20, 50, 100\}$

94. $\{30, 60, 120\}$

95. $\{10, 15, 25\}$

96. $\{22, 44, 121\}$

97. $\{26, 28, 91\}$

98. $\{34, 51, 54\}$

99. $\{35, 40, 72\}$

100. $\{30, 35, 63\}$

101. $\{12, 21, 44\}$

102. $\{20, 28, 45\}$

103. $\{99, 121, 231\}$

104. $\{81, 225, 324\}$

105. $\{48, 120, 144, 192\}$

106. $\{125, 135, 225, 250\}$

107. $\{40, 56, 160, 196\}$

108. $\{35, 49, 63, 126\}$

Applications

Solve.

109. Neville bought 15 boxes of trading cards. Each box has 10 packs of trading cards. Each pack of trading cards contains 20 cards. He adds 132 cards that he already owns to the newly purchased cards. Then, Neville evenly distributes all of the cards to 6 of his friends. How many trading cards would each person get?

a. If you simplify the expression $15 \cdot 10 \cdot 20 + 132 \div 6$ using the order of operations, will you get the correct answer? If not, explain what is wrong with the expression.

b. What is the answer? If necessary, write the corrected expression to get the correct results when following the order of operations.

110. Robert is purchasing shirts for his weekend soccer team. The shirts he wants to buy are normally \$25 each but are on sale for \$10 off. His team has a total of 11 players. How much will he spend to buy the shirts?

a. If you simplify the expression $\$25 - \$10 \cdot 11$ using the order of operations, will you get the correct answer? If not, explain what is wrong with the expression.

b. What is the answer? If necessary, write the corrected expression to get the correct results when following the order of operations.

111. Camila is a seamstress and is creating wedding dresses. She has 126 yards of silk fabric. For each dress, the skirt requires 4 yards of silk and the bodice requires 2 yards of silk. How many dresses can she make with the amount of silk she has?

a. If you simplify the expression $126 \div 4 + 2$ using the order of operations, will you get the correct answer? If not, explain what is wrong with the expression.

b. What is the answer? If necessary, write the corrected expression to get the correct results when following the order of operations.

112. Two astronauts miss connections at their first meeting in space.

a. If one astronaut circles the earth every 15 hours and the other every 18 hours, in how many hours will they meet again at the same place?

b. How many more orbits will each astronaut have to complete between missing their first meeting and making their second meeting?

113. Three neighbors mow their lawns at different intervals during the summer months. The first one mows every 5 days, the second every 7 days, and the third every 10 days.

a. How frequently do they mow their lawns on the same day?

b. How many times does each neighbor mow in between the times when they all mow together?

114. Four women work for the same book company selling textbooks. They leave the home office on the same day and take 8 days, 12 days, 14 days, and 15 days, respectively, to visit schools in their own sales regions.

a. In how many days will they all meet again at the home office?

b. How many sales trips will each have made in this time?

115. A fruit production company has three packaging facilities, each of which uses different-sized boxes as follows: 24 pieces/box, 36 pieces/box, and 45 pieces/box.

a. Assuming that the truck provides the same quantity of uniformly-sized pieces of fruit to all three packaging facilities, what is the minimum number of pieces of fruit that will be delivered so that no fruit will be left over?

b. How many boxes will each facility package?

116. Three swimmers decide to swim laps together, and they will quit when they reach the starting end of the pool together. The first swimmer can swim a lap in 35 seconds, the second will take 40 seconds, and the third takes 42 seconds.

a. How many seconds will it take before they quit?

b. How many laps will each swimmer swim in that interval?

117. Two analog clocks are sitting next to each other. The first clock keeps perfect time, where the minute hand takes 60 minutes to travel completely around the dial. The second clock runs fast and the minute hand makes one complete revolution in 55 minutes.

a. Assuming that both clocks are started so that the minute hands are at 12, how many minutes will it take until both minute hands return to 12 at the same time?

b. How many hours does this represent?

Writing & Thinking

118. Use your calculator to find the following values and discuss, in your own words, any pattern that you notice.

a. 86^0

b. 623^0

c. 9072^0

119. Give one example where addition should be completed before multiplication.

120. Explain how someone might think that $1 + 3^2 = 16$. Then, explain why this would not be correct.

121. List five prime numbers larger than 50.

122. Describe, in your own words, how to find the LCM of a set of counting numbers.

123. **a.** Explain why 1 is not a prime number.

b. Explain why 1 is not a composite number.

Collaborative Learning

124. In groups of three to four students, use a calculator to evaluate 20^{10} and 10^{20}. Discuss what you think is the meaning of the notation on the display.

R.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. If a fraction has a numerator that is equal to or larger than the denominator, it is a/an _____ fraction.
2. A fraction that has a zero in the denominator is considered to be _____.
3. Any whole number can be written in fraction form with denominator _____.
4. Finding a fraction "of" a number requires _____.
5. If all the factors in the numerator or denominator are divided out, then _____ must be used as a factor.
6. Finding _____ factorizations may help in multiplying and reducing at the same time.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

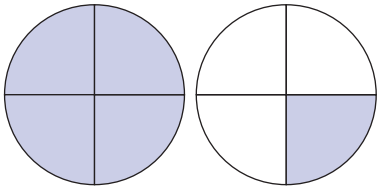
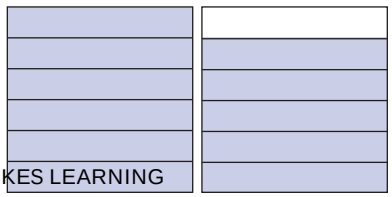
7. In $\frac{11}{13}$, the denominator is 11.
8. $\frac{17}{0}$ is undefined.
9. To find $\frac{1}{2}$ of $\frac{2}{9}$ requires multiplication.
10. The statement $\frac{1}{3}\cdot\frac{2}{5}=\frac{2}{5}\cdot\frac{1}{3}$ is an example of the associative property of multiplication.

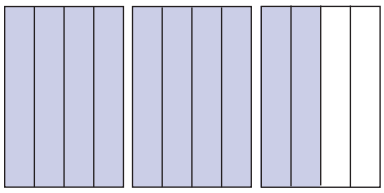
Practice

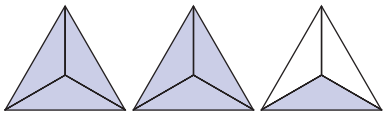
Draw a figure to represent each fraction. .

1. $\frac{1}{3}$
2. $\frac{1}{2}$
3. $\frac{4}{5}$
4. $\frac{3}{4}$

Write a fraction that indicates the shaded parts of each figure. See Example 1.

5. 
6. 

7. 

8. 

Multiply. See Examples 2 through 4.

9. $\frac{1}{7}\cdot\frac{1}{7}$

10. $\frac{2}{5}\cdot\frac{2}{5}$

11. $\frac{0}{4}\cdot\frac{7}{6}$

12. $\frac{2}{1}\cdot\frac{5}{1}$

13. $\frac{3}{5}\cdot\frac{4}{7}$

14. $\frac{2}{3}\cdot\frac{5}{11}$

15. $\frac{5}{8}\cdot\frac{3}{4}$

16. $\frac{7}{6}\cdot\frac{5}{2}$

17. $\frac{1}{2}\cdot\frac{3}{7}\cdot\frac{9}{2}$

18. $\frac{7}{3}\cdot\frac{2}{5}\cdot\frac{1}{9}$

19. $\frac{7}{8}\cdot\frac{7}{9}\cdot\frac{7}{3}$

20. $\frac{8}{5}\cdot\frac{8}{5}\cdot\frac{7}{1}$

21. Find $\frac{2}{3}$ of $\frac{2}{15}$.

22. Find $\frac{4}{7}$ of $\frac{3}{5}$.

23. Find $\frac{1}{3}$ of $\frac{2}{3}$.

24. Find $\frac{1}{4}$ of $\frac{3}{4}$.

Find the missing numerator that will make the fractions equivalent. See Examples 5 and 6.

25. $\frac{3}{4}=\frac{3}{4}\cdot\frac{?}{?}=\frac{?}{12}$

26. $\frac{2}{3}=\frac{2}{3}\cdot\frac{?}{?}=\frac{?}{12}$

27. $\frac{6}{7}=\frac{6}{7}\cdot\frac{?}{?}=\frac{?}{14}$

28. $\frac{5}{8}=\frac{5}{8}\cdot\frac{?}{?}=\frac{?}{40}$

29. $\frac{3}{16}=\frac{3}{16}\cdot\frac{?}{?}=\frac{?}{80}$

30. $\frac{1}{17}=\frac{1}{17}\cdot\frac{?}{?}=\frac{?}{51}$

31. $\frac{7}{26}=\frac{7}{26}\cdot\frac{?}{?}=\frac{?}{52}$

32. $\frac{9}{10}=\frac{9}{10}\cdot\frac{?}{?}=\frac{?}{100}$

33. $\frac{18}{1}=\frac{18}{1}\cdot\frac{?}{?}=\frac{?}{3}$

34. $\frac{1}{5}=\frac{1}{5}\cdot\frac{?}{?}=\frac{?}{75}$

Reduce each fraction to lowest terms. If it is already in lowest terms, simply rewrite the fraction. See Examples 7 and 8.

35. $\frac{3}{9}$

36. $\frac{2}{8}$

37. $\frac{9}{12}$

38. $\frac{6}{20}$

39. $\frac{5}{11}$

40. $\frac{7}{13}$

41. $\frac{0}{25}$

42. $\frac{0}{16}$

43. $\frac{12}{35}$

44. $\frac{27}{56}$

45. $\frac{16}{32}$

46. $\frac{25}{50}$

47. $\frac{42}{63}$

48. $\frac{12}{35}$

49. $\frac{48}{12}$

50. $\frac{72}{36}$

51. $\frac{24}{100}$

52. $\frac{70}{100}$

53. $\frac{150}{135}$

54. $\frac{140}{112}$

Multiply and reduce to lowest terms. See Examples 9 through 11. (**Hint:** Factor before multiplying.)

55. $\frac{1}{3}\cdot\frac{3}{4}$

56. $\frac{3}{7}\cdot\frac{5}{3}$

57. $\frac{2}{3}\cdot\frac{4}{3}$

58. $\frac{3}{5}\cdot\frac{2}{7}$

59. $\frac{5}{16}\cdot\frac{16}{15}$

60. $\frac{14}{9}\cdot\frac{3}{14}$

61. $\frac{7}{8}\cdot\frac{9}{14}$

62. $\frac{8}{10}\cdot\frac{5}{4}$

63. $\frac{2}{21}\cdot\frac{15}{22}$

64. $\frac{3}{16}\cdot\frac{20}{21}$

65. $\frac{15}{27}\cdot\frac{9}{30}$

66. $\frac{25}{9}\cdot\frac{3}{100}$

67. $8\cdot\frac{5}{12}$

68. $9\cdot\frac{7}{24}$

69. $\frac{32}{20}\cdot\frac{13}{9}\cdot\frac{7}{26}$

70. $\frac{20}{32}\cdot\frac{9}{13}\cdot\frac{26}{7}$

71. $\frac{9}{10}\cdot\frac{35}{40}\cdot\frac{25}{15}$

72. $\frac{5}{12}\cdot\frac{56}{42}\cdot\frac{90}{54}$

73. $\frac{17}{100}\cdot\frac{27}{34}\cdot\frac{25}{9}\cdot 6$

74. $\frac{13}{28}\cdot\frac{7}{9}\cdot\frac{45}{39}\cdot 4$

Divide and reduce to lowest terms. See Examples 12 and 13.

75. $\frac{5}{8}\div\frac{3}{5}$

76. $\frac{2}{7}\div\frac{1}{2}$

77. $\frac{2}{3}\div\frac{1}{5}$

78. $\frac{2}{11}\div\frac{1}{7}$

79. $\frac{3}{14}\div\frac{3}{14}$

80. $\frac{5}{8}\div\frac{5}{8}$

81. $\frac{3}{4}\div\frac{4}{3}$

82. $\frac{9}{10}\div\frac{10}{9}$

83. $\frac{15}{20}\div 3$

84. $\frac{14}{20}\div 7$

85. $\frac{25}{40}\div 10$

86. $\frac{36}{80}\div 9$

87. $\frac{7}{8}\div 0$

88. $\frac{15}{64}\div 0$

89. $0\div\frac{5}{6}$

90. $0 \div \frac{1}{2}$

91. $\frac{16}{35} \div \frac{2}{7}$

92. $\frac{15}{27} \div \frac{5}{9}$

93. $\frac{15}{24} \div \frac{25}{18}$

94. $\frac{36}{25} \div \frac{24}{20}$

Applications

Solve.

95. If you had \$20 and you spent \$9 for a hamburger, fries, and a soft drink, what fraction of your money did you spend? What fraction would you still have?

96. In a class of 35 students, 6 students received As on a mathematics exam. What fraction of students received an A? What fraction of students did not receive an A?

97. A software company receives 45 technical support calls in one hour. Twenty-three of the calls are related to customers forgetting their passwords. What fraction of the calls was related to customers forgetting their passwords?

98. A certain brand of plain bagels has 146 calories per bagel. 115 calories come from the carbohydrates in the bagel. What fraction of the calories is from carbohydrates?

99. What fraction of a minute does 43 seconds represent? (**Hint:** There are 60 seconds in a minute.)

100. There are 5280 feet in a mile. What fraction of a mile does 923 feet represent?

101. The product of $\frac{5}{6}$ with another number is $\frac{2}{5}$.

a. Which number is the product?

b. What is the other number?

102. The product of two numbers is 210.

a. If one of the numbers is the fraction $\frac{2}{3}$, do you expect the other number to be larger or smaller than 210?

b. What is the other number?

103. An airplane is carrying 90 passengers. This is $\frac{9}{10}$ of the capacity of the airplane.

a. Is the capacity of the airplane more or less than 90?

b. If you were to multiply 90 times $\frac{9}{10}$ would the product be more or less than 90?

c. What is the capacity of the airplane?

104. The student senate has 75 members, and $\frac{7}{15}$ of these are women. A change in the senate constitution is being considered, and at the present time (before debating has begun), a survey shows that $\frac{3}{5}$ of the women and $\frac{4}{5}$ of the men are in favor of this change.

a. How many women are on the student senate?

b. How many women on the senate are in favor of the change?

c. If the change requires a $\frac{2}{3}$ majority vote in favor to pass, would the constitutional change pass if the vote were taken today?

d. By how many votes would the change pass or fail?

105. The tennis club has 250 members, and they are considering putting in a new tennis court. The cost of the new court is going to involve an assessment of \$200 for each member. Seven-tenths of the members live quite near the club and $\frac{3}{5}$ of them are in favor of the assessment. However, $\frac{2}{3}$ of the members who do not live nearby are not in favor of the assessment.

a. If a vote were taken today, would more than one-half of the members vote for or against the new court?

b. By how many votes would the question pass or fail if more than one-half of the members must vote in favor for the question to pass?

106. There are 3000 students at Mountain High School and $\frac{1}{4}$ of these students are seniors. If $\frac{3}{5}$ of the seniors are in favor of the school forming a debating team and $\frac{7}{10}$ of the remaining students (not seniors) are also in favor of forming a debating team, how many students do not favor this idea?

107. A computer stores data on a hard drive in the form of bits, bytes, and sectors.

- **a.** Each byte is made up of eight bits. What fraction of a byte is a bit?
- **b.** A sector on a hard drive is traditionally 512 bytes. A byte is what fraction of a sector?
- **c.** If a computer stores 159 bytes of data, what fraction of a sector does that amount of data take up?

108. The gas tank of a car holds 14 gallons of gas. What fraction of the tank does 9 gallons of gas take up?

109. A small box will hold 12 books. Kathleen has 35 books to pack into small boxes.

- **a.** Write an improper fraction to describe the number of boxes that will be filled by Kathleen's books.
- **b.** Change the improper fraction from Part **a.** to a mixed number to describe the number of boxes that will be filled by Kathleen's books.

110. A cup holds 8 ounces of liquid. You have 29 ounces of juice to pour into cups.

- **a.** Write an improper fraction to describe the number of cups that will be filled with juice.
- **b.** Change the improper fraction from Part **a.** to a mixed number to describe the number of cups that will be filled with juice.

Writing & Thinking

111. In your own words, list the parts of a fraction and briefly describe the purpose of each part.

Step 2: Steps 2, 3, and 4 can be written in one step.

$$\frac{7}{20}-\frac{3}{28}=\frac{7}{20}\cdot\frac{7}{7}-\frac{3}{28}\cdot\frac{5}{5}$$
$$=\frac{49}{140}-\frac{15}{140}$$
$$=\frac{34}{140}=\frac{\cancel{2}\cdot 17}{\cancel{2}\cdot 70}=\frac{17}{70}$$

Or we can write the fractions vertically.

$$\begin{array}{rcl} \frac{7}{20} & = & \frac{7}{20}\cdot\frac{7}{7} \\ -\frac{3}{28} & = & -\frac{3}{28}\cdot\frac{5}{5} \\ \hline & & \end{array} \quad \begin{array}{rcl} & = & \frac{49}{140} \\ & = & -\frac{15}{140} \\ \hline & & \end{array}$$
$$\frac{34}{140}=\frac{\cancel{2}\cdot 17}{\cancel{2}\cdot 70}=\frac{17}{70}$$

Now work margin exercise 8.

Margin Exercise Answers

1. $\frac{2}{3}$ **2.** $\frac{8}{9}$ **3.** $\frac{7}{15}$ **4.** $\frac{5}{12}$ **5.** $\frac{13}{30}$, \$13,000 **6.** $\frac{1}{2}$ **7.** $\frac{19}{110}$ **8.** $\frac{9}{20}$

R.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. When adding fractions, it is important that the _____ are the same before adding.

2. LCD stands for _____ _____ _____.

3. The LCD is the least common multiple of the _____.

4. When subtracting fractions with the same denominator, _____ the numerators and keep the _____ _____.

5. In subtraction with fractions with different denominators, each fraction is changed to a/an _____ fraction with the LCD as its denominator.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. The final step in adding fractions is to reduce, if possible.

7. The process for finding the LCD is the same as the process for finding the LCM.

8. LCD represents the Least Common Digit.

9. When subtracting fractions, simply subtract the numerators and the denominators.

10. Subtraction of fractions requires that the fractions have the same denominators.

Practice

Add and reduce to lowest terms. See Examples 1 through 5.

1. $\frac{3}{14}+\frac{2}{14}$
2. $\frac{3}{11}+\frac{7}{11}$
3. $\frac{7}{5}+\frac{3}{5}$
4. $\frac{9}{10}+\frac{1}{10}$
5. $\frac{1}{20}+\frac{3}{20}$
6. $\frac{3}{25}+\frac{12}{25}$
7. $\frac{1}{8}+\frac{3}{8}+\frac{2}{8}$
8. $\frac{5}{14}+\frac{4}{14}+\frac{1}{14}$
9. $\frac{7}{45}+\frac{11}{45}+\frac{17}{45}$
10. $\frac{14}{32}+\frac{7}{32}+\frac{1}{32}$
11. $\frac{5}{8}+\frac{3}{4}$
12. $\frac{5}{6}+\frac{2}{3}$
13. $\frac{7}{9}+\frac{3}{5}$
14. $\frac{5}{6}+\frac{2}{7}$
15. $\frac{1}{4}+\frac{5}{6}$
16. $\frac{2}{9}+\frac{5}{6}$
17. $\frac{2}{5}+\frac{7}{20}$
18. $\frac{3}{4}+\frac{1}{12}$
19. $\frac{2}{5}+\frac{3}{10}$
20. $\frac{3}{8}+\frac{5}{16}$
21. $\frac{1}{12}+\frac{2}{3}+\frac{1}{4}$
22. $\frac{5}{18}+\frac{1}{2}+\frac{4}{9}$
23. $\frac{2}{5}+\frac{3}{10}+\frac{3}{20}$
24. $\frac{2}{7}+\frac{4}{21}+\frac{1}{3}$
25. $\frac{1}{4}+\frac{1}{20}+\frac{8}{15}$
26. $\frac{1}{5}+\frac{2}{15}+\frac{1}{6}$
27. $\frac{1}{5}+\frac{1}{10}+\frac{1}{4}$
28. $\frac{1}{5}+\frac{1}{40}+\frac{1}{4}$
29. $\frac{2}{5}+\frac{3}{10}+\frac{3}{20}$
30. $\frac{1}{3}+\frac{5}{36}+\frac{7}{18}$
31. $\frac{3}{10}+\frac{1}{100}+\frac{7}{1000}$
32. $\frac{7}{10}+\frac{5}{100}+\frac{3}{1000}$
33. $6+\frac{1}{100}+\frac{3}{10}$
34. $5+\frac{13}{100}+\frac{7}{10}$

Subtract and reduce to lowest terms. See Examples 8 through 11.

35. $\frac{9}{10}-\frac{3}{10}$
36. $\frac{7}{8}-\frac{5}{8}$
37. $\frac{11}{12}-\frac{7}{12}$

38. $\frac{21}{15}-\frac{11}{15}$

39. $\frac{12}{13}-\frac{4}{13}$

40. $\frac{5}{9}-\frac{1}{9}$

41. $\frac{5}{6}-\frac{1}{3}$

42. $\frac{5}{6}-\frac{1}{2}$

43. $\frac{11}{15}-\frac{3}{10}$

44. $\frac{9}{10}-\frac{3}{4}$

45. $\frac{8}{10}-\frac{3}{15}$

46. $\frac{9}{14}-\frac{2}{21}$

47. $\frac{3}{4}-\frac{2}{3}$

48. $\frac{2}{3}-\frac{1}{4}$

49. $\frac{3}{8}-\frac{1}{16}$

50. $\frac{7}{6}-\frac{2}{3}$

51. $\frac{5}{4}-\frac{3}{5}$

52. $\frac{2}{3}-\frac{2}{7}$

53. $\frac{25}{18}-\frac{21}{27}$

54. $\frac{21}{16}-\frac{15}{24}$

55. $1-\frac{9}{10}$

56. $1-\frac{1}{16}$

57. $2-\frac{9}{16}$

58. $6-\frac{2}{3}$

59. $\frac{14}{35}-\frac{12}{30}$

60. $\frac{20}{35}-\frac{24}{42}$

61. $\frac{76}{100}-\frac{7}{10}$

62. $\frac{54}{100}-\frac{5}{10}$

63. $\frac{1}{10}-\frac{8}{100}$

64. $\frac{3}{100}-\frac{1}{1000}$

Applications

Solve.

65. Three pieces of mail weigh $\frac{1}{2}$ ounce, $\frac{1}{5}$ ounce, and $\frac{3}{10}$ ounce. What is the total weight of the letters?

66. Using a microscope, a scientist measures the diameters of three hairs to be $\frac{1}{1000}$ inch, $\frac{3}{1000}$ inch, and $\frac{1}{100}$ inch. What is the total of these three diameters?

67. A machinist drills four holes in a straight line. Each hole has a diameter of $\frac{1}{10}$ inch and there is $\frac{1}{4}$ inch between the holes. What is the distance between the outer edges of the first and last holes?

68. A notebook contains a piece of cardboard as a back cover that is $\frac{1}{16}$ inch thick. It has a front cover that is $\frac{1}{4}$ inch thick. All together, the sheets of paper between the front and back are $\frac{3}{10}$ inch thick. What is the total thickness of the notebook?

69. A carpenter is installing baseboard and toe molding. If the baseboard is $\frac{3}{8}$ inch thick and the toe molding (to be put in front of the baseboard) is $\frac{1}{4}$ inch thick, what is the total thickness of the two trim pieces?

70. A recipe calls for the following spices: $\frac{1}{2}$ teaspoon of turmeric, $\frac{1}{4}$ teaspoon of ginger, and $\frac{1}{8}$ teaspoon of cumin. What is the total quantity of these three spices?

71. Beth's investment strategy is to put $\frac{1}{6}$ of her paycheck into a savings account and another $\frac{1}{9}$ into a retirement account.

a. What fraction of her salary does Beth invest each month?

b. If she maintains this strategy for 24 paychecks and receives \$900 per paycheck, how much money will she have saved?

72. John has a monthly income of \$3000. $\frac{1}{10}$ of it goes to college savings, $\frac{2}{15}$ to general savings, and $\frac{1}{12}$ to retirement.

a. What fraction of John's income is being saved?

b. How much money is being saved each month?

c. What is the total amount saved in a 12-month year?

73. A $\frac{7}{8}$ inch pipe is to be shortened to $\frac{7}{12}$ inch. How much must be removed?

74. Near the end of the snow season, the road salt supply for a small college had dwindled down to $\frac{7}{10}$ ton. When the next snow storm came, $\frac{1}{2}$ ton of salt was used for the roads. How much road salt was left?

75. Mark has driven to a national park with no gas stations and he wants to drive around some before leaving the park. He knows he can safely make it to the nearest gas station on $\frac{1}{4}$ of a tank of gas. If the tank is currently $\frac{5}{9}$ full, what fraction of a tank of gasoline does he have to use for touring the park?

76. About $\frac{1}{2}$ of all incoming solar radiation is absorbed by the earth, $\frac{1}{5}$ is absorbed by the atmosphere, and $\frac{1}{20}$ is scattered by the atmosphere. The rest is reflected by the earth and clouds.

a. What fraction of solar radiation is absorbed or scattered?

b. What fraction of solar radiation is reflected by the earth and clouds?

77. Jenny has $\frac{3}{4}$ of an apple pie left over from a party last night. Her roommates found it and cut themselves three unequal sized pieces in the following amounts: $\frac{1}{3}$ of a pie, $\frac{1}{4}$ of a pie, and $\frac{1}{6}$ of a pie.

a. What fraction of a full pie did Jenny's roommates take?

b. What fraction of the pie is left over?

78. A moving truck has to make stops at three different apartments to collect items for three different moves. The first apartment takes up $\frac{3}{7}$ of the truck and the second apartment takes up $\frac{3}{14}$ of the truck. What fraction of the space in the moving truck is left to fit in the items from the last apartment?

79. Josh has a large homework assignment to do this weekend. He is able to get $\frac{4}{15}$ of the assignment done Friday after class. If he doesn't want to leave more than $\frac{2}{9}$ of the assignment to do for Sunday, what fraction of the assignment must he complete on Saturday?

80. A pentagon (a five-sided figure) has 4 sides of length $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, and $\frac{1}{6}$ inches. If the perimeter of the pentagon is 1 inch, find the length of the fifth side.

Writing & Thinking

81. Explain how finding the LCM relates to LCDs.

82. Explain the steps to follow when adding or subtracting fractions with unlike denominators

83. Give an example of a situation where you might add or subtract fractions (other than in class).

84. Pick one problem in this section that gave you some difficulty. Explain briefly why you had difficulty and why you think that you can better solve problems of this type in the future.

To Change a Percent to a Decimal Number

1. Move the decimal point two places to the left.
2. Delete the % sign.

PROCEDURE

13. Change each percent to an equivalent decimal number.

a. 40%

b. 29.37%

c. 102%

d. 0.6%

Example 13 Changing Percents to Decimal Numbers

Change each percent to an equivalent decimal number.

a. 72% **b.** 16.2% **c.** 100% **d.** 0.25%

Solution

a. 72% = 0.72 ← % symbol deleted

Understood decimal point

Decimal point moved two places to the left

b. 16.2% = 0.162

c. 100% = 1.00 = 1

d. 0.25% = 0.0025 Note that when moving the decimal point two places to the left, two zeros were added as placeholders.

Now work margin exercise 13.

Margin Exercise Answers

1. 39.0184; thirty-nine and one hundred eighty-four ten-thousandths **2.** 1200.0005 **3.** 0.1205 **4.** 6.44 **5.** 9.251 **6.** 42.077 **7.** 5.962 **8.** $8.68 **9.** 16.0146 **10.** 13.2 **11.** 14.9 **12. a.** 87.1% **b.** 0.35% **c.** 217% **d.** 10% **13. a.** 0.4 **b.** 0.293 **c.** 1.02 **d.** 0.006

R.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Decimal notation uses the _____ _____ system.
2. A _____ _____ consists of a whole number and a fraction written next to each other.
3. When reading decimal numbers, the word "_____" is used in place of the decimal point.
4. When comparing decimal numbers, you should compare each digit moving _____ to _____.

5. To keep digits vertically aligned when adding decimal numbers, _____ may be used.

6. The word _____ comes from the Latin per centum, meaning "per hundred."

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. On a number line, any number to the left of another number is larger than that other number.

8. It is important to align the decimal points vertically when adding decimal numbers.

9. The decimal points should be aligned vertically when multiplying decimal numbers.

10. The first step in division with decimal numbers is to move the decimal point in the divisor to the right so that the divisor is a whole number.

Practice

Write each decimal number in words. See Example 1.

1. 0.9

2. 0.5

3. 20.7

4. 96.3

5. 1.53

6. 2.79

7. 19.102

8. 18.051

9. 800.009

10. 500.005

Write each number in decimal notation. See Examples 2 and 3.

11. three tenths

12. seven tenths

13. seventeen and nine tenths

14. eight hundred and three tenths

15. twenty-three hundredths

16. seventy-two hundredths

17. six and twenty-eight thousandths

18. fourteen and ninety-seven thousandths

19. four thousand five hundred two ten-thousandths

20. seven thousand one hundred sixty-five ten-thousandths

For each pair of decimal numbers, determine which number is larger. See Examples 4 and 5.

21. 0.26, 0.27

22. 0.45, 0.48

23. 0.153, 0.163

24. 4.537, 4.527

25. 23.521, 24.295

26. 110.241, 101.862

27. 0.01, 0.009

28. 4.002, 4.0008

Arrange each set of decimal numbers in order from smallest to largest. Then, graph the numbers on a number line. See Example 6.

29. 0.3, 0.03, 0.33

30. 0.55, 0.05, 0.5

31. 0.2, 0.26, 0.17

32. 1.8, 1.75, 1.86

33. 0.157, 0.2611, 0.192, 0.26

34. 1.432, 1.54, 1.14, 1.5422

Perform the indicated operations. Round any quotient to the nearest hundredth.

35. $0.6+0.4+0.4$

36. $7+5.1+0.8$

37. $0.79+4.92+0.05$

38. $4.005+0.056+0.9$

39. $5.4-3.76$

40. $17.83-9.9$

41. $39.6-13.71$

42. $55.002-53.008$

43.
$$\begin{array}{r} 57.3 \\ 52.08 \\ +38.005 \\ \hline \end{array}$$

44.
$$\begin{array}{r} 1.007 \\ 30.442 \\ +\ 4.992 \\ \hline \end{array}$$

45.
$$\begin{array}{r} 21.007 \\ -\ 1.543 \\ \hline \end{array}$$

46.
$$\begin{array}{r} 30. \\ -\ 6.45 \\ \hline \end{array}$$

47. $(0.2)(0.2)$

48. $8(0.125)$

49.
$$\begin{array}{r} 0.137 \\ \times 0.08 \\ \hline \end{array}$$

50.
$$\begin{array}{r} 6.09 \\ \times 0.11 \\ \hline \end{array}$$

51. $28 \div 5.6$

52. $35 \div 1.64$

53. $2.7\overline{)5.483}$

54. $2.54\overline{)45}$

Change the following decimals to percents. See Example 12.

55. 0.03

56. 0.052

57. 3.0

58. 2.5

59. 1.08

60. 0.5

Change the following percents to decimals. See Example 13.

61. 6%

62. 11%

63. 3.2%

64. 12.5%

65. 120%

66. 80%

Applications

In each exercise, write the decimal numbers that are not whole numbers in words.

67. The tallest unicycle ever ridden was 114.8 feet tall, and was ridden by Sam Abrahams (with a safety wire suspended from an overhead crane) for a distance of 28 feet in Pontiac, Michigan, on January 29, 2004. [1]

68. A penny dated from 1959 through 1982 had an original weight of 3.11 grams. A penny dated 1983 or later had an original weight of 2.5 grams. Write the numbers representing weights in words.

69. One yard is equal to 36 inches. One yard is also approximately equal to 0.914 meter. One meter is approximately equal to 1.09 yards. One meter is also approximately equal to 39.37 inches. (Thus, a meter is longer than a yard by about 3.37 inches.)

70. One foot is equal to 12 inches. One foot is also equal to 30.48 centimeters. One square foot is approximately 0.093 square meters.

71. One quart of water weighs approximately 2.0825 pounds.

72. The number π is approximately equal to 3.14159.

73. The number e is approximately equal to 2.71828.

74. The largest state in the United States is Alaska, which covers approximately 656.4 thousand square miles. The second largest state is Texas, which approximately 268.6 thousand square miles. Alaska is more than 10 times the size of Wisconsin (twenty-third in size), with about 65.5 thousand square miles.

75. World Records: 9.58 seconds for 100 meters (by Usain Bolt, Jamaica, 2009); 19.19 seconds for 200 meters (by Usain Bolt, Jamaica, 2009); 43.18 seconds for 400 meters (by Michael Johnson, USA, 1999). [2]

76. The mean distance from the Sun to Earth is about 92.9 million miles and from the Sun to Venus is 67.24 million miles. One period of revolution of the Earth about the Sun takes 365.2 days, and one period of revolution of Venus about the sun takes 224.7 days.

77. An interesting fact about aging is that the longer you live, the longer you can expect to live. A white male of age 40 can expect to live 35.8 more years; of age 50, can expect to live 26.9 more years; of age 60 can expect to live 18.9 more years; of age 70 can expect to live 12.3 more years; and of age 80 can expect to live 7.2 more years. (This same phenomenon is true of men and women of all races.)

1 Source: http://semcycle.biz/record
2 Source: http://en.wikipedia.org/wiki/List_of_world_records_in_athletics

Writing & Thinking

78. In your own words, state why the word "and" is so commonly misused when numbers are spoken and/or written. Bring an example of this from a newspaper, magazine, or television show to share with the class.

79. Discuss situations where you think it is particularly appropriate (or necessary) to write numbers in English word form.

80. With **a.** and **b.** as examples, explain in your own words how you can tell quickly when one decimal number is larger (or smaller) than another decimal number.

a. The decimal number 2.765274 is larger than the decimal number 2.763895.

b. The decimal number 17.345678 is larger than the decimal number 17.345578.

Solution

a. To find the lowest high temperature, look at the red line (labeled "High") and find the lowest point on that line graph. This shows that the lowest high temperature is 66 °F.

b. Looking directly down from the point located in Part **a.**, we see that the lowest high temperature occurred on Sunday.

c. To find the highest low temperature, look at the blue line (labeled "Low") and find the highest point on that line graph. This shows that the highest low temperature is 70 °F.

d. Looking directly down from the point located in Part **d.**, we see that the highest low temperature occurred on Friday.

e. The high temperature on Tuesday for the week shown was 76 °F and the low temperature was 66 °F. Thus, the difference between the two temperatures is 76 − 66 = 10 °F.

Now work margin exercise 4.

Margin Exercise Answers

1. a. $1013 million **b.** $918 million **c.** $1281 million **d.** 55.8% decrease
2. a. Clemson **b.** 70,000 **c.** 30,000 **3. a.** $13,750 **b.** $5500 **c.** $3850 **d.** $11,000 **4. a.** 80 °F **b.** 60 °F **c.** 64.86 °F **d.** 12 °F

R.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Pictures of numerical information are _____.

2. To emphasize comparative amounts, the best way to present the information is to use _____ graphs.

3. To help understand percentages or parts of a whole, the preferred method is to use _____ graphs.

4. A _____ graph is used to indicate tendencies or trends over time.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

5. Graphs should always be clearly labeled, easy to read, and have appropriate titles.

6. Circle graphs show trends over a period of time.

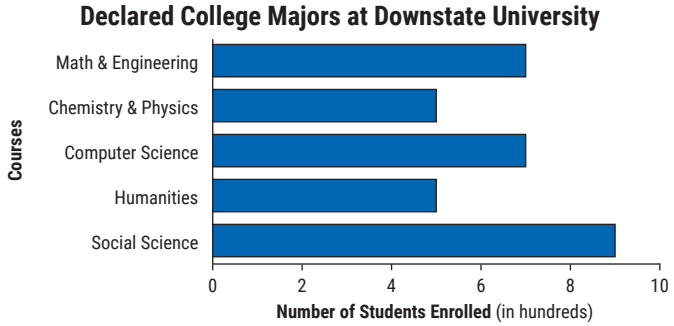
7. All graphs should be clearly labeled, easy to read, and have appropriate titles.

8. Pictographs are used to illustrate percentages as part of a whole.

Applications

Answer the questions using the given graphs.

1. The following bar graph shows the number of students in five fields of study at a university.



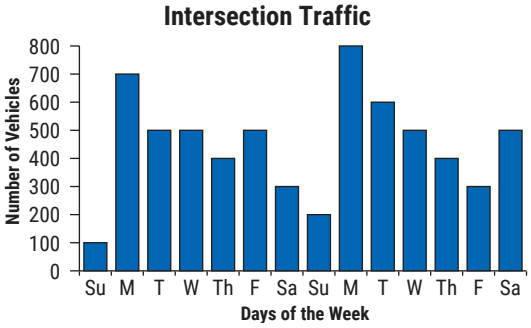
a. Which field(s) of study has the largest number of declared majors?

b. Which field(s) of study has the smallest number of declared majors?

c. How many declared majors are indicated in the entire graph?

d. What percent are computer science majors? Round your answer to the nearest tenth of a percent.

2. The following bar graph shows the number of vehicles that crossed one intersection during a two-week period.



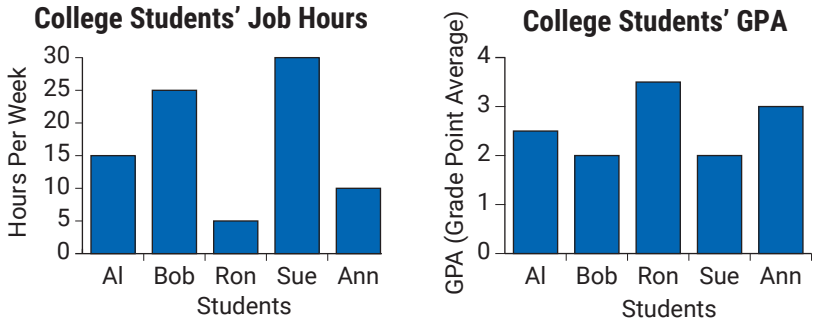
a. On which day did the highest number of vehicles cross the intersection? How many crossed that day?

b. What was the mean number of vehicles that crossed the intersection on the two Sundays?

c. What was the total number of vehicles that crossed the intersection during the two weeks?

d. About what percent of the total traffic was counted on Saturdays? Round your answer to the nearest tenth of a percent.

3. The following bar graphs show the number of hours worked each week and the GPAs of five college students. When comparing the following two graphs, assume that all five students graduated with comparable grades from the same high school.



a. Who worked the most hours per week?

b. Who had the lowest GPA?

c. If Ron spent 30 hours per week studying for his classes, then the length of his total work week is the sum of the time he spent studying and the time he spent working. What percent of his work week did he spend studying? Round your answer to the nearest tenth of a percent.

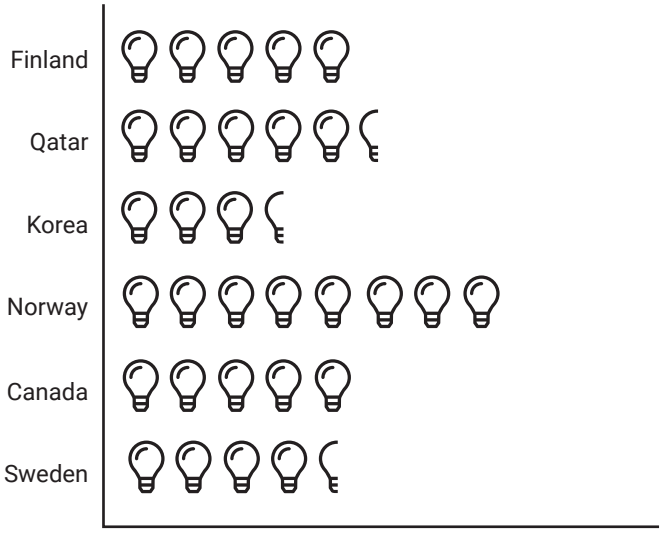
d. Which two students worked the most hours? Which two students had the lowest GPAs? Do you think that this is typical?

e. Do you think that the two graphs shown here could be set as one graph? If so, show how you might do this.

4. The following pictograph shows the average electricity consumption by different countries.

Average Electricity Consumption by Different Countries

Each equals 3000 kilowatt hours (kWh) per person



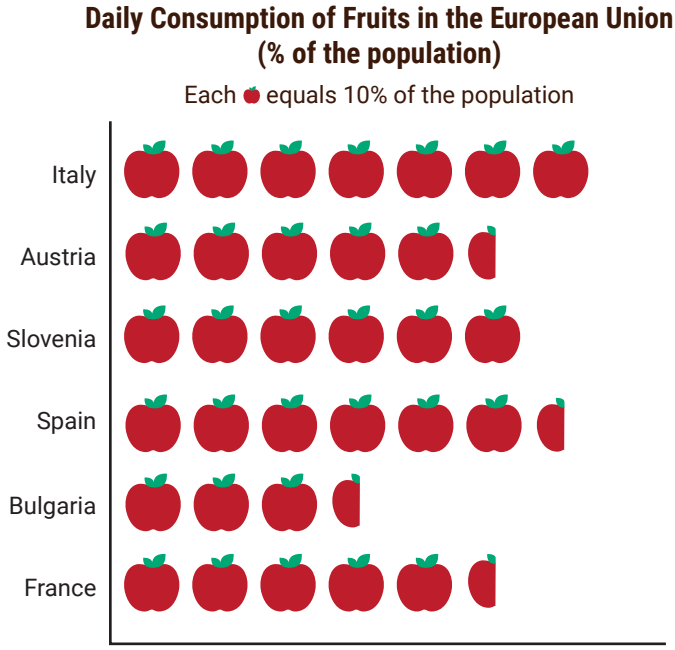
a. Which country has the highest average consumption?

b. Which country has the lowest average consumption?

c. What is the average consumption for Qatar?

d. Which of the three Nordic countries (Finland, Norway, and Sweden) have the average electricity consumption of more than 14,000 kWh per person?

5. The following pictograph shows the daily consumption of fruits in the European Union.



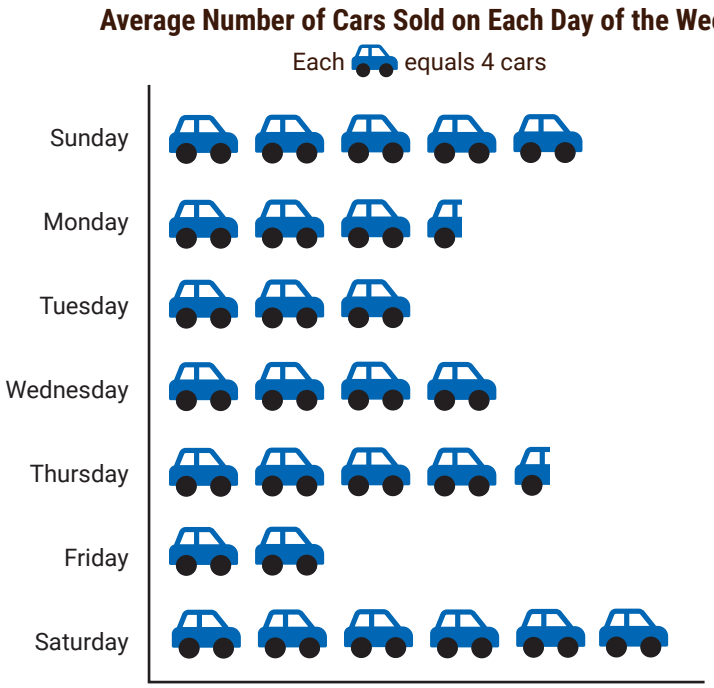
a. Which country has the highest proportion of people who consume fruits daily?

b. Which country has the lowest proportion of people who consume fruits daily?

c. How large is the proportion of people who do not consume fruits daily in Spain?

d. Which countries have the proportion of people consuming fruits daily in between 40% and 60% (endpoints included)?

6. The following pictograph shows the average number of cars sold on each day of the week at a certain dealership.



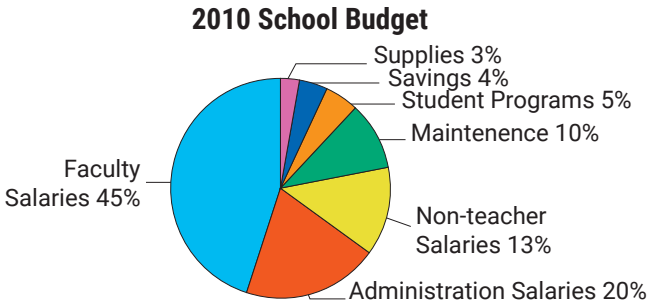
a. On which day is the largest number of cars sold?

b. On which day is the smallest number of cars sold?

c. How many cars on average are sold on Sunday?

d. How many cars on average are sold from Monday through Thursday?

7. The following circle graph represents the various areas of spending for a school with a total budget of $34,500,000.



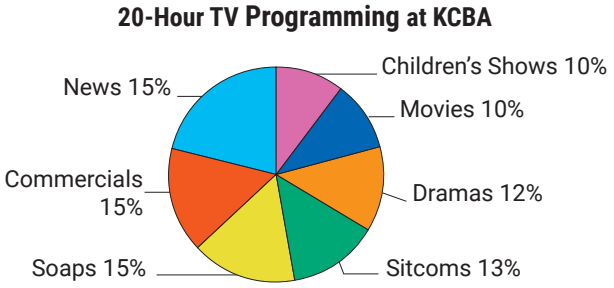
a. What amount will be allocated to each category?

b. What percent will be for expenditures other than salaries?

c. How much will be spent on maintenance and supplies?

d. How much more will be spent on teachers' salaries than on administration salaries?

8. The following circle graph represents the types of shows broadcast on television station KCBA. The station is off the air from 2 a.m. to 6 a.m., so they have only 20 hours of daily programming. Sports are not shown in the graph below because they are considered special events.

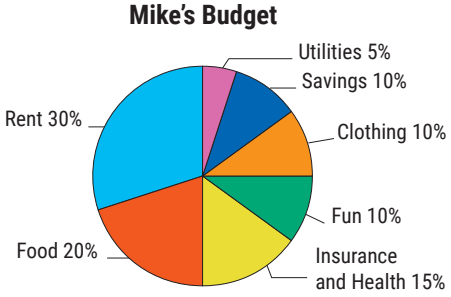


a. In the 20-hour period shown, how much time (in minutes) is devoted daily to each category?

b. What category has the most time devoted to it?

c. How much total time (in minutes) is devoted to drama, soaps, and sitcoms?

9. Mike just graduated from college and decided that he should try to live within a budget. The circle graph shows the categories he chose and the percents he allowed. His beginning take home salary is $24,000.

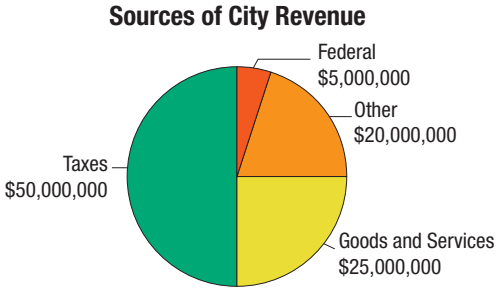


a. How much did he budget for each category?

b. What category was smallest in his budget?

c. What total amount did he budget for food, clothing, and rent?

10. The following circle graph represents the various sources of income for a city government with a total income of $100,000,000.

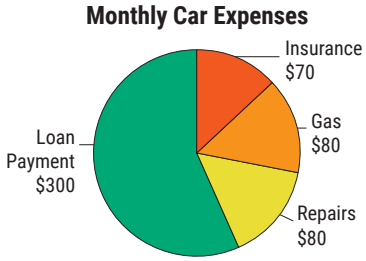


a. What is the city's largest source of income?

b. What percent of income comes from good and services?

c. What is the ratio of income from taxes to the total income?

11. The following circle graph shows Sally's car expenses for the month of June.

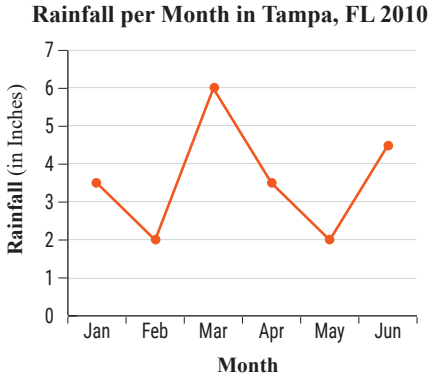


a. What were her total car expenses for the month?

b. What percent of her expenses did she spend on each category? Round your answers to the nearest tenth of a percent.

c. What was the ratio of her insurance expenses to her gas expenses?

12. The following line graph shows the total monthly rainfall in Tampa, Florida for the first 5 months of 2010. [1]



a. Which months had the least rainfall?

b. What was the most rainfall in a month?

c. What month had the most rainfall?

d. What was the mean rainfall over the six-month period (to the nearest hundredth)?

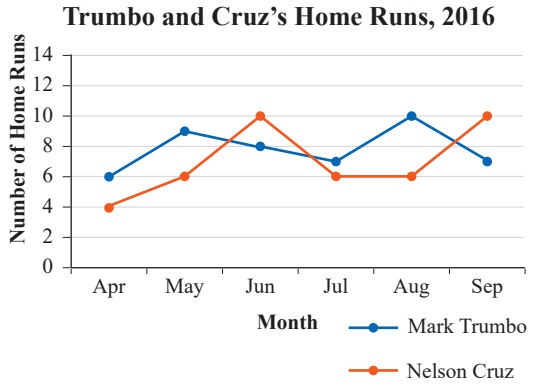
1 Source: weather.gov

13. The following line graph shows the average monthly mortgage rates for June and December for 2012–2016. [2]

a. During what month or months were mortgage rates highest?

b. During what month or months were mortgage rates lowest?

14. The following line graph shows the number of home runs hit each month of the 2016 Season by Mark Trumbo and Nelson Cruz. [3]



a. During which month did Mark hit the most home runs?

b. How much higher was Mark's total for that month than for his lowest month?

c. In what months did Nelson hit fewer home runs than Mark?

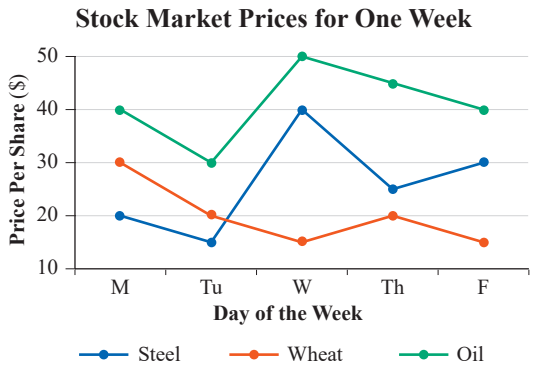
d. What was the difference between Nelson and Mark's home runs in July?

e. What percent of Nelson's total home runs did he hit in May?

f. What percent of Mark's home runs did he hit in April? Round your answer to the nearest percent.

2 Source: http://www.hsh.com/monthly-mortgage-rates.html
3 Source: mlb.com

15. The following line graphs show the stock market prices for oil, steel, and wheat over the course of a week. Assume that on Monday morning you had 100 shares of each of the three stocks shown.



a. If you held the stock all week, on which stock would you have lost money?

b. How much money would you have lost on that stock?

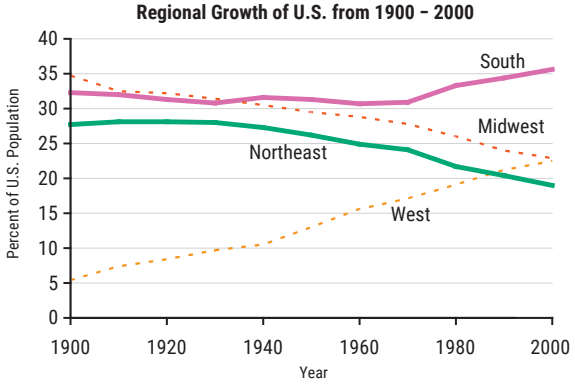
c. On which stock would you have gained money?

d. How much money would you have gained on that stock?

e. On which stock could you have made the most money if you had sold it at the best time?

f. How much money could you have made had you sold that stock at the best time?

16. The following line graph shows the change in the percent of the US population living in each of four major regions over the last century. [4]



a. Approximately what percent of the population was in each of the four regions in 1900?

b. Approximately what percent of the population was in each of the four regions in 2000?

c. Which region seems to have had the most stable percent of the population between 1900 and 2000?

d. What is the difference between the highest and lowest percent for this region?

4 Source: US Census Bureau, decennial census of population, 1900 to 2000

e. Which region has had the most growth?

f. What was its lowest percent and when?

g. What was its highest percent and when?

h. Which region has had the most decline?

Writing & Thinking

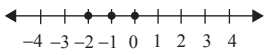
17. State three properties or characteristics that should be true of all graphs so that they can communicate numerical data quickly and easily.

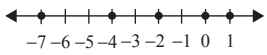
18. List the four types of graphs discussed in the text and briefly give the purpose of each.

19. Give three different and specific examples where graphs are used (outside of a class).

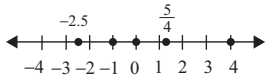
20. Compare and contrast a bar graph and a histogram.

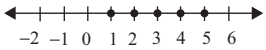
Margin Exercise Answers

1. a. −10 **b.** +8 **2. a.** 

b. 

3. a. 1, 20 **b.** −6, 1, 20 **c.** $-6, -\frac{1}{7}, 1, 20$ **d.** All numbers in S are real numbers.

4. 

5. 

6. a. True **b.** False **c.** True **d.** True **e.** False **7. a.** 4 **b.** 3.3 **c.** −7.4 **8.** True **9.** $z = 3, -3$ **10.** No solution **11.** −3 millimeters and 3 millimeters

1.1 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The set of numbers that includes the whole numbers and their opposites is the set of _____
2. A number that can be written as a fraction is a/an _____ number.
3. Infinite nonrepeating decimal numbers are _____ numbers.
4. A number's distance from 0 on a number line is the number's _____.
5. The _____ of a number is the point that corresponds to the number on a number line.
6. The symbols < and > are known as _____ symbols.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. On a number line, smaller numbers are always to the left of larger numbers.
8. The absolute value of a negative number is a positive number.
9. All whole numbers are also integers.
10. Zero is a positive number.

Practice

Find the opposite of each integer. See Example 1.

1. −3
2. −7
3. 0
4. −0
5. +2
6. +6

Graph each set of real numbers on a real number line. See Examples 2, 4, and 5.

7. $\{1, 2, 5, 6\}$

8. $\{-3, -2, 0, 1\}$

9. $\{2, -3, 0, -1\}$

10. $\{-2, -1, 4, -3\}$

11. $\left\{0, -1, \frac{7}{4}, 3, 1\right\}$

12. $\left\{-2, -1, -\frac{1}{3}, 2\right\}$

13. $\left\{-\frac{3}{4}, 0, 2, 3.6\right\}$

14. $\left\{-3.4, -2, -0.5, 1, \frac{5}{2}\right\}$

15. $\left\{-\frac{7}{2}, -1.5, 1, \frac{4}{3}, 2\right\}$

16. $\left\{-4, -\frac{7}{3}, -1, 0.2, \frac{5}{2}\right\}$

17. all whole numbers less than 4

18. all negative integers greater than -4

19. all whole numbers less than 0

20. all natural numbers less than or equal to -1

List the numbers in the set $A = \left\{-7, -\sqrt{6}, -2, -\frac{5}{3}, -1.4, 0, \frac{3}{5}, \sqrt{5}, \sqrt{11}, 4, 5.9, 8\right\}$ that are described in each exercise. See Example 3.

21. Natural numbers

22. Whole numbers

23. Integers

24. Irrational numbers

25. Rational numbers

26. Real numbers

Fill in each blank with the appropriate symbol that will make the statement true: <, >, or =. See Examples 6 through 8.

27. $4 \_\_\_ 6$

28. $-3 \_\_\_ 1$

29. $-2 \_\_\_ -4$

30. $-8 \_\_\_ 0$

31. $-20 \_\_\_ -19$

32. $-67 \_\_\_ -50$

33. $-(-4.3) \_\_\_ 4.3$

34. $5.6 \_\_\_ -(-8.7)$

35. $-\frac{3}{4} \_\_\_ -1$

36. $-2.3 \_\_\_ -2\frac{3}{10}$

37. $\frac{1}{3} \_\_\_ \frac{1}{2}$

38. $-\frac{1}{2} \_\_\_ -\frac{1}{3}$

39. $|-4| \_\_\_ 4$

40. $|7| \_\_\_ -7$

41. $|-8| \_\_\_ -8$

42. $-15 \_\_\_ |-15|$

Determine whether each statement is true or false. If a statement is false, rewrite it in a form that is a true statement. (There may be more than one way to correct a statement.) See Examples 6 and 8.

43. $0 = -0$

44. $-22 < -16$

45. $-9 > -8.5$

46. $-17 \leq 17$

47. $4.7 \geq 3.5$

48. $|-5| = 5$

49. $-|-7| \geq -|7|$

50. $|-8| \geq 4$

51. $-|-3| < -|4|$

52. $\left|-\frac{5}{2}\right| < 2$

List the possible values for x for each statement. See Examples 9 and 10.

53. $|x| = 5$

54. $|x| = 8$

55. $|x| = 2$

56. $|x| = 0$

57. $|x| = -6$

58. $|x| = -1$

59. $|x| = 23$

60. $|x| = 105$

Choose the response that correctly completes each sentence. Assume that the variables represent integers. In each problem give two examples that illustrate your reasoning.

61. $|a|$ is (never, sometimes, always) equal to a.

62. $|x|$ is (never, sometimes, always) equal to $-x$.

63. $|y|$ is (never, sometimes, always) equal to a positive integer.

64. $|x|$ is (never, sometimes, always) greater than x.

Applications

Solve. Represent each quantity with a signed integer.

65. The Alvin is a manned deep-ocean research submersible that has explored the wreck of the Titanic. The operating depth of the Alvin is 4500 meters below sea level.

66. The Mariana trench is the deepest known location on the Earth's ocean floor. The deepest known part of the Mariana Trench is approximately 11 kilometers below sea level.

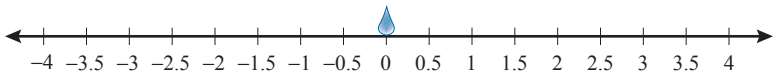
67. Mount Everest is considered to be the highest mountain on Earth. Its peak reaches to a height of approximately 8844 meters.

68. The lowest temperature ever recorded was at the Vostok Station on Antarctica. On July 21, 1983, the temperature was approximately 128 degrees Fahrenheit below zero.

69. Terrence placed a drop of colored water on the center of a white strand of yarn and measured how much the color spread. Before placing the drop, he predicts that the color will spread no more than 3 inches away from the initial drop.

a. Write an absolute value inequality using the variable x to represent the predicted spread.

b. Graph the solution set of integers for the absolute value inequality from part **a.** on the given number line, placing the initial drop at the point 0.

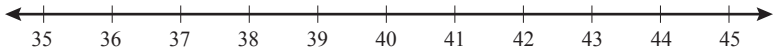


70. A ready-to-assemble bookcase contains wooden boards that have predrilled holes along with the screws and washers needed for assembly. The screws used to assemble the bookcase need to have a length of 38 mm with a tolerance of 2 mm. If the screw is too short, it won't be able to hold the pieces of wood together. If the screw is too long, it might stick out of the other end of the board.

a. What is the largest length the screws can have before they are too long?

b. What is the smallest length the screws can have before they are too short?

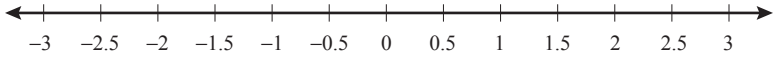
c. Graph the tolerance of the screw. (Graph only the integers in the tolerance range.)



71. A freezer in a biology lab is supposed to be kept at 0 °C. A lab assistant places a thermometer in the freezer and marks down the temperature every half hour. She records the following temperatures:

$$\left\{2°, -\frac{1}{2}°, \frac{1}{2}°, 1\frac{1}{2}°, -\frac{5}{2}°\right\}.$$

a. Graph the set of temperatures on a number line.



b. Which value is the furthest away from 0?

72. During the manufacture of machine parts, certain measurements of the part must stay within a tolerance range or the part will be defective. The quality control manager determines that the maximum amount the length of a screw can vary from its targeted length of 15 millimeters and not be defective is 3 millimeters. If x represents the maximum acceptable difference in length from the target, either positive or negative, then $|x| = 3$.

a. Solve the equation $|x| = 3$ to determine the maximum amounts that the screw length can vary from its target length.

b. What do the answers from Part **a.** mean?

c. Determine the maximum and minimum lengths of a screw with the given tolerance range.

73. A carpenter needs to cut a board 1 meter long to create the backboard for a bookshelf. He has determined that the maximum amount the length of the board can vary from its required length and not cause structural issues is 5 mm. If x represents the maximum acceptable difference in length from the target, either positive or negative, then $|x| = 5$.

a. Solve the equation $|x| = 5$ to determine the maximum amounts that the board length can vary from its target length.

b. What do the answers from Part **a.** mean?

c. Determine the maximum and minimum lengths of a board with the given tolerance range.

Writing & Thinking

74. Give one example each of the use of a positive number, a negative number, and the number zero (outside of a class).

75. Explain, in your own words, how an expression such as $-y$ might represent a positive number.

76. Compare and contrast absolute value with opposites.

10. Multiply.

a. $\frac{0}{-2}$

b. $\frac{12}{0}$

Example 10 Division with 0

Divide.

a. $\frac{0}{-7}$ **b.** $\frac{9}{0}$

Solution

a. $\frac{0}{-7} = 0$ **b.** $\frac{9}{0}$ is undefined.

Now work margin exercise 10.

Margin Exercise Answers

1. a. 15 **b.** −20 **c.** 5.9 **2. a.** 8 **b.** −7 **c.** −2.3 **3.** 16 **4.** 60 ft **5. a.** −11 **b.** 0 **c.** 38 **d.** −4 **e.** 1 **f.** 5.0 **6.** −19 °F **7. a.** −27 **b.** −2 **c.** −90 **d.** 42 **e.** $\frac{2}{15}$ **f.** 0.086 **8.** 11,242 sq ft **9. a.** −20.8 **b.** 5 **c.** −19 **10. a.** 0 **b.** undefined

1.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. If there is no sign in front of a number, the number is understood to be _____.
2. To add two real numbers with _____ signs, subtract their absolute values and use the sign of the number with the larger absolute value.
3. The _____ of a geometric figure is the distance around the figure.
4. Subtraction with real numbers is defined in terms of _____.
5. The _____ of a rectangle is found by multiplying its length by its width.
6. The quotient of two numbers with unlike signs is _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The sum of a positive number and a negative number is always positive.
8. The sum of two positive numbers can equal zero.
9. The expression "15 – 7" can be thought of as "fifteen plus negative seven."
10. If two numbers have the same sign, both the product and the quotient will be negative.
11. The mean of a set of numbers is always positive.

Practice

Simplify.

1. $(-16) + 20$

2. $(-2) + (-9)$

3. $(-8) + (-6) + 5$

4. $\left(-\frac{3}{8}\right) + \frac{7}{8}$

5. $12 - 15$

6. $-4 - (-8)$

7. $(-9) - (-9)$

8. $0 - (-12)$

9. $-\frac{4}{13} - \frac{3}{13}$

10. $\frac{3}{5} - \frac{9}{5}$

11. $(-1.7) + (-5.2)$

12. $(8.5) + (-7.9)$

13. $-7 - (-2) + 6$

14. $-18 - 22 - 41$

15. $-8 + (-7) - (-15)$

16. $9 - (-3) + (-2)$

17. $-\frac{7}{6} + \left(-\frac{5}{6}\right) - \frac{1}{6}$

18. $\left(-\frac{9}{16}\right) + \left(-\frac{7}{8}\right)$

19. $\frac{1}{8} - \left(-\frac{1}{2}\right) + \frac{1}{4}$

20. $\frac{4}{5} + \left(-\frac{2}{3}\right) - \frac{1}{6}$

21. $-\frac{3}{8} - \frac{5}{6} + \left(-\frac{1}{2}\right)$

22. $(-8)(-7)$

23. $(-3)(17)$

24. $(-8)(-1)(-5)(6)(-2)$

25. $(12)\left(-\frac{5}{6}\right)$

26. $\frac{3}{8} \cdot \frac{5}{2}$

27. $-\frac{5}{16} \cdot \frac{3}{4}$

28. $6(5.3)$

29. $\left(-\frac{3}{10}\right)\left(\frac{5}{6}\right)\left(-\frac{8}{7}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{4}\right)$

30. $(-0.8)(4.9)$

31. $(11.7)(2.06)(-1.3)$

32. $(-20) \div (-10)$

33. $\frac{-39}{-13}$

34. $\frac{-91}{-7}$

35. $\frac{52}{13}$

36. $\frac{6}{16} \div 0$

37. $60 \div (-15)$

38. $0 \div \frac{11}{12}$

39. $\frac{28.7}{-7}$

40. $-68.05 \div 5$

41. $-88.64 \div (-8)$

42. $-6.084 \div (-9)$

43. $18 \div 3 \cdot 6 + 3$

44. $7(4-2) \div 7 + 3$

45. $10 \div 2 - 4 \cdot 3^2$

46. $2^2 \cdot 3 \div 3 + 6 \div 3$

47. $-6 \cdot 3 \div (-1) + 4 - 2$

48. $5(-2)\div(-5)+5-3$

49. $\left(4^2+6\right)-2\cdot 19$

50. $\left(5^2-4^2\right)^2-11$

51. $\left[(4+14)\div(3\cdot 3)\right]-5$

52. $\left[8-(5\cdot 6-2)\right]+3$

53. $\left(12\cdot 4\div 2^3\right)-\left[\left(3\cdot 2^3\right)\div(4\cdot 6)\right]$

54. $\left[(3\cdot 0)\div(2\cdot 1)\right]-\left(24-6^2\right)\div\left(4^2-3\cdot 4\right)$

55. $\left(3\cdot 2^3\right)\div(3\cdot 4)+(2\cdot 3+4)\div(6-1)$

56. $-6+(-2)(12\cdot 2\div 3)4$

57. $14-\left[11\cdot 4-\left(2\cdot 3^2+1\right)\right]$

58. $6+3\left[-4-2(3-1)\right]$

59. $7-\left[4\cdot 3-(4-3\cdot 2)\right]$

60. $-2\left[6+4(1+7)\right]\div 4$

61. $\dfrac{(-3)(-6)}{5-(-4)}-2$

62. $\dfrac{4-(-10)}{-2-5}\div(-2)$

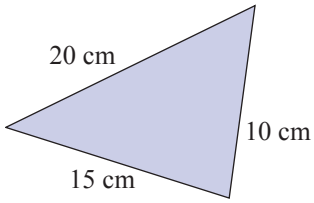
63. $\dfrac{16-(-4)}{-3+9}\div\dfrac{10^2+10}{-5\cdot 11}$

64. $\dfrac{3^3-(-27)}{2\cdot 3^2}+\dfrac{-6\cdot 5}{-2\cdot 5}$

Applications

Calculate the perimeter of each geometric figure. See Example 4.

65.



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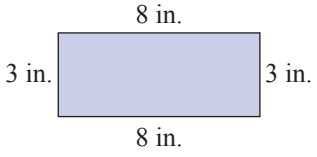

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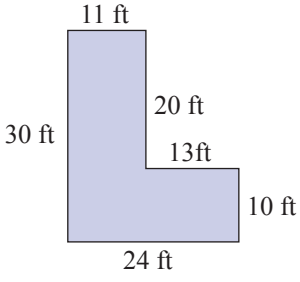

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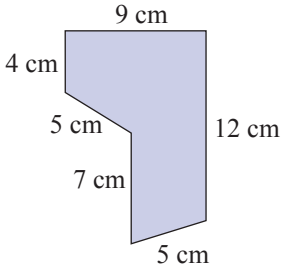

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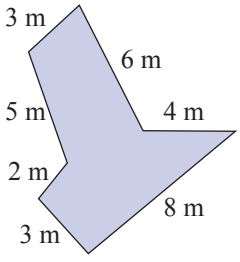
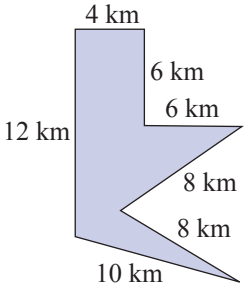
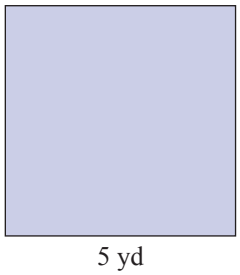


70.

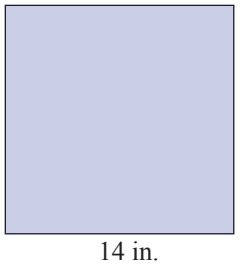


Calculate the area of each rectangle. See Example 8.

71.



72.



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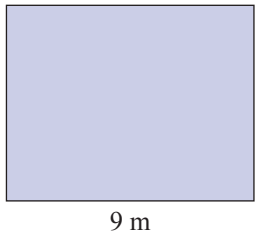

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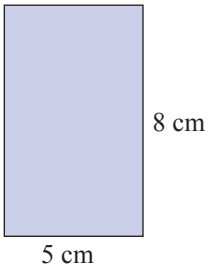
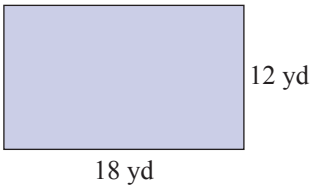


75.



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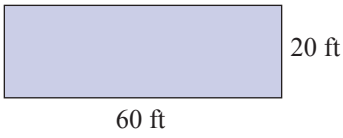


Solve.

77. During the first hour of trading, a stock trader has stock worth $1973.27. During the second hour, the trader loses $797.53. During the third hour, he gains $925.87. What was the net worth of the trader's stock after the first three hours of trading?

78. A college student opens a checking account with a deposit of $1000.00. She withdraws $252.68 to pay for textbooks. Later that evening, she writes a check for $116.89 for groceries. The next day, she deposits a graduation gift of $75.25 cash. What is her final account balance?

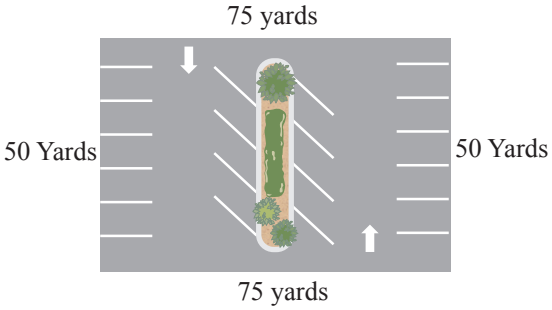
79. In a 5-day week the NASDAQ stock market posted a gain of 38 points, a loss of 65 points, a loss of 32 points, a gain of 10 points, and a gain of 15 points. If the NASDAQ started the week at 2050 points, what was the market at the end of the week?

80. In ten running plays in a football game, the tailback gained 5 yards, lost 3 yards, gained 15 yards, gained 7 yards, gained 12 yards, lost 4 yards, lost 2 yards, gained 20 yards, lost 5 yards, and gained 6 yards. What was his cumulative yardage in the game?

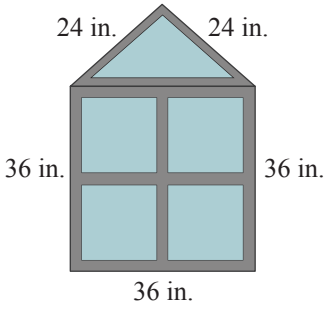
81. A hiker, beginning at an altitude of 970 ft, ascends a peak 5260 ft. Next, he descends 3130 ft and climbs another peak 1570 ft. After a brief rest, he continues his ascent another 2190 ft. Finally he descends 4040 ft. What is his final altitude?

82. A commercial fishing boat casts a net and brings in 258 fish. The fishermen find that 77 of the fish are too small to sell and throw them back. They cast their net again and bring in 401 more fish. Of these, 98 are too small to sell. How many fish do the fishermen have left to sell?

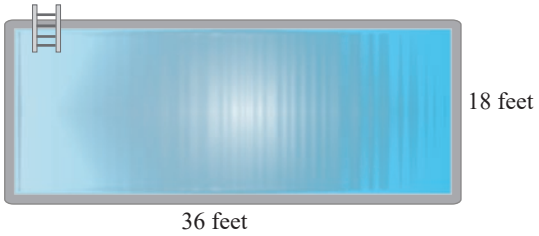
83. Find the perimeter of a parking lot that is in the shape of a rectangle 50 yards wide and 75 yards long.



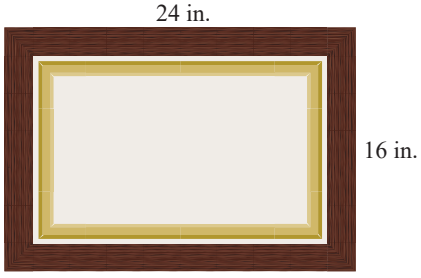
84. A window is in the shape of a triangle placed on top of a square. The length of each of two equal sides of the triangle is 24 inches and the third side is 36 inches long. The length of each side of the square is 36 inches long. Find the perimeter of the window.



85. A rectangular pool measures 36 feet long by 18 feet wide. Find the area of the pool in square feet.



86. A painting is mounted in a rectangular frame (16 inches by 24 inches) and hung on a wall. How many square inches of wall space will the framed painting cover?



87. Cheyenne has been commissioned by her city to paint a mural on the side of a brick building and must calculate the area of the wall so she can plan and scale her mural. The side of the building measures 120 feet tall and 84 feet wide. Find the area of the mural.

88. A rectangular lot for a house measures 210 feet long by 175 feet wide. Find the area of the lot in square feet.

Example 9 Application: Calculating an Average

On an English exam, two students scored 95, five scored 86, one scored 82, one scored 78, and six scored 75. What was the average score for the class?

Solution

There were fifteen students in the class. We can multiply as follows rather than add all fifteen scores.

$$\begin{array}{rrrrr} {\scriptstyle 1}95 & {\scriptstyle 3}86 & 82 & 78 & {\scriptstyle 3}75 \\ \times\ 2 & \times\ 5 & \times\ 1 & \times\ 1 & \times\ 6 \\ \hline 190 & 430 & 82 & 78 & 450 \end{array}$$

Next, we add the five products to find the sum of all of the scores.

$$\begin{array}{r} {\scriptstyle 3\ 1}190 \\ 430 \\ 82 \\ 78 \\ +450 \\ \hline 1230 \end{array}$$ Sum

Finally, divide by 15 because the total represents 15 scores.

$$\begin{array}{r} 82 \\ \hline 15\overline{)1230} \\ -1200 \\ \hline 30 \\ -30 \\ \hline 0 \end{array}$$ Average

The average score for the class was 82.

Now work margin exercise 9.

9. In a recent high school basketball game, two of the starters scored 18 points each, two scored 10 points, and one scored 4 points. What was the average number of points scored among the starters?

Margin Exercise Answers

1. 1001 visitors **2.** 322 pounds **3.** 120 pencils **4.** $25,125 **5.** $868 **6.** 21 **7. a.** Season 2 **b.** Season 5 **c.** 34 home runs **8.** 29,000 people **9.** 12 points

1.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The words "product" or "double" indicate that the operation of _____ is involved.

2. To find the average of a set of numbers, the first step is to _____ the given numbers.

3. When solving word problems, the very first thing to do is to _____ the problem carefully.

4. The words "sum" and "increased by" are key words to indicate the problem requires _____.

5. In solving word problems you should always _____ your work. Then make sure that the answer seems _____.

6. "Difference" and "less than" indicate that _____ will be involved in the problem.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Averages are found by performing addition and then division.

8. The sum of 312 and 4 is 1248.

9. The word "quotient" indicates multiplication.

10. After reading a problem carefully, the next step might be to make a diagram or draw a figure.

Applications

Solve.

1. Steven is calculating how many calories are in his lunch. He has a hamburger that has 354 calories, a medium fry that has 365 calories, and a chocolate milk shake that has 384 calories. How many total calories is his meal?

2. Shawna is refurnishing her living room. She buys a sofa and love seat set for $1549, a coffee table for $245, an end table for $99, and a rug for $479. How much did she pay for all of the items to redecorate her living room?

3. Meghan is taking inventory of syringes at a clinic. She finds 240 in the first supply closet, 115 in the second supply closet, and 65 in the third supply closet. How many syringes does the clinic have in these three closets?

4. Dennis is filling out tax forms and needs to total his charitable donations. During the year, he donated $234 to Goodwill, $345 to the Red Cross, and $260 to Child's Play. What is the total amount of donations that Dennis will report on his taxes?

5. Marcus is moving from Cleveland, Ohio, to Houston, Texas, which is a 1299 mile drive. In one day, he drives 651 miles. How many more miles must he drive to reach Houston, Texas?

6. Amy is at an arcade and wins 3483 tickets. She spends 2975 of the tickets on prizes from the redemption area. How many tickets does Amy have left?

7. Tommy and Liz invited 250 people to their wedding. So far, 186 people have returned their RSVP. How many people have not sent in their RSVP?

8. Kyle is reading a book that has 978 pages. He has read 382 pages so far. How many pages does Kyle need to read to finish the book?

9. A secretary is making an inventory of office supplies. She finds 12 full boxes of blue pens which have 25 pens in each box. How many pens will she indicate on the inventory?

10. Sean has a part-time job as a data entry clerk. He earns $11 per hour and works for 22 hours each week. How much money does Sean make per week?

11. A jeweler makes a necklace that requires 128 beads. He receives an order for 6 necklaces. How many beads will he need to fulfill the order?

12. A juice bar sells fresh squeezed orange juice. To make an 8 ounce glass of orange juice, they use 4 oranges. The juice bar typically sells 86 glasses of orange juice per day. How many oranges do they typically use each day?

13. During a food drive, people donated 208 cans of baked beans. The cans of baked beans will be equally distributed to 8 food kitchens in the area. How many cans of baked beans will each food kitchen receive?

14. Claire took out a 6 month loan for $2700 to cover car repairs. If she makes equal monthly payments, how much will she pay towards the loan each month?

15. Sixteen families want to raise money to build a playground on an empty lot for the neighborhood children. The total cost to buy the empty lot and build the playground is $18,800. How much money will each family need to raise if they all raise the same amount of money?

16. A homeless shelter is holding a fundraising event with a goal of $8250 to make repairs to their building. There are 11 companies participating in the fundraiser and each company plans to raise the same amount of money. How much money does each team need to raise to reach the goal?

17. Mark bought a new bicycle for $950. He also paid an additional $61 in sales tax. If he traded in his old bicycle for $320 and contributed that amount to the total for the new bicycle, how much does he owe?

18. To purchase a new dining room set for $1200, Mrs. Steel had to pay an additional $72 in sales tax. If she made an initial payment of $486, how much did she still owe?

19. David bought a new phone for $250. The store had a deal that gave him $10 off. He also bought 3 video games for $48 each. What did he pay total for the phone and video games?

20. Vivian is shopping for groceries. She estimates that the total of the groceries will be $140. She has $8 worth of coupons to cash in. While in the checkout line, she also decides to purchase two magazines, which cost $3 each. How much will Vivian pay upon checkout?

21. To purchase a new refrigerator for $1200 including tax, Mr. Kline paid $240 down and the remainder in six equal monthly payments. What were his monthly payments?

22. To purchase a new 47-inch LED flat screen TV with a surround sound system that sells for $1300 including tax, Mr. Daley paid $200 down and the remainder in five equal monthly payments. How much were his monthly payments?

23. For a class in statistics, Anthony bought a new graphing calculator for $95, special graphing paper for $8, a USB flash drive for $10, a textbook for $105, and a workbook for $37. How much did he spend for this class?

24. Lynn decided to take up surfing. She bought a new surfboard for $675, a wet suit for $130, a beach towel for $12, and a new swimsuit for $57. How much money did she spend? (Sales tax was included in the prices.)

25. Pat needed art supplies for a new course at the local community college. She bought a portfolio for $32, a zinc plate for $44, etching ink for $12, and three sheets of rag paper for a total of $6. She received a student discount of $9. How much did she spend on art supplies?

26. Michael is purchasing supplies for his upcoming classes. He buys a textbook for $180, a calculator for $90, a pack of pens for $5, and four notebooks for a total of $11. The bookstore is offering a discount of $20 off when students spend at least $200. How much did Michael's supplies cost?

27. Paula is training for a marathon and decided to buy some new clothes. She bought a pair of running shoes for $84, two pairs of socks for $5 a pair, one pair of shorts for $26, and two shirts for $15 each. If taxes are included in the prices, how much did she spend?

28. Miguel decided to go shopping for school clothes before college started in the fall. How much did he spend if he bought four pairs of pants for $21 per pair, five shirts for $18 each, three pairs of socks for $4 a pair, and two pairs of shoes for $38 a pair?

29. Alan wants to buy a new car. He could buy a red one for $8500 plus $510 in sales tax and $135 in fees, or he could buy a blue one for $8700 plus $522 in sales tax and $140 in fees. If the manufacturer is giving a $250 rebate on the blue model, which car would be cheaper for Alan? How much cheaper would it be?

30. Alexis is looking to join a gym with classes and personal training for one year. Gym 1 charges $90 to activate the membership, then charges $350 for the year for membership (including classes and personal training), and an additional $28 in taxes and fees. Gym 2 has no membership activation charge, but costs $380 for membership for the year, $75 for classes and personal training, and $21 in taxes and fees. Which gym membership will be cheaper for Alexis? How much cheaper will it be for the year-long membership?

Find the average of each set of numbers. See Example 7.

31. 56, 64, 38, 58

32. 102, 113, 97, 100

33. 512, 618, 332, 478

34. 436, 520, 630, 422

35. 6, 7, 8, 4, 4, 5, 8, 6

36. 5, 4, 5, 6, 5, 8, 9, 6

Solve.

37. The Lee family spent the following amounts for groceries: $338 in June; $307 in July; $318 in August. What was the average amount they spent for groceries in these three months?

38. If Rina's cell phone bills for the past five months have been $56, $63, $52, $85, and $49, what was her average cell phone bill for the past 5 months?

39. In one month, an airline pilot spent the following number of hours in preparation for and flying each of 12 flights: 6, 8, 9, 6, 7, 7, 7, 5, 6, 6, 6, and 11 hours. What was the average amount of time the pilot spent per flight?

40. Over one week, a hospital had the following number of patients in the ICU (intensive care unit) each day: 22, 19, 23, 19, 17, 21, 19. What was the average number of patients in the ICU each day during the week?

41. Bill wanted to compare car insurance rates to find out the average amount he should pay for car insurance. If he looked at four different companies and the monthly rates were $164, $107, $131, and $98, what is the average monthly rate of car insurance?

42. A salesman sold items from his sales list for $972, $834, $1005, $1050, and $799. What was the average price per item?

43. During a sports trivia game, one team scored 35 points, three teams scored 23 points, two teams scored 18 points, and two teams scored 14 points. What was the average score of the teams?

44. On a history exam, two students scored 95, six scored 90, three scored 80, and one scored 50. What was the class average?

45. Ms. Lee bought 150 shares of stock in Microsoft at $75 per share. Two months later, she bought another 100 shares at $79 per share. What average price per share did she pay? If she sold all 250 shares at $77 per share, what was her profit?

46. Three families, each with two children, had incomes of $56,000. Two families, each with four children, had incomes of $62,000. Four families, each with two children, had incomes of $45,000. One family had no children and an income of $37,000. What was the average income per family?

47. Lacie owns a shoe store. During the first week of a month, Lacie sold 41 pairs of shoes and received two shipments, one with 26 pairs of shoes and one with 10 pairs of shoes. During the second week of the month, Lacie received a shipment of 20 pairs of shoes and sold 35 pairs of shoes. During the third week of the month, Lacie sold 38 pairs of shoes and received two shipments, one with 22 pairs of shoes and one with 19 pairs of shoes. During the final week of the month, Lacie received three shipments, one with 16 pairs of shoes, one with 24 pairs of shoes, and one with 14 pairs of shoes, and she sold 29 pairs of shoes.

a. What was the average weekly difference between Lacie's shipments and her sales?

b. What was Lacie's inventory at the end of the month if she began the month with an inventory of 246 pairs of shoes?

48. During July, Mr. Rodriguez made deposits in his checking account of $400 and $750 and wrote checks totaling $625. During August, his deposits were $632, $322, and $798, and his checks totaled $978. In September, his deposits were $520, $436, $200, and $376, and his checks totaled $836.

a. What was the average monthly difference between his deposits and his withdrawals?

b. What was his bank balance at the end of September if he had a balance of $500 on July 1?

49. The five longest rivers in the world are given in the following table. What is the average length of these rivers?

Five Longest Rivers in the World

River	Length (miles)
Nile	4132
Amazon	3980
Yangtze	3917
Mississippi/Missouri	3902
Yenisei	3434

50. The 10 largest cities in South Carolina have the following approximate populations. What is the average population of these cities?

10 Largest Cities in South Carolina

City	Population
Columbia	129,333
Charleston	115,638
North Charleston	97,601
Rock Hill	69,210
Mount Pleasant	66,420
Greenville	61,782
Sumter	59,180
Summerville	45,240
Spartanburg	40,387
Hilton Head	34,249

Writing & Thinking

51. State the basic strategy for solving word problems.

52. Make up three word problems that include key words to indicate operations such as addition, subtraction, multiplication and division. Underline the key words.

53. Give an example where you might use average (other than in a class).

54. Discuss how you used mathematics to solve some problem in your life this week. (You know you did!)

8. Simplify:

$$\left|-\frac{5}{3}\cdot\frac{3}{10}\right|-\frac{1}{9}\div\left(\frac{4}{3}\right)^2$$

Example 8 Using the Order of Operations with Real Numbers

Simplify: $\left|-\frac{3}{5}\cdot\frac{5}{6}\right|-\frac{1}{4}\div\left(\frac{5}{2}\right)^2$

Solution

$$\left|-\frac{3}{5}\cdot\frac{5}{6}\right|-\frac{1}{4}\div\left(\frac{5}{2}\right)^2=\left|-\frac{3}{5}\cdot\frac{5}{6}\right|-\frac{1}{4}\div\left(\frac{25}{4}\right)$$ Exponents

$$=\left|-\frac{\cancel{3}}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{6}_{2}}\right|-\frac{1}{\cancel{4}}\cdot\frac{\cancel{4}}{25}$$ Divide, then multiply and reduce.

$$=\left|-\frac{1}{2}\right|-\frac{1}{25}$$

$$=\frac{1}{2}-\frac{2}{50}$$ Evaluate the absolute value.

$$=\frac{25}{50}-\frac{2}{50}$$ Find the LCD.

$$=\frac{23}{50}$$ Subtract.

Now work margin exercise 8.

Completion Example Answers

6. $6(25-16)-2\cdot 27=6(9)-2\cdot 27=54-54=0$

Margin Exercise Answers

1. a. 9 **b.** 19 **c.** -6 **d.** -12; **2.** –37 **3.** –22 **4.** 10 **5.** –14 **6.** 3 **7.** 12.6 **8.** $\frac{7}{16}$

1.4 Exercises

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. When following the rules for order of operations, simplify within _____ _____ first.
2. Start by simplifying the _____ grouping symbol and working outward.
3. When performing multiplication and division, move from _____ to _____.
4. When performing addition and subtraction, perform the operations in the order they _____, moving left to right.
5. A negative sign in front of a variable means the variable is being multiplied by ____.
6. Parentheses, brackets, and braces are known as _____ symbols.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If there are no grouping symbols, multiplication should always be performed before addition.

8. When following the rules for order of operations, powers indicated by exponents should be evaluated last.

9. The square root symbol is a grouping symbol.

10. A well-known mnemonic device for remembering the rules for order of operations is SADMEP.

Practice

Evaluate each expression. See Example 1.

1. 12^2

2. 17^2

3. 20^2

4. 8^2

5. $\sqrt{36}$

6. $\sqrt{81}$

7. $\sqrt{225}$

8. $\sqrt{169}$

9. $\sqrt{196}$

10. $\sqrt{361}$

11. $\sqrt{900}$

12. $\sqrt{3600}$

Simplify.

13. a. $24 \div 4 \cdot 6$

b. $24 \cdot 4 \div 6$

14. a. $20 \div 5 \cdot 2$

b. $20 \cdot 5 \div 2$

15. $15 \div (-3) \cdot 3 - 10$

16. $20 \cdot 2 \div 2^2 + 5(-2)$

17. $3^2 \div (-9) \cdot \left(4 - 2^2\right) + 5(-2)$

18. $4^2 \div (-8)(-2) + 3\left(2^2 - 5^2\right)$

19. $14 \cdot 3 \div (-2) - 6(4)$

20. $6(13-15)^2 \cdot 8 \div 2^2 + 3(-1)$

21. $-10 + 15 \div (-5) \cdot 3^2 - 10^2$

22. $16 \cdot 3 \div \left(2^2 - 5\right)$

23. $9 \cdot \sqrt{52 - 3} \div 7$

24. $4 \cdot \sqrt{25} + 21 \div 3$

25. $5\left(\sqrt{289} - 7\right)$

26. $2 \cdot \sqrt{3^2 + 7} - 3$

27. $2 - 5\left[(-20) \div (-4) \cdot 2 - 40\right]$

28. $9 - 6\left[(-21) \div 7 \cdot 2 - (-8)\right]$

29. $(7-10)\left[49 \div (-7) + 20 \cdot 3 - (-10)\right]$

30. $(9-11)\left[(-10)^2 \cdot 2 + 6(-5)^2 - 10^2 + 3 \cdot 5\right]$

31. $8-9\left[(-39)\div(-13)+7(-2)-(-2)^2\right]$

32. $6-20\left[(-15)\div 3\cdot 5+6\cdot 2\div 3\right]$

33. $|16-20|\left[32\div|3-5|-5^2\right]$

34. $|10-30|\left[4^2\cdot|5-8|\div(-2)^2+|17-18|\right]$

35. $(-10)+(-2)+|2-4|$

36. $|16-20|+(-10)^2+5^2$

37. $\frac{3}{8}\cdot\frac{4}{5}+\frac{1}{15}$

38. $\frac{1}{4}\cdot\frac{12}{15}+\frac{2}{7}$

39. $\frac{1}{3}\div\frac{1}{2}-\frac{5}{6}\cdot\frac{3}{4}$

40. $\frac{2}{9}\div\frac{14}{3}-\frac{1}{6}\cdot\frac{4}{7}$

41. $\left(\frac{5}{6}\right)^2\div\frac{5}{12}-\frac{3}{8}$

42. $\left(\frac{2}{5}\right)^2\cdot\frac{5}{8}+\frac{1}{5}\div\frac{3}{4}$

43. $\frac{7}{6}\cdot 2^2-\frac{2}{3}\div\frac{1}{2}$

44. $\frac{3}{4}\div 3^2-4\left(\frac{1}{2}\right)^2$

45. $\left(-\frac{3}{4}\right)\div\left(-\frac{3}{5}\right)\cdot\frac{7}{8}+\frac{3}{16}$

46. $\left(-\frac{2}{3}\right)\div\frac{7}{12}-\frac{2}{7}+\left(-\frac{1}{2}\right)^2$

47. $\left(\sqrt{225}\div 5+4\right)^2$

48. $\sqrt{3\cdot 27}-\left(5^2-7\right)$

49. $-\sqrt{8^2+(9+7)\cdot 5}$

50. $6\div\left[\left(-\sqrt{400}-\sqrt{256}\right)\div 12\right]$

51. $\left(-\frac{9}{10}\right)+\frac{5}{8}\cdot\frac{4}{5}\div\frac{6}{10}+\frac{2}{3}$

52. $\frac{5}{8}\div\frac{5}{2}+\left(-\frac{1}{2}\right)^2\cdot\frac{2}{5}$

53. $-0.7-8.5\div 1.7$

54. $-0.4-2.6\cdot 1.5$

55. $(3.1+1.1)\div(5.7-6.9)$

56. $(3.2-6.5)\cdot 2^2$

57. $-15\div\left(\frac{1}{4}-\frac{7}{8}\right)$

58. $-12\div\left(\frac{1}{2}+\frac{1}{10}\right)$

59. $(-5-7)\div -4-8$

60. $4(-2)^2-10\div 5+1$

Solve.

61. Find the average of the five numbers: –7, 8, –3, 5, and 2.

62. Find the average of the six numbers: –1, –2, –3, 3, 2, and 1.

63. If the square of $\frac{7}{8}$ is subtracted from the square of $\frac{3}{4}$, what is the difference?

64. Find the quotient if the sum of $\frac{1}{5}$ and $\frac{2}{15}$ is divided by the difference between $\frac{7}{8}$ and $\frac{3}{4}$.

Use a calculator to evaluate each expression.

65. $3.4 \div 4 + 5 \cdot 8.32$

66. $8.1 \div 5 + 16.3 \cdot 7$

67. $0.75 \div 1.5 + 7 \cdot 3.1^2$

68. $1.05 \div (-3) \cdot 3.7 - 1.1^2$

69. $6.32 \cdot 8.4 \div 16.8 + 3.5^2$

70. $(82.7 + 16.2) \div (14.83 - 19.83)^2$

Applications

Solve.

71. Madeline sells homemade aprons online and needs to determine how to charge for each apron. To create each apron, she spends \$8.50 on supplies and it takes her $1\frac{1}{4}$ hours to cut and sew each one. Madeline wants to charge \$11 per hour of work plus the cost of supplies.

a. Write an expression to describe how much each apron will cost.

b. Evaluate the expression to determine the selling cost of each apron.

c. Madeline will sew a name or initials onto the apron for an additional charge of \$1.75 per letter. If Kathy orders an apron and wants her name sewn onto it, how much will the apron cost?

72. The Matthews family, a family of 4, is planning a trip to New York City. During their visit, they want to see the Broadway play *Matilda*. The tickets cost \$102 each. The Matthews purchase the tickets online and the website charges a service fee of \$7.50 per ticket. The website is running a sale where the Matthews can get 10% off of their entire purchase.

a. Write an expression to describe how much of a discount the Matthews will receive on their purchase.

b. What is the final purchase price of the tickets?

73. Dennis overdrew his checking account and ended up with a balance of –\$42. The bank charged a \$35 overdraft fee and an additional \$5 fee for every day the account was overdrawn. Dennis left his account overdrawn for 3 days.

a. Write an expression to show the balance of Dennis's checking account after 3 days.

b. Simplify the expression in Part **a.** to find the balance of Dennis's checking account after 3 days.

74. Camila is a seamstress and is creating bridesmaid dresses. She has 115 yards of satin fabric. For each dress, the skirt requires 3 yards of satin and the bodice requires 1.5 yards of satin. She plans to make 20 dresses.

a. Write an expression to show how much fabric Camila will have left over after making the dresses.

b. Simplify the expression in Part **a.** to determine how much fabric Camila will have left over.

c. Camila wants to make shawls from the leftover fabric. Each shawl requires 1.25 yards of satin. Can she make 15 shawls?

75. During harvest season, farmers donate fresh food to a local food kitchen. To make sure the food doesn't spoil, the food kitchen distributes the food between themselves and 5 other food kitchens in the area. One farmer donates $12\frac{1}{2}$ pounds of potatoes, another farmer donates $15\frac{3}{4}$ pounds of potatoes, and a third farmer donates $11\frac{3}{4}$ pounds of potatoes. The food kitchen finds that $1\frac{1}{4}$ pounds of the donated potatoes are rotten.

a. Write an expression to show how many pounds of potatoes each food kitchen will receive.

b. Simplify the expression from Part **a.** to determine how many pounds of potatoes each food kitchen will receive.

Writing & Thinking

76. Explain, in your own words, why the following expression cannot be evaluated.

$$\left(24-2^4\right)+6(3-5)\div\left(3^2-9\right)$$

77. Consider any number between 0 and 1. If you square this number, will the result be larger or smaller than the original number? Is this always the case? Explain.

78. Consider any number between -1 and 0. If you square this number, will the result be larger or smaller than the original number? Is this always the case? Explain.

Example 2 Identifying Properties of Addition and Multiplication

For each of the following equations, state the property illustrated, and show that the statement is true for the value given for the variable by substituting the value in the equation and evaluating.

a. $x+14=14+x$ given that $x=-4$

b. $(3\cdot 6)x=3(6x)$ given that $x=5$

c. $12(y+3)=12y+36$ given that $y=-2$

Solutions

a. The commutative property of addition is illustrated.

$$(-4)+14=10 \text{ and } 14+(-4)=10$$

b. The associative property of multiplication is illustrated.

$$(3\cdot 6)\cdot 5=18\cdot 5=90 \text{ and } 3\cdot(6\cdot 5)=3\cdot 30=90$$

c. The distributive property is illustrated.

$$12(-2+3)=12(1)=12 \text{ and } 12(-2)+36=-24+36=12$$

Now work margin exercise 2.

2. State the property illustrated and show that the statement is true for the value given for the variable.

a. $x+21=21+x$ given that $x=-7$

b. $(5\cdot 4)x=5(4x)$ given that $x=2$

c. $11(y+3)=11y+33$ given that $y=-4$

Margin Exercise Answers

1. a. associative property of multiplication **b.** distributive property **c.** zero factor law **d.** associative property of addition **e.** commutative property of multiplication **f.** additive identity **g.** additive inverse **2. a.** commutative property of addition $(-7)+21=14$ and $21+(-7)=14$ **b.** associative property of multiplication $(5\cdot 4)\cdot 2=40$ and $5\cdot(4\cdot 2)=40$ **c.** distributive property $11(-4+3)=-11$ and $11(-4)+33=-11$

1.5 Exercises

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The multiplicative inverse of a number is its _____.
2. The _____ _____ of all numbers is 1.
3. Zero multiplied by a number or variable is an example of the _____-_____ law.
4. The distributive property involves two operations, _____ and _____.
5. The additive inverse of a number is the _____ of that number.
6. In the term $8x$, the 8 is the _____ of the variable.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Changing the order of the numbers in an addition problem is allowed because of the associative property of addition.

8. The equation is an example of the associative property of multiplication.

9. The additive identity of all numbers is 1.

10. The commutative property works for division and subtraction.

Practice

Complete the expressions using the given property. Do not simplify.

1.	$7+3=$ _____	commutative property of addition
2.	$(6\cdot 9)\cdot 3=$ _____	associative property of multiplication
3.	$19\cdot 4=$ _____	commutative property of multiplication
4.	$18+5=$ _____	commutative property of addition
5.	$6(5+8)=$ _____	distributive property
6.	$16+(9+11)=$ _____	associative property of addition
7.	$2\cdot(3x)=$ _____	associative property of multiplication
8.	$3(x+5)=$ _____	distributive property
9.	$3+(x+7)=$ _____	associative property of addition
10.	$9(x+5)=$ _____	distributive property
11.	$6\cdot 0=$ _____	zero-factor law
12.	$6\cdot 1=$ _____	multiplicative identity
13.	$0+(x+7)=$ _____	additive identity
14.	$0\cdot(-13)=$ _____	zero-factor law
15.	$2(x-12)=$ _____	distributive property
16.	$(-5)+5=$ _____	additive inverse
17.	$6.3+(-6.3)=$ _____	additive inverse
18.	$3\cdot\frac{1}{3}=$ _____	multiplicative inverse

State the name of each property illustrated. See Example 1.

19. $5+16=16+5$

20. $5\cdot 16=16\cdot 5$

21. $32 \cdot 1 = 32$

22. $32 + 0 = 32$

23. $5 + (3 + 1) = (5 + 3) + 1$

24. $5 + (3 + 1) = (3 + 1) + 5$

25. $13(y + 2) = (y + 2) \cdot 13$

26. $13(y + 2) = 13y + 26$

27. $6(2 \cdot 9) = (2 \cdot 9) \cdot 6$

28. $6(2 \cdot 9) = (6 \cdot 2) \cdot 9$

29. $5 \cdot \frac{1}{5} = 1$

30. $14 \cdot \frac{1}{14} = 1$

31. $7.1 + (-7.1) = 0$

32. $(-9) + 9 = 0$

33. $1 \cdot 14.2 = 14.2$

34. $(5 \cdot 3) \cdot (-7) = 5(3 \cdot (-7))$

35. $5.68 \cdot 0 = 0 \cdot 5.68 = 0$

36. $0 + 5.68 = 5.68$

37. $2 + (x + 6) = (2 + x) + 6$

38. $2(x + 6) = 2x + 12$

First evaluate each expression using the rules for order of operations and then use the distributive property to evaluate the same expression. The value must be the same.

39. $6(3 + 8)$

40. $7(8 - 5)$

41. $10(2 - 9)$

42. $13(5 + 3)$

For each of the following equations, state the property illustrated, and show that the statement is true for the value of $x = 4$, $y = -2$, or $z = 3$ by substituting the corresponding value in the equation and evaluating. See Example 2.

43. $6 \cdot x = x \cdot 6$

44. $19 + z = z + 19$

45. $8 + (5 + y) = (8 + 5) + y$

46. $(2 \cdot 7) \cdot x = 2 \cdot (7x)$

47. $5(x + 18) = 5x + 90$

48. $(2z + 14) + 3 = 2z + (14 + 3)$

49. $(6 \cdot y) \cdot 9 = 6 \cdot (y \cdot 9)$

50. $11 \cdot x = x \cdot 11$

51. $z + (-34) = -34 + z$

52. $3(y + 15) = 3y + 45$

53. $2(3 + x) = 2(x + 3)$

54. $(y + 2)(y - 4) = (y - 4)(y + 2)$

55. $5 + (x - 15) = (x - 15) + 5$

56. $z + (4 + x) = (4 + x) + z$

57. $(3x) \cdot 5 = 3 \cdot (x \cdot 5)$

58. $(x + y) + z = x + (y + z)$

Applications

Solve

59. Jessica works part-time at a retail store and makes \$11 an hour. During one week, she worked $6\frac{1}{2}$ hours on Monday and $4\frac{1}{4}$ hours on Thursday.

a. Determine the amount of money she earned during the week by evaluating the expression $\$11 \cdot \left(6\frac{1}{2} + 4\frac{1}{4}\right)$.

b. Rewrite this expression to remove the parentheses using one of the properties talked about in this section.

c. What property did you use in Part **b.** to rewrite the expression?

60. Robin went to the grocery store to buy a few items she needed in order to cook dinner. She bought milk for $3.99, rolls for $2.25, a package of steaks for $12.01, and some marinade for $1.75. Before getting to the checkout line, Robin remembered that she only had $20 in her purse. Did she have enough money to buy the food items if the store does not charge sales tax on food?

a. Write an expression to find the total of Robin's food purchases. Do not simplify.

b. Robin doesn't have a calculator to determine the total cost of her items. She wants to make sure that she has enough money to buy them. Rearrange the expression from Part **a.** so that she could quickly find the total using mental math.

c. What properties did you use in Part **b.** to rewrite the expression?

d. Did Robin have enough money to purchase all of the items?

61. Jordan didn't balance his checking account during the week and ended up overdrawing his account. He had a starting balance of $85.04 and wrote checks for two bills for the amounts of $28.79 and $50.00. He also used his debit card to purchase lunch for $12.16. In order to avoid an overdraft fee, Jordan must deposit enough money today to bring his balance back to a minimum of zero.

a. Write an expression to find the current balance of Jordan's checking account. Do not simplify.

b. Evaluate the expression from Part **a.** to determine the current balance of Jordan's checking account.

c. Write an equation to show Jordan's current checking account balance plus the amount he must deposit today to bring the balance to zero.

d. What property is illustrated in Part **c.**?

Writing and Thinking

62. a. The distributive property illustrated as $a(b+c) = ab + ac$ is said to "distribute multiplication over addition." Explain, in your own words, the meaning of this phrase.

b. What would an expression that "distributes addition over multiplication" look like? Explain why this would or would not make sense.

Now evaluate.

$$x-2=(5)-2$$
$$=3$$

Now work margin exercise 6.

7. Simplify and evaluate $4x^2-3x+2x^2+4x$ for $x=-2$.

Completion Example 7 Simplifying and Evaluating Expressions

Simplify and evaluate $5x^2-2x^2+x+3x^2-2x$ for $x=-1$.

Solution

First, simplify the expression by combining like terms.

$$5x^2-2x^2+x+3x^2-2x=(\_\_\_\_\_)x^2+(\_\_\_\_)x$$
$$=\_\_x^2-\_\_x$$

Now, substitute -1 for x and evaluate.

$$6x^2-x=6(\_\_\_\_)^2-(\_\_\_\_)$$
$$=6(\_\_)-(\_\_\_\_)$$
$$=\_\_+\_\_$$
$$=\_\_$$

Now work margin exercise 7.

Completion Example Answer

7. $(5-2+3)x^2+(1-2)x=6x^2-1x$; $6(-1)^2-(-1)=6(1)-(-1)=6+1=7$

Margin Exercise Answers

1. $\frac{5}{3}$, -1, and 0 are like terms; $10x$, $-3x$, and $-5.65x$ are like terms; and $4x^2z$ and $2x^2z$ are like terms.
2. a. $12x$ **b.** $1.1z+5$ **c.** $7x^2+5y$ **d.** $10a+8$ **e.** Already simplified **f.** $8x$ **3. a.** For $x=10$, $x^2=100$ For $x=-5$, $x^2=25$ **b.** For $x=10$, $-x^2=-100$ For $x=-5$, $-x^2=-25$. **4.** $7x+6$; -22
5. $-3ab-a$; 12 **6.** $5y-16$; 4 **7.** $6x^2+x$; 22

1.6 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. If no number is written next to a variable, the coefficient is understood to be the number _____.

2. Any constant, variable, or product or quotient of a constant and/or variable is a _____ .

3. A single number is a/an _____ .

4. In a term, the number being multiplied by the variable is the numerical _____ of that term.

5. To combine like terms, add or subtract the _____ and keep the common variable expression.

6. When substituting, _____ must be used around negative numbers.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A variable that does not appear to have an exponent has an exponent of 1.

8. In the term $-9x$, nine is being subtracted from x.

9. In the term "$12a$," 12 is the constant.

10. Like terms have the same coefficients.

Practice

Identify the like terms in each list of terms. See Example 1.

1. $-5, 3, 7x, 8, 9x, 3y$

2. $-2x^2, -13x^3, 5x^2, 14x^2, 10x^3$

3. $5xy, -x^2, -6xy, 3x^2y, 5x^2y, 2x^2$

4. $3ab^2, -ab^2, 8ab, 9a^2b, -10a^2b, ab, 12a^2$

5. $24, 8.3, 1.5xyz, -1.4xyz, -6, xyz, 5xy^2z, 2xyz^2$

6. $-35y, 1.62, -y^2, -y, 3y^2, \frac{1}{2}, 75y, 2.5y^2$

Simplify each expression by combining like terms. See Example 2.

7. $8x + 7x$

8. $3y + 8y$

9. $5x + (-2x)$

10. $7x + (-3x)$

11. $6y^2 - y^2$

12. $16z^2 - 5z^2$

13. $3x - 5x + 12x$

14. $2a + 14a - 25a$

15. $6c - 13c + 5c$

16. $40x - 30x - 10x$

17. $4x + 2 + 3x$

18. $3x - 1 + x$

19. $5x^2 - 3x^2 + 2x$

20. $-2x^2 - x^2 - x$

21. $7x^2 - 4x^2 + 20$

22. $14y^3 - 25 + 8y^3$

23. $2x^2 - 2y + 5x^2 + 6x^2$

24. $4a + 2a - 3b - a$

25. $4x + 7 - 8 + 3x$

26. $-5x - 1 + 8 + 9x$

27. $2n^2 - 6n + 1 - 4n^2 + 8n - 4$

28. $3n^2 + 2n - 5 - n^2 + n - 4$

29. $3 - 5x^2 + 4x^2 + 20x + 42 - 17x$

30. $13x + 12x^2 + 15x - 35 - 41 - 2x^2$

31. $3(n+1)+n$

32. $2(n-4)+n+1$

33. $5(a-b)+2a-3b$

34. $4a-3b+2(a+2b)$

35. $3(2x+y)+2(x-y)$

36. $4(x+5y)+3(2x-7y)$

37. $y-\dfrac{4y+5y}{3}$

38. $z-\dfrac{3z+5z}{4}$

39. $\dfrac{2x+3x}{3}+x$

40. $\dfrac{2y+4y}{5}-2y$

Evaluate the expression for $x = 2$ and $y = -3$. See Example 3.

41. $-x^2$

42. $-y^2$

43. $(-x)^2$

44. $(-y)^2$

45. $-x$

46. $-y$

Simplify each expression and then evaluate the expression for $x = 4$, $y = 3$, $a = -2$, and $b = -1$. See Examples 4, 6, and 7.

47. $5y + 4 - 2y$

48. $7b - 17 - b$

49. $3(y-1)+2(y+2)$

50. $4(y+3)+5(y-2)$

51. $3.1a^2 - 0.9a^2 + 4a - 5.3a^2$

52. $8.3x^2 - 5.7x^2 + x^2 + 2$

53. $2.4(x+1)+1.3(x-1)$

54. $1.3(y+2)-2.6(8-y)$

55. $\dfrac{3a+5a}{-2}+12a$

56. $8a+\dfrac{5a+4a}{9}$

57. $\dfrac{-4b-2b}{-3}+\dfrac{2b+5b}{7}$

58. $\dfrac{5b+3b}{4}+\dfrac{-4b-b}{-5}$

Simplify each expression and then evaluate the expression for $x = -2$ and $y = -1$. See Examples 4 and 7.

59. $2x^2 - 3x^2 + 5x - 8 + 1$

60. $5x^2 - 4x + 2 - x^2 + 3$

61. $y^2 + 2y^2 + 2y - 3y$

62. $y^2 + y^2 - 8y + 2y - 5$

63. $y^3 + 3y^3 + 5y - 4y^2 + 1$

64. $7y^3 + 4y^2 + 6 + y^2 - 12$

65. $2(x^2-3x-5)+3(x^2+5x-4)$

66. $5(y^2-4y+3)-2(y^2-2y+10)$

Simplify each expression and then evaluate the expression for $a = -1$, $b = -2$, and $c = 3$. See Example 5.

67. $a^2 - a + a^2 - a$

68. $a^3 - 2a^3 - 3a + a - 7$

69. $5ab - 7a + 4ab + 2b$

70. $2ab + 4b - 3a + ab - b$

71. $14(a+7) - 15(b+6) + 2(c-3)$

72. $12(a-3) + 8(b-2) - 3(c+4)$

73. $20(a+b+c) - 10(a+b+c)$

74. $16(a-b+c) + 16(-a+b-c)$

Applications

Solve.

75. An apartment management company owns a property with 100 units. The company has determined that the profit made per month from the property can be calculated using the equation $P = -10x^2 + 1500x - 6000$, where x is the number of units rented per month. How much profit does the company make when 80 units are rented?

76. A ball is thrown upward from an initial height of 96 feet with an initial velocity of 16 feet per second. After t seconds, the height of the ball can be described by the expression $-16t^2 + 16t + 96$. What is the height of the ball after 3 seconds?

77. A moving company starts the week with 72 bundles of small boxes and 50 bundles of medium boxes. During the week, they use 25 bundles of small boxes and 32 bundles of medium boxes. At the end of the week, they buy 125 bundles of medium boxes. The total number of boxes at the end of the week can be modeled by the expression $72s + 50m - 25s - 32m + 125m$, where s represents the number of boxes in a bundle of small boxes and m represents the number of boxes in a bundle of medium boxes.

a. Simplify the expression by combining like terms.

b. How many boxes are in stock at the end of the week if there are 40 boxes in a small bundle of boxes and 30 boxes in a medium bundle of boxes?

78. During a sale, all newly released video games are priced the same and all Blu-ray discs are priced the same. During the first day of the sale, Mitchell buys 4 video games and 6 Blu-ray discs. The next day he buys 2 more video games and returns 2 of the Blu-ray discs. The amount of money Mitchell spends can be modeled by $4v + 6d + 2v - 2d$, where v represents the cost of each video game and d represents the cost of each Blu-ray disc.

a. Simplify the expression by combining like terms.

b. How much did Mitchell spend if each video game costs \$35 and each Blu-ray disc costs \$19?

79. The ocean water temperature at Conimicut Lighthouse in Rhode Island was 39.9 °F on February 2, 2020. Determine the temperature in degree Celsius knowing that $C = \frac{5}{9}(F - 32)$. Round your answer to the nearest hundredth.

Writing & Thinking

80. Define constant and variable. Explain why those particular words are used.

81. Discuss like and unlike terms and give an example of each.

82. The text recommends simplifying an expression (combining like terms) before evaluating. Do you think this is necessary?
Evaluate the expression $4x^2 - 5(x + 2) + 3x + 10 + 2x$ for $x = 3$:

a. by substituting and then evaluating.

b. by first simplifying and then evaluating.

Which method would you recommend? Why?

83. Explain the difference between -13^2 and $(-13)^2$.

Margin Exercise Answers

1. a. $7x$ **b.** $5+n$ **c.** $4(y+2)$ **d.** $2x+3$ **e.** $9x-4$ **f.** $\frac{3}{n}$ **2. a.** $12f$ **b.** $25+0.33x$ **3. a.** The product of ten and a number **b.** Four times a number increased by seven **c.** Seven times the difference between a number and five **4.** Answers will vary. For example, a number plus three times that number is equal to nineteen. What is the number?

1.7 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. A phrase is considered _____ if its meaning is not clear or if it has two or more possible interpretations.
2. Phrases such as "a number" or "the number" imply the use of a/an _____.
3. Key words such as "decreased by" and "minus" indicate the operation of _____.
4. The key words "cube of" and "square of" mean _____ are involved.
5. "Twice" and "three times" indicate the operation of _____.
6. "Divide" and "quotient" specify that _____ should be used.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The order in which the values are given is particularly important when working with subtraction and division problems.
8. "More than" and "increased by" are key phrases specifying the operation of subtraction.
9. Division is indicated by the phrase "five less than a number."
10. Key phrases for parentheses can be used to limit ambiguity in English phrases.

Practice

Write the algebraic expressions described by the English phrases. Choose your own variable. See Example 1.

1. six added to a number
2. seven more than a number
3. four less than a number
4. a number decreased by thirteen
5. the quotient of twice a number and ten

6. the difference between a number and three, all divided by seven

7. four subtracted from the product of six and a number

8. eight minus twice a number

9. the sum of four times a number and twice the same number

10. the sum of nine times a number and the same number

11. fifteen decreased by twice a number

12. twenty decreased by the product of four and a number

13. three times a number, less five times the same number

14. seven times a number, decreased by twice the number

15. nine times the sum of a number and two

16. three times the difference between a number and eight

17. thirteen less than the product of four and the sum of a number and one

18. four more than the product of eight and the difference between a number and six

19. eight more than the product of three and the sum of a number and six

20. six less than twice the difference between a number and seven

21. four less than the product of three and the difference between seven and a number

22. nine more than twice the sum of seventeen and a number

23. eighteen less than the quotient of a number and two

24. seven increased by the quotient of a number and five

Translate each pair of English phrases into algebraic expressions.

25. **a.** six less than a number

b. six less a number

26. **a.** twenty less than a number

b. twenty less a number

27. **a.** five less than three times a number

b. five less three times a number

28. **a.** six less than four times a number

b. six less four times a number

Write the algebraic expression described by the English phrases using the given variables. See Example 2.

29. the number of hours in d days

30. the cost of x graphing calculators if one calculator costs \$115

31. the cost of x gallons of gasoline if the cost of one gallon is \$3.15

32. the number of seconds in m minutes

33. the number of days in y years (Assume 365 days in a year.)

34. the cost of x pounds of candy priced at \$4.95 a pound

35. the number of days in t weeks and three days

36. the number of minutes in h hours and twenty minutes

37. the points scored by a football team on t touchdowns and one field goal (a touchdown is 7 points and a field goal is 3 points)

38. the amount of vacation days an employee has after w weeks if she gets 0.2 vacation days for every week she works

39. the cost of renting a car for one day and driving m miles if the rate is \$20 per day plus 15 cents per mile

40. the cost of purchasing a fishing rod and reel if the rod costs x dollars and the reel costs \$8 more than twice the cost of the rod

41. the perimeter of a rectangle if the width is w centimeters and the length is three centimeters less than twice the width

42. the area of a square with side length of c centimeters

Translate each algebraic expression into an equivalent English phrase. (There may be more than one correct translation.) See Examples 3 and 4.

43. $4x$

44. $-9x$

45. $x+5$

46. $x-12$

47. $4x-7$

48. $3x+5$

49. $7(x+1)$

50. $3(x+2)$

51. $-2(x-8)$

52. $10(x+4)$

53. $5(2x+3)$

54. $3(4x-5)$

55. $\dfrac{6}{x-1}$

56. $\dfrac{9}{x+3}$

57. $6x+x-1$

58. $5x-x+2$

59. $8+2(x-1)$

60. $5-3(x+1)$

Translate each pair of expressions into equivalent English phrases. (There may be more than one correct translation.)

61. $3x+7;\quad 3(x+7)$

62. $4x-1;\quad 4(x-1)$

63. $7x-3;\quad 7(x-3)$

64. $5(x+6);\quad 5x+6$

Writing & Thinking

65. Explain why translating addition and multiplication problems from English into algebra may be easier than changing subtraction or division problems. (Consider the properties previously studied.)

66. Explain the difference between $5(n+3)$ and $5n+3$ when converting from algebra to English.

67. Make up your own word problem that might use the given equation in its solution. Be creative! Translate the variable into something like "a strange number," or "the age of a dog," or "an amount invested."

a. $2x+3=-4$

b. $3x-2=-5$

c. $n+(n+1)=25$

d. $n+(n+2)=135$

e. $2x+3x=x$

Margin Exercise Answers

1. a. False **b.** True **c.** True **d.** True **2.** $x = 17$ **3.** $x = -12$ **4.** $x = 3.7$ **5.** $x = \frac{9}{8}$ **6.** $z = -5$ **7.** $z = 4.2$ **8.** $x = 11$ **9.** $x = 4$ **10.** $x = 15$ **11.** $x = \frac{6}{5}$ **12.** The original price of the wool coat was $100.59.

2.1 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. A/An _____ is a statement that two algebraic expressions are equal.
2. If an equation contains a variable, any number that gives a true statement when substituted for the variable is a/an _____ of the equation.
3. The _____ principle of _____ involves adding the same algebraic expression to both sides of an equation.
4. The objective of solving linear equations is to get the variable (with a coefficient of +1) on one side of the equation and any _____ on the other side.
5. Multiplying by the reciprocal of the coefficient of the variable is the same as _____ by the coefficient.
6. If both sides of an equation are multiplied by the same nonzero constant, the _____ principle of equality can be used.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. When an algebraic expression is added to both sides of an equation, the new equation has the same solutions as the original equation.
8. The process of finding the solution set to an equation is called simplifying the equation.
9. A linear equation in x is also called a first-degree equation in x.
10. Equations with the same solutions are said to be equivalent equations.

Practice

Determine whether or not the given number is a solution to the given equation by substituting and then evaluating. See Example 1.

1. $x + 4 = 2$ given that $x = -2$
2. $z + (-12) = 6$ given that $z = 18$
3. $x - 3 = -7$ given that $x = 4$

4. $x-2=-3$ given that $x=1$

5. $-10+x=-14$ given that $x=-4$

6. $-9-x=-14$ given that $x=5$

7. $-26+|x|=-8$ given that $x=-18$

8. $42+|z|=-30$ given that $z=-72$

9. $|x|-|-3|=25$ given that $x=-28$

10. $|-2|+|x|=13$ given that $x=-11$

Solve each equation.

11. $x-6=1$

12. $x-10=9$

13. $y+7=3$

14. $y+12=5$

15. $x+15=-4$

16. $x+17=-10$

17. $22=n-15$

18. $36=n-20$

19. $6=z+12$

20. $18=z+1$

21. $x-20=-15$

22. $x-10=-11$

23. $y+3.4=-2.5$

24. $y+1.6=-3.7$

25. $x+3.6=2.4$

26. $x+2.7=3.8$

27. $x+\frac{1}{20}=\frac{3}{5}$

28. $n-\frac{2}{7}=\frac{3}{14}$

29. $5x=45$

30. $9x=108$

31. $32=4y$

32. $51=17y$

33. $\frac{3x}{4}=15$

34. $\frac{5x}{7}=65$

35. $\frac{y}{5}=2$

36. $\frac{x}{3}=-4$

37. $-1=\frac{x}{8}$

38. $0=\frac{x}{15}$

39. $7x-8x=13-25$

40. $10n-11n=20-14$

41. $3n-2n+6=14$

42. $7n-6n+13=22$

43. $1.7y+1.3y=6.3$

44. $2.5y+7.5y=4.2$

45. $\frac{3}{4}x=\frac{5}{3}$

46. $\frac{5}{6}x=\frac{5}{3}$

47. $7.5x=-99.75$

48. $-14=0.7x$

49. $1.5y-0.5y+6.7=-5.3$

50. $2.6y-1.6y-5.1=-2.9$

51. $10x-9x-\frac{1}{2}=-\frac{9}{10}$

52. $6x - 5x + \frac{3}{4} = -\frac{1}{12}$

53. $1.4x - 0.4x + 2.7 = -1.3$

54. $3.5y - 2.5y - 6.3 = -1.0 - 2.5$

55. $\frac{7x}{4} - \frac{3x}{4} + \frac{7}{8} = \frac{3}{2}$

56. $\frac{5n}{2} - \frac{3n}{2} + \frac{4}{5} = \frac{7}{5} - \frac{1}{10}$

57. $6.2 = -3.5 + 7n - 6n$

58. $-7.2 = 1.3n - 0.3n - 1.0$

59. $1.7x = -5.1 - 1.7$

60. $3.2x = 2.8 - 9.2$

Applications

Solve.

61. The Japanese writing system consists of three sets of characters, two with 81 characters (which all Japanese students must know), and a third, Kanji, with over 50,000 characters (of which only some are used in everyday writing). If a Japanese student knows 2107 total characters, solve the equation $x + 2(81) = 2107$ to determine the number of Kanji characters the student knows.

62. A nurse must give a patient 800 milliliters of intravenous solution over 4 hours. This can be represented by the equation $4x = 800$, where x represents the amount of solution the patient receives per hour in milliliters.

a. Why was multiplication chosen in the equation?

b. Solve the equation to determine the value of x.

c. What does the answer to Part **b.** mean? Write a complete sentence.

63. John is making a garden in his backyard. He buys enough topsoil to cover 300 square feet. John wants the garden to go along the side of his garage, which is 24 feet in length. To determine how wide the garden needs to be, John uses the equation $24x = 300$, where x is the width of the garden in feet.

a. Why was multiplication chosen in this equation?

b. Solve the equation to determine the value of x.

c. What does the answer to Part **b.** mean? Write a complete sentence.

64. A university enrolls both undergraduate and graduate students in all programs of study. There are a total of 28,000 students enrolled. Of this total, 17,500 students are undergraduates. Solve the equation $17{,}500 + x = 28{,}000$ to determine how many graduate students are enrolled in the university.

65. The diameter of the Milky Way is approximately 23,585 times the distance from the sun to the nearest star, Proxima Centauri. Considering that the Milky Way is roughly 100,000 light years across, solve the equation, $23{,}585x = 100{,}000$ to find the number of light years from the sun to this star. (Round your answer to the nearest hundredth.)

66. A group of students at Homestate University decide to start a math club. They create a Facebook page for their club, and their goal is get 5000 "likes" for their page. Three months after they launch their club and Facebook page, they have received a total of 3500 likes. Solve the equation $3500 + x = 5000$ to determine how many more likes they need to get to reach their goal.

67. An author is determined to have his first novel published by the publisher of George Orwell's 1984, his favorite book. However, his contract with the publisher requires his novel to be at least 75,000 words, and he has only written 63,500 words. Solve the equation, $63{,}500 + x = 75{,}000$ to determine how many more words he must write.

68. The best pizza parlor in town slices their large pizzas so that each pizza contains 8 slices. Joe's fraternity hosts a pizza party for its members and guests, and the fraternity orders large pizzas from the best pizza parlor in town. By the end of the party, 400 slices of pizza had been eaten and all of the pizza boxes were empty. Solve the equation $8x = 400$ to determine how many pizzas were ordered for the party.

69. During rush week at Homestate University, the fraternities and sororities pledge a combined total of 450 freshmen. These 450 freshmen represent $\frac{1}{5}$ of the school's total enrollment. Solve the equation $\frac{1}{5}x = 450$ to determine the total number of students enrolled at Homestate University.

70. The inventory manager's computer crashed and he did not have a backup of his data. The company manager is requesting an inventory report for the week for a specific item. The inventory manager knows that there are currently 1472 of that item in stock. During the week, a shipment arrived with 1500 of the item. The company also shipped out 975 of the item during the week. This situation can be represented by $x + 1500 - 975 = 1472$, where x is the number of items in the inventory at the beginning of the week.

a. Why were the operations of addition and subtraction chosen in this equation?

b. Solve the equation to determine the value of x.

c. What does the answer to Part **b.** mean? Write a complete sentence.

71. Clara has $4200 saved to use as a down payment on the new car she is buying that costs $15,750. She will have to get a loan to pay for the rest of the cost. This situation can be modeled by $4200 + x = 15{,}750$, where x is the amount of the loan in dollars.

a. Why was the operation of addition chosen in this equation?

b. Solve the equation to determine the value of x.

c. What does the answer to Part **b.** mean? Write a complete sentence.

72. A sculptor has decided to begin a project to make scale models of famous landmarks out of stone. His first model will be of one of the moai, giant human figures carved from stone on Easter Island. If his model is to be $\frac{1}{12}$ scale, and the original moai weighs 75 tons, solve the equation $12x = 75$ to determine how many tons his completed sculpture will weigh.

Use a calculator to help solve the following equations.

73. $y + 32.861 = -17.892$

74. $x - 41.625 = 59.354$

75. $17.61x - 16.61x + 27.059 = 9.845$

76. $14.83y - 8.65 - 13.83y = 17.437 + 1.0$

77. $2.637x = 648.702$

78. $-0.3057y = 316.7052$

79. $-x = 145.6 + 17.89 - 10.32$

80. $-y = 143.5 + 178.462 - 200$

Writing & Thinking

81. **a.** Is the expression $6 + 3 = 9$ an equation? Explain.

b. Is 4 a solution to the equation $5 + x = 10$? Explain.

2.2 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The first step in solving linear equations that simplify to the form $ax + b = c$ is to combine _____ terms on both sides of the equation.
2. When solving a linear equation that has been simplified to the form $ax + b = c$, use the _____ principle of equality and add the _____ of the constant b to both sides of the equation.
3. Once you have a variable term on one side of the equation and a constant term on the other, use the _____ principle of equality and multiply both sides of the equation by the reciprocal of the coefficient of the variable.
4. When you multiply both sides of the equation by the reciprocal of the coefficient of the variable, the coefficient of the variable will become _____.
5. Check your answer by _____ it in for the variable in the original equation.
6. Every linear equation is a/an _____ equation.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The first step in solving $2x + 3 = 9$ is to add 3 to both sides.
8. To solve an equation that has been simplified to $4x = 12$, you need to multiply both sides by $\frac{1}{4}$, or divide both sides by 4.
9. If an equation has no solution, it is called an identity.
10. The most general form of a linear equation is $ax + b = cx + d$.

Practice

Solve each equation.

1. $3x + 11 = 2$
2. $4y - 8 = -12$
3. $6x + 10 = 22$
4. $9x - 5 = 13$
5. $1 - 3y = 4$
6. $5 - 2x = 9$
7. $-5x + 2.9 = 3.5$
8. $3x + 2.7 = -2.7$
9. $10 + 3x - 4 = 18$
10. $14 = 9x + 5 + 8$
11. $5y - 3y + 2 = 2$
12. $6y + 8y - 7 = -7$

13. $3y+9y-13=11$

14. $-20=5y+y+16$

15. $4n-10n+35=1-2$

16. $-5n-3n+2=34$

17. $2n+12+n=0$

18. $5.4x-0.2x=0$

19. $0=5.1x+0.3x$

20. $\frac{1}{2}x+7=\frac{7}{2}$

21. $\frac{3}{5}x+4=\frac{9}{5}$

22. $\frac{3}{2}=\frac{1}{3}x+\frac{11}{3}$

23. $\frac{11}{8}=\frac{1}{5}x+\frac{4}{5}$

24. $\frac{7}{2}-5-\frac{5}{2}x=9$

25. $\frac{8}{3}+2-\frac{7}{3}x=6$

26. $\frac{5}{8}x-\frac{1}{4}x+\frac{1}{2}=\frac{3}{10}$

27. $\frac{1}{2}x+\frac{3}{4}x-\frac{5}{3}=\frac{5}{6}$

28. $\frac{7}{8}=\frac{3}{4}x-\frac{5}{8}$

29. $\frac{1}{10}=\frac{4}{5}x+\frac{3}{10}$

30. $\frac{y}{7}+\frac{y}{28}+\frac{1}{2}=\frac{3}{4}$

31. $\frac{5y}{6}-\frac{7y}{8}-\frac{1}{12}=\frac{1}{3}$

32. $x+1.2x+6.9=-3.0$

33. $33=y+3-0.4y$

34. $2.5x+0.5x-3.5=2.5$

35. $4.7-0.5x-0.3x=-0.1$

36. $6.4+1.2x+0.3x=0.4$

37. $3x+2=x-8$

38. $5x+1=2x-5$

39. $4n-3=n+6$

40. $2y+5=8y+10$

41. $3x+11=8x-4$

42. $14n=3n$

43. $1.6x=0.8x$

44. $6y-2.1=y-2.1$

45. $2(z+1)=3z+3$

46. $6x-3=3(x+2)$

47. $16y+23y-3=16y-2y+2$

48. $5x-2x+4=3x+x-1$

49. $6.5+1.2x=0.5-0.3x$

50. $x-0.1x+0.8=0.2x+0.1$

51. $\frac{2}{3}x+1=\frac{1}{3}x-6$

52. $\frac{4}{5}n+2=\frac{2}{5}n-4$

53. $\frac{y}{5}+\frac{3}{4}=\frac{y}{2}+\frac{3}{4}$

54. $\frac{5n}{6}+\frac{1}{9}=\frac{3n}{2}+\frac{1}{9}$

55. $\frac{3}{8}\left(y-\frac{1}{2}\right)=\frac{1}{8}\left(y+\frac{1}{2}\right)$

56. $\frac{1}{2}\left(\frac{x}{2}+1\right)=\frac{1}{3}\left(\frac{x}{2}-1\right)$

57. $x+\frac{2}{3}x-2x=\frac{x}{6}-\frac{1}{8}$

58. $3x+\frac{1}{2}x-\frac{2}{5}x=\frac{x}{10}+\frac{7}{20}$

59. $3(1+9x)=6(2-4x)$

60. $4(5-x)=8(3x+10)$

61. $5-3(2x+1)=4(x-5)+6$

62. $-2(y+5)-4=6(y-2)+2$

63. $0.2(x+3)=0.1(x-5)$

64. $0.4(x+3)=0.3(x-6)$

65. $\frac{1}{2}(4-8x)=\frac{1}{3}(4x+7)-3$

66. $3+\frac{1}{4}(x-4)=\frac{2}{5}(2+3x)$

67. $0.12n+0.25n-5.895=4.3n$

68. $0.15n+32n-21.0005=10.5n$

69. $0.7(x+14.1)=0.3(x+32.9)$

70. $0.8(x-6.21)=0.2(x-24.84)$

Determine whether each equation is a conditional equation, an identity, or a contradiction. See Examples 11 through 13.

71. $2(3x-1)+5=3$

72. $-2x+13=-2(x-7)$

73. $5x+13=-2(x-7)+3$

74. $7(x-1)=-3(3-x)+4x$

75. $3(x-2)+4x=6(x-1)+x$

76. $5(x+1)=3(x+1)+2(x+1)$

77. $8x-20+x=-3(5-2x)+3(x-4)$

78. $5.2x+3.4x=0.2(x-0.42)$

Applications

Solve.

79. The tickets for a concert featuring the new hit band, Flying Sailor, sold out in 2.5 hours. If there were 35,000 tickets sold, solve the equation $35{,}000-2.5x=0$ to find the number of tickets sold per hour.

80. Salim is a student in a course on the modern British novel. He is given an assignment to read a 350 page novel in 7 days. He reads the first 38 pages of the novel on the day he receives his assignment and decides to finish the novel by reading the same amount of pages of each day until the assignment is due. Solve the equation $6x + 38 = 350$ to determine how many pages Salim should read each day.

81. All snacks (candy, popcorn, and soda) cost \$3.50 each at the local movie theater. Admission tickets cost \$7.50 each. After a long week, Carlos treats himself to a night at the movies. His movie night budget is \$25 and he spends all his movie money. Solve the equation $\$3.50x + \$7.50 = \$25.00$ to determine how many snacks Carlos can buy.

82. The Political Science Club at Homestate University is planning to host an election night party for members and guests. The club plans to serve cookies and estimates it will need a total of 1500 cookies in 6 varieties for the party. The club orders 300 chocolate chip cookies and an equal number of cookies in each of the remaining 5 varieties. Solve the equation $5x + 300 = 1500$ to determine how many cookies of each remaining variety will be ordered.

83. All courses in the Homestate University graduate school are worth 3 credits. To earn a master's degree, a student must earn a total of 36 credits. The student's thesis work counts as 6 credits. Solve the equation $3x + 6 = 36$ to determine how many courses a student must take to earn a master's degree.

84. Jeff, who lives in England, is reading a letter from his pen pal in the United States. His pen pal says that the temperature was 97.7° Fahrenheit that day, making it too hot to play soccer outside. Jeff doesn't know how hot this is, because he is used to temperatures in Celsius. Help Jeff solve the equation, $1.8C + 32 = 97.7$ to determine the temperature in degrees Celsius.

85. The tallest man-made structure in the world is the Burj Khalifa in Dubai, which stands at 2717 feet tall. The tallest tree in the world is a Mendocino tree in California. If 7 of these trees were stacked on top of each other, they would still be 59.1 feet shorter than the Burj Khalifa. Solve the equation, $7x + 59.1 = 2717$ to determine the height of the tree.

86. Starbucks sells cake pops individually and in packages of 4. At the beginning of the day, Starbucks had 114 cake pops in stock. They sold 34 individual cake pops and several packages of cake pops. At the end of the day, there were 8 cake pops left. This situation can be modeled by the equation $114 - 34 - 4x = 8$, where x is the number of packages of cake pops sold.

a. Explain what each term in the equation $114 - 34 - 4x = 8$ represents in the situation.

b. Solve the equation to determine the value of x.

c. What does the answer to Part **b.** mean? Write a complete sentence.

87. The lowest temperature of the night was reported to be 24 °F. The weather report mentioned that the temperature has steadily risen 1.5 degrees per hour since the lowest temperature of the day and it is currently 30 °F. This situation can be modeled by the equation $24 + 1.5x = 30$, where x is the time in hours since the lowest temperature was recorded.

a. Explain what each term in the equation $24 + 1.5x = 30$ represents in the situation.

b. Solve the equation to determine the value of x.

c. What does the answer to Part **b.** mean? Write a complete sentence.

88. Caitlyn and Steve are planning their wedding reception and must decide between two catering halls. The first site, A Wedding Space, rents for $800 for one day and charges $50 per person for dinner. The second venue, A Wedding Place, costs $1000 to rent for one day and charges $40 per person for the same dinner. Solve the equation $800 + 50x = 1000 + 40x$ to determine how many guests they can invite so that the cost they pay will be the same at both wedding catering halls.

89. The value of a new car depreciates at a rate of about \$250 per month. Suppose a car originally costs \$30,000. The car was bought with a \$1000 down payment and a loan with 0% financing for 60 months with payments of \$200 a month. Solve the equation $30{,}000 - 250t = 29{,}000 - 200t$ to determine how many months it will take for the value of the vehicle to equal the amount owed on the loan?

90. Heidrick has won a chance to "Eat the Elite" at the Zombie Dash 7-mile race. He will receive a 2.2-mile head start. The elite runners run at a pace of 12.6 miles per hour and Heidrick runs at a pace of 6 miles per hour. Solve the equation $12.6t = 6t + 2.25$ to determine how many hours it will take the elite runners catch up to Heidrick? Enter your answer in hours rounded to the nearest hundredth.

91. A company has two packaging options for shipping quantities of a certain inventory item. Option A uses 20 boxes and there are 5 items unpacked. Option B requires more filler and uses 23 boxes, where each box holds 2 less items than Option A and there are only 3 items unpacked. This situation can be represented by $20x + 5 = 23(x - 2) + 3$, where x is the number of items that can fit in the box used for Option A.

a. What does $20x + 5$ represent in the equation?

b. What does $x - 2$ represent?

c. Solve the equation for x.

d. Check the solution.

e. What does the answer from part **c.** mean? Write a complete sentence.

92. Two advertisement flyers have the same area. The first flyer has a length of 12 inches and a width of x inches. The second flyer has a length of 4 inches and a width that is 10 inches more than x. This situation can be represented by $12x = 4(10 + x)$, where x is the width of the first flyer.

a. What does $12x$ represent in the equation?

b. What does $10 + x$ represent?

c. Solve the equation for x.

d. Check the solution.

e. What does the answer from part **c.** mean? Write a complete sentence.

93. The manager of a café wants to list a price for the weekly featured combo that includes tax. He wants to sell a medium house-blend coffee with a pastry for a total of \$5.45. He doesn't know which pastry to sell with the coffee to avoid losing money on the combo. The medium coffee costs \$2.75 and the tax is 9%. He uses the equation $1.09(2.75 + x) = 5.45$ to determine the price of the pastry, which is represented by the variable x.

a. What does the sum $2.75 + x$ represent?

b. Solve the equation for x.

c. Which of the following pastries would you choose to be a part of the combo? Explain why you made your choice.

cherry pie for \$2.50, coffee cake for \$2.25, bagel for \$2.00

94. A farmer is putting a shed on his property. He has two designs. One uses wood and would cost \$2 per square foot plus an extra \$8400 in materials. The other design is metal and would cost \$4 per square foot plus an additional \$8800. Both sheds are the same size, and the wood shed costs $\frac{3}{4}$ what the metal shed costs. Solve the equation $2x + 8400 = \frac{3}{4}(4x + 8800)$ to determine how many square feet the shed will be.

95. An ice cream shop is having a special "Ice Cream Sunday" event in which they are giving away giant mixed sundaes of 3 scoops of vanilla ice cream and 2 scoops of chocolate. If they have 24 gallons of chocolate and 36 gallons of vanilla to start with, solve the equation, $36 - \frac{1}{20}(3x) = 24 - \frac{1}{20}(2x)$ to determine how many sundaes they will have made when they run out of ice cream. (For this problem, we assume a gallon equals 20 scoops.)

Writing & Thinking

96. Answer each question.

a. Simplify the expression $3(x+5)+2(x-7)$.

b. Solve the equation $3(x+5)+2(x-7)=31$.

c. How are the methods you used to answer questions **a.** and **b.** similar? How are they different?

97. Write an equation to represent each situation, using x to represent Ryan's current age. Determine whether each equation is a conditional equation, an identity, or a contradiction, and explain why that makes sense for the situation represented.

a. In 6 years, Ryan will be 20 years old.

b. In 6 years, Ryan will be 8 years older than he is now.

c. In 6 years, Ryan will be 3 years older than he will be 3 years from now.

Completion Example Answers

9.

$$3x - y = 15$$

$$3x - y - 3x = 15 - 3x \quad \text{Subtract } 3x \text{ from both sides.}$$

$$-y = 15 - 3x$$

$$-1(-y) = -1(15 - 3x) \quad \text{Multiply both sides by } -1 \text{ (or divide both sides by } -1\text{).}$$

$$y = -15 + 3x \quad \text{Simplify using the distributive property.}$$

or $y = 3x - 15$

Margin Exercise Answers

1. \$2020 **2.** $F = 122°\text{F}$ **3.** 1,012,500 lb **4.** 30° **5.** $I = \frac{P}{V}$ **6.** $t = \frac{I}{Pr}$ **7.** $x = \frac{5}{2}y - 3$

8. a. $y = \frac{400 - 25z}{16}$ or $y = -\frac{25}{16}z + 25$ **b.** $z = \frac{400 - 16y}{25}$ or $z = -\frac{16}{25}y + 16$ **9.** $x = 4 - 2y - 3z$

2.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. Formulas are general rules or principles stated _____.

2. The _____ _____ earned by investing money is equal to the product of the principle times the rate of interest times the time in one year or part of a year.

3. The distance traveled equals the product of the rate of speed and the _____.

4. The _____ of a rectangle is equal to twice the length plus twice the width.

5. If you know values for all but one variable in a formula, you can _____ those values and find the value of the unknown variable by solving the equation.

6. If you want to use a formula in another form, treat the variables just as you would _____ in solving linear equations.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. When using formulas, typically it does not matter if capital or lower case letters are used: $A = a$, $C = c$, etc.

8. If the perimeter and length are known, $P = 2l + 2w$ can be used to find the width of a rectangle.

9. Rate of interest is stated as an annual rate in percent form.

Applications

Solve. See Exmple 1. (Note: Use 365 days in a year and 30 days in each month.)

Simple Interest

1. You want to borrow \$4000 at 12% for only 90 days. How much interest would you pay?

2. For how many days must you leave \$1000 in a savings account at 5.5% to earn \$11.00 in interest?

3. What principal would you need to invest to earn \$450 in simple interest in 6 months if the interest rate was 9%?

4. After 30 days, Gustav received \$25 in simple interest on his savings account of \$12,000. What was the interest rate?

5. A savings account of \$3500 is left for 9 months and draws simple interest at a rate of 7%.
   a. How much interest is earned?
   b. What is the balance in the account at the end of the 9 months?

6. Tim just deposited \$2562.50 to pay off a 3 month loan of \$2500.
   c. How much of what he deposited was interest on the loan?
   d. What rate of interest was he charged?

Solve using substitution. See Examples 1 through 4.

Velocity

If an object is shot upward with an initial velocity v_0 in feet per second, the velocity v in feet per second is given by the formula $v = v_0 - 32t$, where t is time in seconds. (v_0 is read "v sub zero." The $_0$ is called a subscript.)

7. An object projected upward with an initial velocity of 106 feet per second has a velocity of 42 feet per second. How many seconds have passed?

8. Find the initial velocity of an object if the velocity after 4 seconds is 48 feet per second.

Medicine

In nursing, one procedure for determining the dosage for a child is

$$\text{child's dosage} = \frac{\text{age of child in years}}{\text{age of child} + 12} \cdot \text{adult dosage.}$$

9. If the adult dosage of a drug is 20 milliliters, how much should a 3-year-old child receive?

10. If the adult dosage of a drug is 340 milligrams, how much should a 5-year-old child receive?

Investments

The total amount of money in an account with P dollars invested in it is given by the formula $A = P + Prt$, where r is the rate expressed as a decimal and t is time (one year or part of a year).

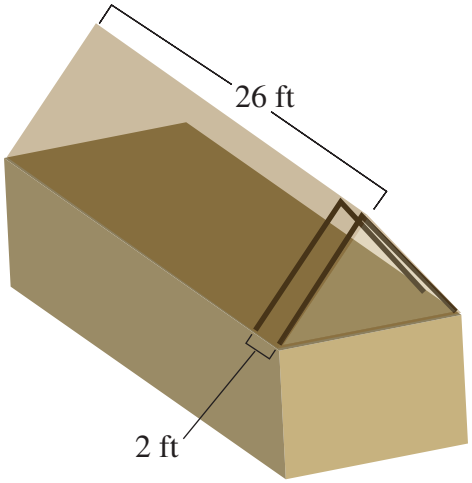
11. If \$1000 is invested at 6% interest, find the total amount in the account after 6 months.

12. How long will it take an investment of \$600, at an annual rate of 5%, to be worth \$615?

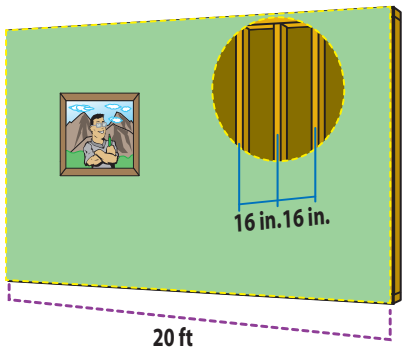
Construction

The number N of rafters in a roof or studs in a wall can be found by the formula $N = \frac{L}{d} + 1$, where L is the length of the roof or wall and d is the center-to-center distance from one rafter or stud to the next. Note that L and d must be in the same units.

13. How many rafters will be needed to build a roof 26 ft long if they are placed 2 ft apart center-to-center?



14. A wall has studs placed 16 in. apart center-to-center. If the wall is 20 ft long, how many studs are in the wall?



15. How long is a wall if it requires 22 studs placed 16 in. apart center-to-center?

16. What should the center-to-center distance be if you are building a 33 ft long roof using 12 rafters?

Cost

The total cost C of producing x items can be found by the formula $C = ax + k$, where a is the cost per item and k is the fixed costs (rent, utilities, and so on).

17. Find the total cost of producing 30 items if each costs \$15 and the fixed costs are \$580.

18. The total cost to produce 80 dolls is $1097.50. If each doll costs $9.50 to produce, find the fixed costs.

19. It costs a company $3.60 to produce a calculator. Last week the total costs were $1308. If the fixed costs are $480 weekly, how many calculators were produced last week?

20. Each week an electronics company builds 60 smartphones for a total cost of $5340. If the fixed costs for a week are $750, what is the cost to produce each smartphone?

21. In 1981, the U.S. demand for wheat was given by $q = 3550 - 266p$, where q is measured in millions of bushels and p is the price in dollar per bushel. Determine the price per bushel when the demand hit 2630 million bushels.

Profit

The profit P is given by the formula $P = R - C$, where R is the revenue and C is the cost.

22. Find the revenue (income) of a company that shows a profit of $3.2 million and costs of $1.8 million.

23. Find the revenue of a company that shows a profit of $3.2 million and costs of $5.7 million.

Depreciation

Many items decrease in value as time passes. This decrease in value is called depreciation. One type of depreciation is called linear depreciation. The value V of an item after t years is given by $V = C - Crt$, where C is the original cost and r is the rate of depreciation expressed as a decimal.

24. If you buy a car for $6000 and it depreciates linearly at a rate of 10% per year, what will be its value after 6 years?

25. A contractor buys a 4-year-old piece of heavy equipment valued at $20,000. If the original cost of this equipment was $25,000, find the rate of depreciation.

Distance, Rate, Time

The distance traveled d is given by the formula $d = rt$, where r is the rate of speed and t is the time it takes.

26. How long will a truck driver take to travel 350 miles if he averages 50 mph?

27. What is the average rate of speed of a biker who bikes 21.92 miles in 68.5 minutes?

28. What is Jonathan's average rate of speed if he hikes 10.4 miles in 6.4 hours?

29. How long will it take a train traveling at 40 mph to go 140 miles?

Solve each formula for the indicated variable. See Examples 5 through 9.

30. $P = a + b + c$; solve for b.

31. $P = 3s$; solve for s.

32. $F = ma$; solve for m.

33. $C = \pi d$; solve for d.

34. $A = lw$; solve for w.

35. $P = R - C$; solve for C.

36. $R = np$; solve for n.

37. $v = k + gt$; solve for k.

38. $I = A - P$; solve for P.

39. $L = 2\pi rh$; solve for h.

40. $A = \dfrac{m+n}{2}$; solve for m.

41. $P = a + 2b$; solve for a.

42. $I = Prt$; solve for t.

43. $R = \dfrac{E}{I}$; solve for E.

44. $P = a + 2b$; solve for b.

45. $c^2 = a^2 + b^2$; solve for b^2.

46. $\alpha + \beta + \gamma = 180°$; solve for β.

47. $y = mx + b$; solve for x.

48. $V = lwh$; solve for h.

49. $v = -gt + v_0$; solve for t.

50. $A = \dfrac{1}{2}bh$; solve for b.

51. $R = \dfrac{E}{I}$; solve for I.

52. $V = \pi r^2 h$; solve for h.

53. $A = \dfrac{R}{2L}$; solve for L.

54. $K = \dfrac{mv^2}{2g}$; solve for g.

55. $x + 4y = 4$; solve for y.

56. $2x + 3y = 6$; solve for y.

57. $3x - y = 14$; solve for y.

58. $5x + 2y = 11$; solve for x.

59. $-2x + 2y = 5$; solve for x.

60. $A = \dfrac{1}{2}h(b+c)$; solve for b.

61. $A = P(1+rt)$; solve for r.

62. $R = \dfrac{3(x-12)}{8}$; solve for x.

63. $-2x - 5 = -3(x+y)$; solve for x.

64. $3y - 2 = x + 4y + 10$; solve for y.

65. $V = \dfrac{1}{3}\pi r^2 h$; solve for h.

Determine a formula for each of the following situations.

66. Each ticket for a concert costs \$$t$ per person and parking costs \$9.00. What is the total cost per car C if there are n people in a car?

67. A-to-Z Truck Rentals charges \$25 per day plus \$0.75 per mile for a 10-foot rental truck. What would you pay per day for renting the truck from A-to-Z if you were to drive the truck x miles in one day?

68. Top-of-the-Line computer company knows that the cost (labor and materials) of producing a computer is \$325 per computer per week and the fixed overhead costs (lighting, rent, etc.) are \$5400 per week. What are the company's weekly costs of producing n computers per week?

69. If the Top-of-the-Line computer company (see Exercise 67) sells its computers for $683 each, what is its profit per week if it sells the same number n that it produces? (Remember that profit is equal to revenue minus costs, or $P = R - C$.)

Solve.

70. Samantha uses a credit promotion at a home improvement store where she doesn't have to pay any interest on her purchase as long as she pays off the entire balance within 6 months. She purchases $8000 in merchandise. If she fails to pay off the balance within 6 months, then she will be charged $600 in interest. Samantha lost the paper work and wants to determine the interest rate on her purchase.

a. Which formula from Table 1 fits this situation?

b. Match the variables in the formula from Part **a.** to the information provided.

c. The formula from Part **a.** needs to be solved for which variable?

d. What is the interest rate on her purchase? (Remember to convert to a percent.)

71. In a physics lab, a ball is rolled down an incline that has a machine at the bottom which calculates the force of impact. The ball has a mass of 1.5 kilograms. After several trials, the average force of impact is calculated to be 12.75 kg · m/s^2. The researchers need to determine the average acceleration of the ball at the moment it struck the machine.

a. Which formula from Table 1 fits this situation?

b. Match the variables in the formula from Part **a.** to the information provided.

c. The formula from Part **a.** needs to be solved for which variable?

d. What was the average acceleration of the ball in m/s^2?

72. Charles is experimenting with a new sail design for his sailboat and needs to keep the total area of the triangular sail to 150 square feet. The base of the sail must be exactly 3 times the height of the sail.

a. What geometric formula for area should be used?

b. Write an expression for the base of the formula using the variable h for height.

c. Substitute the expression from Part **b.** into the area formula for the base.

d. Solve this formula for the height squared.

e. What would you have to do to both sides of the equation in Part **d.** to solve the formula for the height?

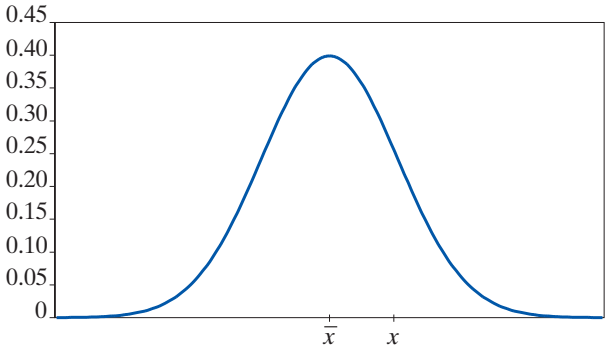
f. Substitute 150 for the area of the sail in the formula from Part **d.** and solve for the height of the sail.

g. What is the length of the base of the sail?

Writing & Thinking

73. The formula $z = \dfrac{x - \bar{x}}{s}$ is used extensively in statistics. In this formula, x represents one value in a set of data, $\bar{x}$ represents the average (or mean) of those numbers in the set, and s represents a value called the standard deviation of the numbers. (The standard deviation is a positive number and is a measure of how "spread out" the numbers are.) The values for z are called z-scores, and they measure the number of standard deviation units a number x is from the average $\bar{x}$.

a. If $\bar{x} = 70$, what will be the z-score for $x = 70$? Does this z-score depend on the value of s? Explain.



b. For what values of x will the corresponding z-scores be negative?

c. Calculate your z-score on each of the last two test scores in this class. (Your instructor will give you the average and standard deviation for each test.) What do these scores tell you about your performance on the two exams?

74. Suppose that, for a particular set of exam scores, $\bar{x} = 72$ and $s = 6$. Find the z-score that corresponds to each of the following scores.

a. 78

b. 66

c. 81

Check

Step 1:

$$\begin{array}{rl} \$600 & \text{Marked selling price} \\ \times\ 0.30 & \text{Discount \%} \\ \hline \$180 & \text{Discount} \end{array}$$

Step 2:

$$\begin{array}{rl} \$600 & \text{Marked selling price} \\ -\ \$180 & \text{Discount} \\ \hline \$420 & \text{Actual selling price} \end{array}$$

Step 3:

$$\begin{array}{rl} \$420 & \text{Actual selling price} \\ -\ \$350 & \text{Cost} \\ \hline \$70 & \text{Profit} \end{array}$$

As a double check,

$$\begin{array}{rl} \$350 & \text{Cost} \\ \times\ 0.20 & \text{Profit \%} \\ \hline \$70 & \text{Profit} \end{array}$$

Now work margin exercise 8.

Margin Exercise Answers

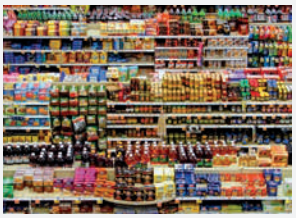
1. 64 **2.** −5 **3.** $2800 **4.** Bagpipes: $288.85, Kilt: $218.45 **5.** 2 hours **6.** First part took $\frac{4}{3}$ or $1\frac{1}{3}$ hours; Second part took $\frac{8}{3}$ or $2\frac{2}{3}$ hours. **7.** 53 **8.** $2300

2.4 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. Word problems are designed to teach you to _____ carefully and to _____ clearly.
2. Word problems generally do not give specific _____.
3. After solving a word problem, you should check to see if your _____ seems reasonable.
4. The average of a set of numbers can be found by _____ the numbers and then _____ by the quantities of numbers in the set.
5. To calculate distance, find the product of the _____ and _____.
6. When solving distance-rate-time problems, a _____ or _____ showing the known and unknown values is helpful.



Comparing Wholesale and Retail Prices

In business, understanding the difference between the wholesale price and the retail price can mean success or failure. The wholesale price is the amount that a store pays to purchase items. The retail price is the amount the customer pays to purchase the item. The difference between the retail price and wholesale price is called the retail markup. This markup is used to pay the overhead of the business (things like utilities, employee salary, insurance, and so on). Profit or loss is determined by the amount of money left after paying the overhead.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. In the distance-rate-time formula $d = r \cdot t$, the value t stands for the time spent traveling.

8. The concept of average can be used to find unknown numbers.

9. The first step to solving word problems is to draw any type of figures or diagram that might be helpful.

10. Translating English phrases into algebraic expressions can be used to solve number problems.

Practice

Read each problem carefully, translate the various phrases into algebraic expressions, set up an equation, and solve the equation. See Examples 1 and 2.

1. Five less than a number is equal to 13 decreased by the number. Find the number.

2. Three less than twice a number is equal to the number. What is the number?

3. Thirty-six is 4 more than twice a certain number. Find the number.

4. Fifteen decreased by twice a number is 27. Find the number.

5. Seven times a certain number is equal to the sum of twice the number and 35. What is the number?

6. The difference between twice a number and 3 is equal to 6 decreased by the number. Find the number.

7. Fourteen more than 3 times a number is equal to 6 decreased by the number. Find the number.

8. Two added to the quotient of a number and 7 is equal to −3. What is the number?

9. The quotient of twice a number and 5 is equal to the number increased by 6. What is the number?

10. Three times the sum of a number and 4 is equal to −9. Find the number.

11. Four times the difference between a number and 5 is equal to the number increased by 4. What is the number?

12. When 17 is added to 6 times a number, the result is equal to 1 plus twice the number. What is the number?

13. If the sum of twice a number and 5 is divided by 11, the result is equal to the difference between 4 and the number. Find the number.

14. If the difference between a number and 21 is divided by 2, the result is 4 times the number. What is the number?

15. Twice a number increased by 3 times the number is equal to 4 times the sum of the number and 3. Find the number.

16. Twice the difference between a number and 10 is equal to 6 times the number plus 16. What is the number?

Applications

Solve.

17. A mathematics student bought a calculator and a textbook for a course in statistics. If the textbook costs $67.51 more than the calculator, and the total cost for both was $329.49, what was the cost of each item?

18. The total cost of a computer flash drive and an all-in-one printer was $96.94, including tax. If the cost of the flash drive was $58.96 less than the printer, what was the cost of each item?

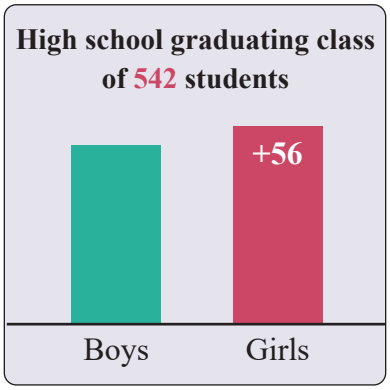
19. A real estate agent says that the current value of a 25-year old home is $90,000 more than twice its value when it was new. If the current value is $310,000, what was the value of the home when it was new?

20. On average the number of electric guitars sold in Texas each year is 91,399, which is seven times the average number of guitars sold each year in Wyoming. How many electric guitars, on average, are sold each year in Wyoming?

21. A classic car is now selling for $1500 more than three times its original price. If the selling price is now $12,000, what was the car's original price?

22. On August 24, the Fernandez family received 19 pieces of mail, consisting of magazines, bills, letters, and ads. If they received the same number of magazines as letters, three more bills than letters, and five more ads than bills, how many magazines did they receive?

23. A high school graduating class is made up of 542 students. If there are 56 more girls than boys, how many boys are in the class?



24. Lucinda bought two buckets of golf balls for the driving range. She gave the pro-shop clerk a 50-dollar bill and received $10.50 in change. What was the cost of one bucket of golf balls? (Tax was included.)

25. A guitar manufacturer spent $158 million on the production of acoustic and electric guitars last year. If the amount the company spent producing acoustic

guitars was $68 million more than it spent on producing electric guitars, how much did the company spend producing electric guitars?

26. The cost to rent a ballroom at a convention center for one day is $800 for the first 2 hours. Every additional hour that the ballroom is in use costs an additional $50 per hour, which is used to cover security fees. If you owe $1450 for a one-day rental of the ballroom, how many hours did you use the ballroom?

27. The cost to rent a party room at an arcade is a fixed price per hour plus $15 per child. If a 3-hour room rental with 20 children costs $330, what is the fixed price per hour for the room rental?

28. A-to-Z Truck Rentals charges $19.99 per day plus 65¢ per mile driven to rent a pick-up truck. For a one day trip, Louis paid a rental fee of $127.24. How many miles did he drive?

29. Dana finds the perfect dress for the Freshman Dance on sale at Belk. If she paid $95.96 for the dress on sale (before tax) and the dress was marked 20% off, find the original price of the dress.

- **a.** Is the 20% marked off the price she paid, or the original price?
- **b.** How do you represent the original price; the amount of the discount?
- **c.** Set up an equation and solve for the original price of the dress.

30. Two bicyclists, Chantelle and Taylor, start from opposite ends of a 19-mile-long bike path. Taylor rides her bike 6 mph faster than Chantelle, and the cyclists meet in 30 minutes. How fast was each of them riding? Draw a picture to represent the described problem.

- **a.** Use a table to organize the information given in the problem.
- **b.** Write an equation based on your diagram and/or table.
- **c.** Solve the equation and relate your answer to the original problem.

31. Two planes which are 2475 miles apart fly toward each other. Their speeds differ by 75 mph. If they pass each other in 3 hours, what is the speed of each?

32. Marcus drives from Chicago to Detroit in 6 hours. On the return trip, his speed is increased by 10 mph and the trip takes 5 hours. Find his rate on the return trip. How far apart are the towns?

33. Tim and Barb have 8 hours to spend on a mountain hike. They can walk up the trail at an average rate of 2 mph and can walk down at an average rate of 3 mph. How long should they plan to hike uphill before turning around?

34. The Reeds are moving across Texas. Mr. Reed leaves $3\frac{1}{2}$ hours before Mrs. Reed. If his average speed is 40 mph and her average speed is 60 mph, how long will Mrs. Reed have to drive before she overtakes Mr. Reed?

35. After traveling for 40 minutes, Mr. Koole had to slow to $\frac{2}{3}$ his original speed for the rest of the trip due to heavy traffic. The total trip of 84 miles took 2 hours. Find his original speed.

36. A train leaves Cincinnati at 2:00 p.m. A second train leaves the same station in the same direction at 4:00 p.m. The second train travels 24 mph faster than the first. If the second train overtakes the first at 7:00 p.m., what is the speed of each of the two trains?

37. Maria runs through the countryside at a rate of 10 mph. She returns along the same route at 6 mph. If the total trip took 1 hour 36 minutes, how far did she run in total?

38. Mr. Kent drove to a conference. The first half of the trip took 3 hours due to traffic. Traffic let up for the second half of the trip and he was able to increase his speed by 20 mph to make sure he got there on time. Find his rates of speed if he traveled 2 hours at the second rate.

39. Jayden walked to his friend's house at a rate of 4 mph to borrow his friend's bicycle. Coming back home, he rode the bicycle at an average rate of 12 mph. The total time for the round trip was 1 hour 30 minutes. How far away does Jayden's friend live?

40. Once a week, Felicia walks/runs for a total of 6 miles. Felicia spends twice as much time walking as she does running. If she walks at a rate of 4 mph and runs three times faster than she walks, what is the time for each part?

41. Achilles is racing a tortoise and gives him a 2-hour head start. The tortoise runs at a pace of 10 miles per hour and Achilles runs at a pace of 25 miles per hour. How long will it take Achilles to catch up to the tortoise?

a. Fill out the $d = r \cdot t$ table. Let the variable t represent the amount of time that the tortoise has traveled.

	Rate (mph)	·	Time (min)	=	Distance (miles)
Tortoise					
Achilles					

b. When Achilles catches up to the tortoise, they will have traveled the same distance. Set up a linear equation using the information in the table.

c. Solve the equation from Part **b.** for the variable.

d. How long will it take Achilles to catch up to the tortoise?

e. If the race is 35 miles long, will Achilles pass the tortoise before crossing the finish line? Show work to support your answer.

42. Marissa has five exam scores of 75, 82, 90, 85, and 77 in her chemistry class. What score does she need on the final exam to have an average grade of 80 (and thus earn a grade of B)? (All exams have a maximum of 100 points.)

43. Gerald had scores of 80, 92, 89, and 95 on four exams in his algebra class. What score will he need on his fifth exam to have an overall average grade of 90? (All exams have a maximum of 100 points.)

44. While riding her bike to the park and back home five times, Stacey timed herself at 60 min, 62 min, 55 min (the wind was helping), 58 min, and 63 min. She had set a goal of having an average time of 60 minutes for her rides. How many minutes will she need on her sixth ride to attain her goal?

45. For every 4-week period, Lauren wants to make an average of 6 phone calls per week from her prepaid cell phone. The first week she made 9 phone calls; the second week she made 6 phone calls, and the third week 5 phone calls. How many phone calls does Lauren need to make in the fourth week to make sure she stays on track with her goal?

46. While growing up, Jason was allowed to watch TV an average of 3 hours a day over a one-week period. One particular week he watched 1 hour, 2 hours, 1 hour, 3 hours, 3 hours, and 5 hours. How many hours could Jason watch the seventh and last day of the week and still obey his parents?

47. A college student realized that he was spending too much money on video games. For the remaining 5 months of the year, his goal is to spend an average of $50 a month towards his hobby. How much can he spend in December, taking into consideration that the other 4 months he spent $70, $25, $105, $30, respectively?

48. Wade has scores of 59, 68, 76, 84, and 69 on the first five tests in his social studies class. He knows that the final exam counts as two tests. What score will he need on the final to have an average of 70? (All tests and exams have a maximum of 100 points.)

49. A statistics student has grades of 86, 91, 95, and 76 on four hour-long exams. What score must he receive on the final exam to have an average grade of 90 if:

a. the final is equivalent to a single hour-long exam (100 points maximum)?

b. the final is equivalent to two hour-long exams (200 points maximum)?

50. Kevin consulted a dietician who told him to consume an average of 2100 calories per day based on his age, current weight, activity level, and weight goals. Kevin kept track of his calorie intake for several days. He consumed 2050 calories on Monday, 2200 calories on Tuesday, 2300 calories on Wednesday, and 2400 calories on Thursday. How many calories would he need to consume on Friday to have an average calorie intake of 2100 for the five days?

a. Set up an equation to solve for the amount of calories Kevin would need to consume on Friday. Use the variable x to represent the number of calories needed.

b. Solve the equation from Part **a.** for the variable.

c. It is recommended that active men consume more than 1500 calories per day to avoid triggering "starvation mode" in the body. Can Kevin stay above this calorie amount and meet his recommended average for the 5 days?

d. Do you think this is a smart way for Kevin to adjust his average calorie intake? If not, what are some alternatives?

Writing & Thinking

The following problem is given with an incorrect answer. Explain how you can tell that the answer is incorrect without needing to solve the problem or do any algebra; then, solve the problem correctly.

51. Kareem can paddle his kayak at 6 mph in still water. He decides to go kayaking on the local river. He paddles downriver (with the current) for 2 hours; then he turns around and paddles upriver (against the current) for 2.5 hours, returning to his starting point. How fast is the current in the river? **Incorrect answer: 27 mph**

2.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. When two quantities are compared by division, the comparison is called a _____ .
2. A unit rate is a rate with denominator _____.
3. To find a unit price (or price per unit), divide the _____ by the number of units.
4. A proportion is true if the _____ _____ are equal
5. To solve a word problem using a proportion, the first step is to _____ the unknown quantity.
6. When setting up a proportion to solve a word problem, the same _____ should be compared in the same order.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The units in the numerator and denominator of a ratio must be the same, or need to be able to be converted to the same units.
8. To make a unit rate, divide the numerator by the denominator.
9. A proportion is a statement that two ratios are being multiplied.
10. When using proportions to solve a word problem, there is only one correct way to set up the proportion.

Practice

Write each ratio as a fraction in lowest terms. See Example 1.

1. 18 to 28
2. 2.1 to 3.5
3. $\frac{2}{3}$ to $\frac{5}{6}$
4. $\frac{10}{9}$ to $\frac{5}{12}$
5. \$5.50 to \$8.00
6. \$12.75 to \$18.50
7. $3\frac{1}{2}$ donuts to 12 donuts
8. 3 months to $9\frac{1}{2}$ months

Write each rate as a fraction in lowest terms. See Example 4.

9. \$200 in profit to \$500 invested
10. \$200 in profit to \$1000 invested
11. 2 teachers to 24 students
12. 3 teachers to 54 students

13. 1500 children to 450 families

14. 900 children to 480 families

15. 25 scholarships to 210 students

16. 12 scholarships to 160 students

Write each rate as a unit rate. See Example 5.

17. 270 miles to 4.5 hours

18. 245 miles to 3.5 hours

19. 50 miles to 2 gallons of gas

20. 60 miles to 5 gallons of gas

21. 1260 words to 30 minutes

22. 840 words to 15 minutes

23. $612 to 48 hours

24. $399 to 28 hours

Find the unit price (to the nearest tenth of a cent) of each set of items and tell which one is the better buy. See Example 6.

25. ***Coffee beans:*** 1.75 oz at $1.99, 12 oz at $7.99

26. ***Coffee:*** 11.5 oz at $3.99, 39 oz at $8.99

27. ***Socks:*** 5 pairs for $12.50, 3 pairs for $8.25

28. ***Tennis balls:*** 3-pack for $2.99, 12-pack for $10.98

29. ***Sliced ham:*** 12 oz at $3.99, 8 oz at $2.89

30. ***Frozen orange juice:*** 16 fl oz at $2.99, 12 fl oz at $2.19

31. ***Mayonnaise:*** 8 oz at $1.59, 16 oz at $2.69, 32 oz at $3.89, 64 oz at $6.79

32. ***Aluminum foil:*** 200 sq ft at $7.39, 75 sq ft at $3.69, 50 sq ft at $3.19, 25 sq ft at $1.49

Determine whether each proportion is true or false. See Example 7.

33. $\frac{3}{6}=\frac{4}{8}$

34. $\frac{9}{8}=\frac{7}{6}$

35. $\frac{2}{5}=\frac{4}{10}$

36. $\frac{3}{5}=\frac{6}{10}$

37. $\frac{2}{3}=\frac{66}{100}$

38. $\frac{1}{3}=\frac{33}{100}$

39. $\frac{12}{18}=\frac{14}{21}$

40. $\frac{11}{22}=\frac{17}{34}$

41. $\frac{7}{16}=\frac{3\frac{1}{2}}{8}$

42. $\frac{10}{17}=\frac{5}{8\frac{1}{2}}$

43. $\frac{6}{1.56}=\frac{2}{0.52}$

44. $\frac{8.5}{6.5}=\frac{4.5}{3.5}$

Solve each proportion. See Example 8.

45. $\frac{5}{4}=\frac{x}{8}$

46. $\frac{3}{6}=\frac{6}{x}$

47. $\frac{1}{2}=\frac{x}{100}$

48. $\frac{3}{5}=\frac{R}{100}$

49. $\frac{3}{5}=\frac{60}{D}$

50. $\frac{8}{B}=\frac{6}{30}$

51. $\frac{x}{1}=\frac{1\frac{1}{4}}{5}$

52. $\frac{1}{5}=\frac{x}{7\frac{1}{2}}$

53. $\frac{150}{300}=\frac{R}{100}$

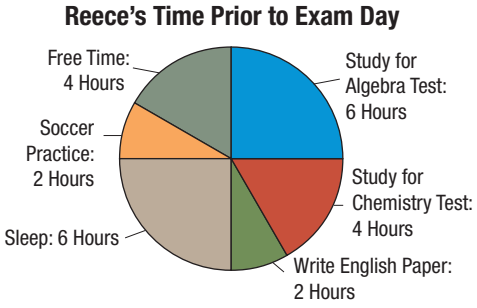
54. $\frac{98}{100}=\frac{B}{35}$

55. $\frac{3}{2}=\frac{B}{4.5}$

56. $\frac{7}{x}=\frac{2}{6.5}$

Applications

The circle graph shown here illustrates how Reece spent his time the day before his final exams. Use the information from the graph to answer the following questions.



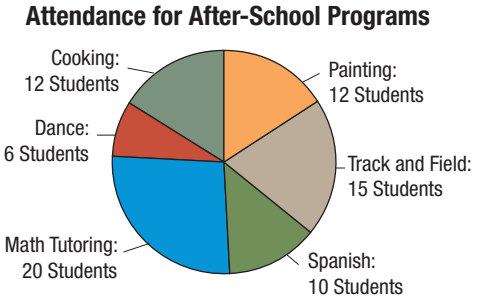
57. What is the ratio of time Reece spent studying for his algebra exam to time spent studying for his chemistry exam?

58. What is the ratio of time Reece spent at soccer practice to time spent sleeping?

59. What is the ratio of time Reece spent writing his English paper to the total number of hours in the day?

60. What is the ratio of time Reece spent in free time to the total number of hours in the day?

Iliana is organizing an after school program for kids in her school. The circle graph shown here illustrates the number of kids that have signed up for each session. Use the information from the graph to answer the following questions.



61. What is the ratio of children in Cooking to children in Track and Field?

62. What is the ratio of children in Dance to children in Spanish?

63. What is the ratio of children in Math Tutoring to total children in the program?

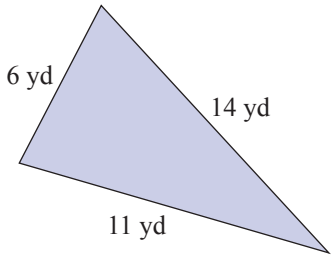
64. What is the ratio of children in Painting to total children in the program?

Solve.

65. The lengths of the sides of a triangle are 6 yd, 11 yd, and 14 yd.

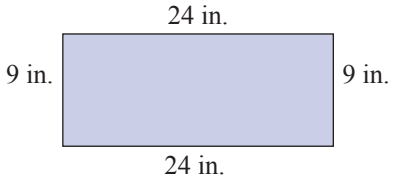
a. Find the ratio of the length of the longest side to the length of the shortest side.

b. Find the ratio of the length of the shortest side to the length of the longest side.



66. The length of a rectangle is 24 in. and its width is 9 in.

a. Find the ratio of the length to the width.



b. Find the ratio of the width to the length.

67. If a car consumes 9 gallons of gasoline after being driven 225 miles, what is the unit rate of the distance driven to the gasoline consumed?

68. A certain selective college has 18,400 applicants for 400 openings. What is the unit rate of applicants to openings?

69. A store sells 42-gallon trash bags in two different sizes. If you can buy a box of 50 trash bags for $25.98 or a box of 24 trash bags for $14.98, which option is the better deal per trash bag?

70. A landscaping company needs to purchase grass seed to plant a new lawn. Brand A sells 225 square feet worth of seed for $29.97. Brand B sells 400 square feet worth of seed for $55.97. Which option should they go with?

71. A carton of 18 free-range organic eggs costs $5.94. What is the cost per egg?

72. A 24-pack of 16.9-ounce bottles of water is sold for $3.84. What is the price per bottle of water?

73. A receptionist at a luxury spa sold 32 memberships during her 5-day work week. She is required to sell an average of 6 memberships per work day.

a. Find the rate of memberships per day that the receptionist sold.

b. Does the average number of membership sales by the receptionist match her daily quota? If not, is it higher or lower?

74. A motorist drives along an interstate highway for 3 hours and travels a total of 189 miles. The posted speed limit is 65 miles per hour. Does the motorist's average speed (in miles per hour) match the posted speed limit? If not, is it higher or lower?

75. A nurse sets up a 750 milliliter bag of intravenous solution. The bag empties in 5 hours. The patient's doctor prescribed the solution to be administered at a rate of 150 milliliters per hour. Does the rate that the nurse set the bag at match the prescribed rate? If not, is it higher or lower?

76. One propane heater will operate for $2\frac{2}{5}$ hours on $4\frac{2}{3}$ pounds of propane, while a second heater will operate for $4\frac{4}{5}$ hours on $9\frac{1}{3}$ pounds of propane. Determine if they operate at the same efficiency.

77. One olive press will extract 3.2 cups of oil from 15.6 pounds of olives, while the second press will extract 9.5 cups of oil from 48.8 pounds of olives. Assuming that the olives came from the same batch, determine if the two presses operate at the same efficiency.

78. When making a small "Energizing" scented candle, 5 drops of pepper scent and 3 drops of lemongrass scent are added to the wax. When making a large "Energizing" scented candle 35 drops of peppermint scent and 21 drops of lemongrass scent are added to the wax. Determine if the ratio of peppermint scent to lemongrass scent is the same for each size candle.

79. While competing in a speed run to complete the first level of a video game, Alex scores a total of 2352 points in 98 seconds. Shannon scored a total of 2314 points in 89 seconds. Determine if the ratio of points per second is the same for Alex and Shannon.

80. An English teacher must read and grade 27 essays. If the teacher takes 20 minutes to read and grade 3 essays, how much time will he need to grade all 27 essays?

81. A salesman figured he drove 560 miles every two weeks. How far would he drive in three months (12 weeks)?

82. Two units of a certain gas weigh 175 grams. What is the weight of 5 units of this gas?

83. A typist can type 8 pages of a manuscript in 56 minutes. How long will this typist take to type 300 pages?

84. Sales tax is figured at 6¢ for every \$1.00 of merchandise purchased. What is the purchase price on an item that had sales tax of \$2.04?

85. A sales woman makes \$8 for every \$100 worth of product that she sells. What will she make if she sells \$5000 worth of product?

86. If you can drive 286 miles in $5\frac{1}{2}$ hours, how long will it take you to drive 468 miles at the same rate of speed?

87. An architect drew plans for a city park using a scale of $\frac{1}{4}$ inch to represent 25 feet. How many feet would 2 inches represent?

88. One bag of dichondra lawn food contains 20 pounds of fertilizer and its recommended coverage is 4000 square feet. If you want to cover a lawn that is in the shape of a rectangle 120 feet by 160 feet, how many pounds of lawn food do you need?

Writing & Thinking

89. When finding price per unit, will monetary units be located in the numerator or the denominator of the rate?

90. A problem states:

a. "On a map from Detroit to Chicago, the distance is 4 inches. The actual distance is 300 miles. What is the distance on the map for an actual distance of 750 miles?"

b. When Traci set up this problem, her proportion was: $\frac{4 \text{ inches}}{300 \text{ miles}} = \frac{750 \text{ miles}}{x \text{ inches}}$. Explain and correct the problem/s with Traci's proportion.

2.6 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. If two variables are inversely proportional, an increase in the value of one variable must be accompanied by a/an _____ in the other.
2. If a variable varies (directly or inversely) with more than one other variable, this variation is said to be a _____ variation.
3. When two variables vary directly, an increase in one variable indicates a/an _____ in the other.
4. When two variables vary so that their product is constant, the two variables vary _____.
5. If there is a combined variation that is all direct variation, it is a _____ variation.
6. The letter k often represents the constant of _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The number of hamburgers eaten varies inversely with calories consumed.
8. The equation $y = \frac{k}{x}$ represents direct variation.
9. Distance and time varies directly, which means they are directly proportional.
10. The circumference of a circle varies directly with its radius.

Practice

Use the information given to find the unknown value. See Examples 1 and 3.

1. If y varies directly as x, and $y = 3$ when $x = 9$, find y if $x = 7$.
2. If y is directly proportional to x^2, and $y = 3$ when $x = 2$, what is y when $x = 8$?
3. If y varies inversely as x, and $y = 5$ when $x = 8$, find y if $x = 20$.
4. If y is inversely proportional to x, and $y = 5$ when $x = 4$, what is y when $x = 2$?
5. If y varies inversely as x^2, and $y = -8$ when $x = 2$, find y if $x = 3$.
6. If y is inversely proportional to x^3, and $y = 40$ when $x = \frac{1}{2}$, what is y when $x = \frac{1}{3}$?
7. If y is directly proportional to the square root of x, and $y = 6$ when $x = \frac{1}{4}$, what is y when $x = 9$?

8. If y is directly proportional to the square of x, and $y = 80$ when $x = 4$, what is y when $x = 6$?

9. z varies jointly as x and y, and $z = 60$ when $x = 2$ and $y = 3$. Find z if $x = 3$ and $y = 4$.

10. z varies jointly as x and y, and $z = -6$ when $x = 5$ and $y = 8$. Find z if $x = 12$ and $y = 15$.

11. z varies jointly as x and y^2, and $z = 63$ when $x = 5$ and $y = 3$. Find z if $x = \frac{10}{3}$ and $y = 2$.

12. z varies jointly as x^2 and y, and $z = 20$ when $x = 2$ and $y = 3$. Find z if $x = 4$ and $y = \frac{7}{10}$.

13. z varies directly as x and inversely as y^2. If $z = 5$ when $x = 1$ and $y = 2$, find z if $x = 2$ and $y = 1$.

14. z varies directly as x^3 and inversely as y^2. If $z = 24$ when $x = 2$ and $y = 2$, find z if $x = 3$ and $y = 2$.

15. z varies directly as $\sqrt{x}$ and inversely as y. If $z = 24$ when $x = 4$ and $y = 3$, find z if $x = 9$ and $y = 2$.

16. z varies directly as x^2 and inversely as $\sqrt{y}$. If $z = 108$ when $x = 6$ and $y = 4$, find z if $x = 4$ and $y = 9$.

17. s varies directly as the sum of r and t and inversely as w. If $s = 24$ when $r = 7$ and $t = 8$ and $w = 9$, find s if $r = 9$ and $t = 3$ and $w = 18$.

18. s varies directly as r and inversely as the difference of t and u. If $s = 36$ when $r = 12$ and $t = 9$ and $u = 6$, find s if $r = 18$ and $t = 11$ and $u = 8$.

19. L varies jointly as m and n and inversely as p. If $L = 6$ when $m = 7$ and $n = 8$ and $p = 12$, find L if $m = 15$ and $n = 14$ and $p = 10$.

20. W varies jointly as x and y and inversely as z. If $W = 10$ when $x = 6$ and $y = 5$ and $z = 2$, find W if $x = 12$ and $y = 6$ and $z = 3$.

Applications

Solve.

21. The distance a free-falling object falls is directly proportional to the square of the time it falls (before it hits the ground). If an object fell 256 feet in 4 seconds, how far would it have fallen by the end of 5 seconds?

22. The length a hanging spring stretches varies directly with the weight placed on the end. If a spring stretches 5 in. with a weight of 10 lb, how far will the spring stretch if the weight is increased to 12 lb?

23. The total price (P) of gasoline purchased varies directly as the number of gallons purchased. If 10 gallons are purchased for \$23.40, what will be the price of 15 gallons?

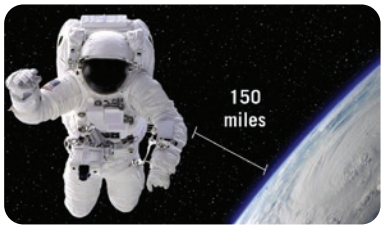
24. Research shows that the value of gold and the value of the dollar are inversely proportional. In 2016, gold cost \$1200 per ounce and the dollar had a rating of 93 on the US dollar index. In 2017, the cost of gold was \$1300 per ounce. What was the 2017 rating of the dollar? (Round your answer to the nearest hundredth.)

25. The circumference of a circle varies directly as the diameter. A circular pizza pie with a diameter of 1 foot has a circumference of 3.14 feet. What will be the circumference of a pizza pie with a diameter of 1.5 feet?

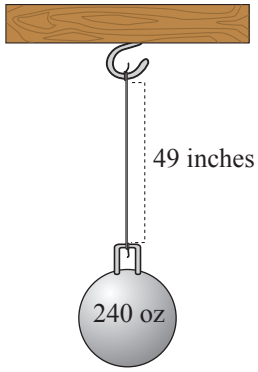
26. The area of a circle varies directly as the square of its radius. A circular pizza pie with a radius of 6 in. has an area of 113.04 in.^{2} What will be the area of a pizza pie with a radius of 9 in.?

27. Several triangles have the same area. In this set of triangles, the height and base are inversely proportional. In one such triangle, the height is 5 m and the base is 12 m. Find the height of the triangle in this set with a base of 10 m.

28. If an astronaut weighs 250 pounds on the surface of the earth, what will the astronaut weigh 150 miles above the earth? Assume that the radius of the earth is 4000 miles, and round to the nearest tenth.



29. The elongation (E) in a wire when a mass (m) is hung at its free end varies jointly as the attached mass and the length (l) of the wire and inversely as the cross-sectional area (A) of the wire. The elongation is 0.0055 cm when a mass of 120 g is attached to a wire 330 cm long with a cross-sectional area of 0.4 cm^2. Find the elongation if a mass of 160 g is attached to the same wire.



30. When a mass of 240 oz is suspended by a wire 49 in. long whose cross-sectional area is 0.035 in.^{2}, the elongation of the wire is 0.016 in. Find the elongation if the same mass is suspended by a 28 in. wire of the same material with a cross-sectional area of 0.04 in.^{2} (See Exercise 29.)

31. The safe load (L) of a wooden beam supported at both ends varies jointly as the width (w) and the square of the depth (d) and inversely as the length (l). A beam 4 in. wide, 6 in. deep, and 12 ft long supports a load of 4800 lb safely. What is the safe load of a beam of the same material that is 6 in. wide, 10 in. deep, and 15 ft long?

32. A wooden beam 2 in. wide, 8 in. deep, and 14 ft long holds up to 2400 lb. What load would a beam 3 in. wide, 6 in. deep, and 15 ft long, of the same material, support? (See Exercise 31.)

33. The gravitational force of attraction (F) between two bodies varies directly as the product of their masses (m_1 and m_2) and inversely as the square of the distance (d) between them. The gravitational force between a 5-kg mass and a 2-kg mass 1 m apart is 1.5×10^{-10} N. Find the force between a 24-kg mass and a 9-kg mass that are 6 m apart. (N represents a unit of force called a Newton.)

34. In Exercise 33, what is the force if the distance between the 24 kg mass and the 9 kg mass is cut in half?

35. The total price (P) of gasoline purchased varies directly as the number of gallons purchased. If 10 gallons are purchased for \$39.80, what will be the price of 15 gallons?

36. The distance that an object falls is directly proportional to the square of the time that has passed since the object started to fall. A rock falls a distance of 64 feet in 2 seconds. How long will it take the rock to fall a distance of 100 feet?

37. For a certain type of wooden beam that carries a load at its center, the safe load (SL) varies jointly as the width w and the cube of the depth (d) and inversely as the square of the length (l). A wooden beam that is 4 inches wide, 6 inches deep, and 12 feet long can safely support a load of 2400 pounds.

- **a.** Set up the variation equation.
- **b.** Determine the constant of variation.
- **c.** How much weight can a wooden beam that is 5 inches wide, 6 inches deep, and 10 feet long safely support?

Lifting Force

The lifting force (or lift) (L) in pounds exerted by the atmosphere on the wings of an airplane is related to the area (A) of the wings in square feet and the speed (or velocity) (v) of the plane in miles per hour by the formula $L = kAv^2$, where k is the constant of variation.

38. If the lift is 9600 lb for a wing area of 120 ft^2 at a speed of 80 mph, find the lift of the same wing at a speed of 100 mph.

39. The lift for a wing of area 280 ft^2 is 34,300 lb when the plane is traveling at 210 mph. What is the lift if the speed is decreased to 180 mph?

40. The lift for a wing with an area of 144 ft^2 is 10,000 lb when the plane is traveling at 150 mph. What is the lift if the speed is decreased to 120 mph?

41. A plane traveling 140 mph with wing area 195 ft^2 has 12,500 lb of lift exerted on the wings. Find the lift for the same plane traveling at 168 mph.

Pressure

Boyle's Law states that if the temperature of a gas sample remains the same, the pressure (P) of the gas is related to the volume (V) by the formula $P = \frac{k}{V}$, where k is the constant of variation.

42. A pressure of 1600 lb per ft^2 is exerted by 2 ft^3 of air in a cylinder. If a piston is pushed into the cylinder until the pressure is 1800 lb per ft^2, what will be the volume of the air? Round to the nearest tenth.

43. The volume of gas in a container is 300 cm^3 when the pressure on the gas is 20 g per cm^2. What will be the volume if the pressure is increased to 30 g per cm^2?

44. The pressure in a canister of gas is 1360 g per $in.^2$ when the volume of gas is 5 $in.^3$ If the volume is reduced to 4 $in.^3$, what is the pressure?

45. A scuba diver is using a diving tank that can hold 6 liters of air. If the tank has a pressure rating of 220 bar when full, what is the pressure rating when the volume of gas is 4 liters?

Electricity

The resistance (R) (in ohms), in a wire is given by the formula $R = \frac{kL}{d^2}$, where k is the constant of variation, L is the length of the wire and d is the diameter.

46. The resistance of a wire 500 ft long with a diameter of 0.01 in. is 20 ohms. What is the resistance of a wire 1500 ft long with a diameter of 0.02 in.?

47. The resistance is 2.6 ohms when the diameter of a wire is 0.02 in. and the wire is 10 ft long. Find the resistance of the same type of wire with a diameter of 0.01 in. and a length of 5 ft.

48. Tristan's car stereo uses a 5-ft audio wire with diameter 0.025 in. and resistance of 1.6 ohms. What is the resistance of 8 ft of the same type of audio wire?

49. Nicole purchased a spool of wire with diameter 0.01 in. for the speakers in her home audio system. If the resistance of 15 ft of this wire is 6 ohms, what is the resistance of 25 ft of the wire?

Levers

If a lever is balanced with weight on opposite sides of its balance point, then the following proportion exists:

$\frac{W_1}{W_2} = \frac{L_2}{L_1}$ or $W_1 L_1 = W_2 L_2$ where $L_1 + L_2 = L$, the total length of the lever.

50. How much weight can be raised at one end of a bar 8 ft long by the downward force of 60 lb when the balance point is $\frac{1}{2}$ ft from the unknown weight?

51. Where should the balance point of a bar 12 ft long be located if a 120-lb force is to raise a load weighing 960 lb?

52. Find the location of the balance point of a 25-ft board that can raise a 300-lb package with a downward force of 75 lb.

53. How much weight can be raised on one end of a 17-meter board by 90 kilograms, if the balance point is 5 meters from the unknown weight?

Writing & Thinking

54. Explain, in your own words, the meaning of the following terms.

a. Direct variation

b. Joint variation

c. Inverse variation

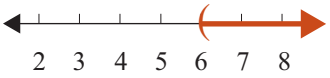
d. Combined variation

Discuss an example of each type of variation that you have observed in your daily life.

Completion Example Answers

11.

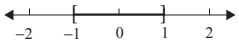
$2x+5<3x-(7-x)$	Write the inequality.
$2x+5<3x-7+x$	Distribute the negative sign.
$2x+5<4x-7$	Combine like terms.
$2x+5-2x<4x-7-2x$	Add $-2x$ to both sides.
$5<2x-7$	Simplify.
$5+7<2x-7+7$	Add 7 to both sides.
$12<2x$	Simplify.
$\frac{12}{2}<\frac{2x}{2}$	Divide both sides by 2.
$6<x$	Simplify.



x is in $(6,\infty)$

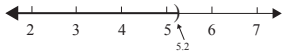
Margin Exercise Answers

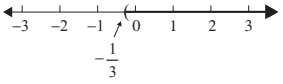
1. 

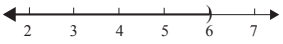
2. 

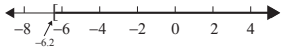
3. $-2 \le x < 1$ is a half-open interval.

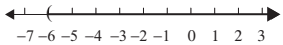
4. $[8,\infty)$ is a half-open interval.

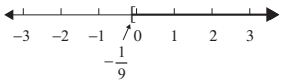
5. $y < 5.2$ 

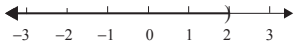
6. $x > -\frac{1}{3}$ 

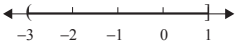
7. $x < 6$ 

8. $x \ge -6.2$ 

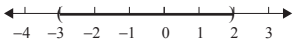
9. $(-6,\infty)$ 

10. $\left[-\frac{1}{9},\infty\right)$ 

11. $(-\infty,2)$ 

12. 

$(-3,1]$

13. 

$(-3,2)$

14. The maximum final dosage that can be administered must be less than 400 milligrams.

15. Ashley can buy at most 8 rose centerpieces.

2.7 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. If a and b are real numbers where $a < b$, the set of all real numbers between a and b is called a/an _____ of real numbers.

2. In a/an _____ interval, neither endpoint is included.

3. In a/an _____ interval, both end points are included.

4. Linear inequalities are inequalities that relate two _____ _____.

5. If A and B are algebraic expressions and C is a real number, then the _____ principle for solving linear inequalities states that $A < B$ and $A + C < B + C$ are equivalent.

6. If A and B are algebraic expressions and C is a real number, then the _____ principle for solving linear inequalities states that $A < B$ and $AC < BC$ are equivalent.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If only one end-point is included in an interval, it is called a half-open interval.

8. When both sides of a linear inequality are multiplied by a negative constant, the sense of the inequality should stay the same.

9. To check the solution set of a linear inequality, every solution in the solution set must be checked in the original inequality.

10. The infinity symbol ∞ does not represent a specific number.

Practice

Graph each interval on a real number line. See Example 1.

1. $(-1,\infty)$

2. $[-2,4)$

3. $(-\infty,5]$

4. $[0,3]$

5. $[-5,-1]$

6. $(3,8)$

7. $[-7,-4)$

8. $(-\infty,-6]$

Graph each interval on a real number line and tell what type of interval it is. See Examples 2 through 4.

9. $x \le -3$

10. $x \ge -0.5$

11. $x > 4$

12. $x < -\frac{1}{10}$

13. $0 < x \le 2.5$

14. $-1.5 \le x < 3.2$

15. $-2 \le x \le 0$

16. $-1 \le x \le 1$

17. $4 > x \ge 2$

18. $0 > x \ge -5$

Solve each inequality and graph the solution set. Write each solution set using interval notation.

19. $x + 1 > 5$

20. $x - 3 < 2$

21. $3 + x \le 7$

22. $5 + x \ge 11$

23. $3 < 4 + x$

24. $9 > 6 + x$

25. $4 \ge x - 3$

26. $12 \le x + 8$

27. $4x > 16$

28. $3x < 27$

29. $5x \le 15$

30. $-2x \ge 6$

31. $10 > -5x$

32. $12 < 8x$

33. $14 \ge 2x$

34. $9 \le -3x$

35. $2x + 3 < 5$

36. $4x - 7 \ge 9$

37. $14 - 5x < 4$

38. $23 < 7x - 5$

39. $6x - 15 > 1$

40. $9 - 2x < 8$

41. $5.6 + 3x \ge 4.4$

42. $12x - 8.3 < 6.1$

43. $1.5x + 9.6 < 12.6$

44. $0.8x - 2.1 \ge 1.1$

45. $2 + 3x \ge x + 8$

46. $x - 6 \le 4 - x$

47. $3x - 1 \le 11 - 3x$

48. $5x + 6 \ge 2x - 2$

49. $4 - 2x < 5 + x$

50. $4 + x > 1 - x$

51. $x - 6 > 3x + 5$

52. $4 + 7x \le 4x - 8$

53. $\frac{x}{2} - 1 \le \frac{5x}{2} - 3$

54. $\frac{x}{4} + 1 \le 5 - \frac{x}{4}$

55. $\frac{x}{3} - 2 > 1 - \frac{x}{3}$

56. $\frac{5x}{3} + 2 > \frac{x}{3} - 1$

57. $6x + 5.91 < 1.11 - 2x$

58. $4.3x + 21.5 \ge 1.7x + 0.7$

59. $6.2x - 5.9 > 4.8x + 3.2$

60. $0.9x - 11.3 < 3.1 - 0.7x$

61. $4(6 - x) < -2(3x + 1)$

62. $-3(2x - 5) \le 3(x - 1)$

63. $-(3x + 8) \ge 2(3x + 1)$

64. $6(3x + 1) < 5(1 - 2x)$

65. $11x + 8 - 5x \ge 2x - (4 - x)$

66. $1 - (2x + 8) < (9 + x) - 4x$

67. $5 - 3(4 - x) + x \le -2(3 - 2x) - x$

68. $x - 2(x + 3) \ge 7 - (4 - x) + 11$

69. $\frac{2(x-1)}{3} < \frac{3(x+1)}{4}$

70. $\frac{3(x-2)}{2} \ge \frac{4(x-1)}{3}$

71. $\frac{x-2}{4} > \frac{x+2}{2} + 6$

72. $\frac{x+4}{9} \le \frac{x}{3} - 2$

73. $\frac{2x+7}{4} \le \frac{x+1}{3} - 1$

74. $\frac{4x}{7} - 3 > \frac{x-6}{2} - 4$

75. $-4 < x + 5 < 6$

76. $2 \le -x + 2 \le 6$

77. $3 \ge 4x - 3 \ge -1$

78. $13 > 3x + 4 > -2$

79. $1 \le \frac{2}{3}x - 1 \le 9$

80. $-2 \le \frac{1}{2}x - 5 \le -1$

81. $14 > -2x - 6 > 4$

82. $-11 \ge -3x + 2 > -20$

83. $-1.5 < 2x + 4.1 < 3.5$

84. $0.9 < 3x + 2.4 < 6.9$

Represent each of the following statements as an inequality involving a variable x, and graph its solution set on a number line.

85. You must be at least 58 inches in height to ride this roller coaster.

86. There are fewer than 12 days left before final exams.

87. Gifts worth \$5 or less do not need to be declared.

88. Arsenic levels over 10 parts per billion may be dangerous.

Applications

Solve.

89. *A* statistics student has grades of 82, 95, 93, and 78 on four hour-long exams. He must average 90 or higher to receive an *A* for the course. What scores can he receive on the final exam and earn an *A* if:

a. The final is equivalent to a single hour-long exam (100 points maximum)?

b. The final is equivalent to two hourly exams (200 points maximum)?

90. To receive a grade of B in a chemistry class, Melissa must average 80 or more but less than 90. If her five hour-long exam scores were 75, 82, 90, 85, and 77, what score does she need on the final exam (100 points maximum) to earn a grade of B?

91. A car salesman makes \$1000 each day that he works and makes approximately \$250 commission for each car he sells. If a car salesman wants to make at least \$3500 in one day, how many cars does he need to sell?

92. Allison is going to the post office to buy 34¢ stamps and 3¢ adjustment stamps. Since the current postage rate is 49¢, she will need 5 times as many 3¢ adjustment stamps as 34¢ stamps. If she has \$12.25 to spend, what is the largest number of 34¢ stamps she can buy?

93. WildLily Florist is creating arrangements for a wedding this weekend. The large arrangements use 8 flowers and the small arrangements use 5 flowers.

a. Let x represent the number of large arrangements. Write an algebraic expression for the number of small arrangements, if there are 15 tables that need an arrangement.

b. Write an algebraic expression representing the total number of flowers used in the 15 arrangements.

c. If the bride has paid for 100 flowers, use an inequality to determine the maximum number of the 15 arrangements that can be large.

94. John's algebra test consists of 19 questions, 13 equations and 6 word problems. Each equation is worth 4 points, and each word problem is worth 8 points. Assume there is no partial credit on this test.

a. Let w be the number of word problems John gets correct. Write an expression for the number of points John will get from the word problem part of his test.

b. Assuming John gets every equation correct, write an inequality that will help determine the fewest number of word problems he can get correct and still make an 80 on the test. What is the fewest number he can get correct?

c. Let x be the number of equations John gets correct. Write an expression for the number of points John will get from the equation part of his test.

d. Assuming John gets every word problem correct, write an inequality that will help determine the fewest number of equations he can get correct and still get an 80 on the test. What is the fewest number of equations he can get correct?

95. Dr. Smiley has an attendance clause in his course syllabus that a student loses 5 points on his or her final grade average for every unexcused absence the student has after his or her first three unexcused absences. If Kara must have a 70 to pass the course, determine the largest number of unexcused absences Kara can have and still have any chance to pass the course.

96. Tracy needs to purchase 25 pastries for the PTA Teachers' Breakfast. Bear Claws cost \$1.75 each and Apple Turnovers cost \$2.15 each. If Tracy's budget is \$50, find the maximum number of Apple Turnovers that Tracy can purchase.

97. The maximum occupancy for a concert in Thompson-Boling Arena is 24,000 people. However, for every 15 tickets sold, there must be one worker present (security, food service, admissions, etc…). Determine the maximum number of tickets than can be sold.

98. Phineas wants to build a nuclear-powered submarine to take his friends on a tour of the Arctic Circle. At least twice as much titanium must be used in the construction of the shell of the sub as the amount of stainless steel used in its construction. The cost of titanium is \$500 per lb and the cost of steel is \$300 per lb. If Phineas has only \$1,000,000 to spend on metal for the sub, determine the greatest number of lbs of metal (both together) that can be used to construct his submarine. (Round your answer to the nearest pound.)

99. Nicole has just moved to Orlando and discovered that Florida residents can purchase 4-day tickets to Disney World for $55 per day. Annual passes (with certain restrictions) for Florida residents are $390. Nicole is trying to decide if she thinks she will go to the park enough times to make it worth buying an annual pass. Use the formula $55x \leq 390$, where x is the number of days spent visiting at Disney World, to determine how many times she would have to go in order for the annual pass to be the better deal.

100. Fernando has already consumed 270 grams of carbohydrates and is on a diet that restricts his carbohydrate consumption to no more than 300 grams of carbohydrates per day. A serving of 6 crackers has 21 grams of carbohydrates. Solve the inequality $\frac{21}{6}c + 270 \leq 300$ to determine how many crackers (c) Fernando can eat without going over his goal. Write your answer as a whole number.

101. Jeph is in charge of buying office supplies for the nonprofit organization he works for. He has $400 to spend. He needs to buy a printer that costs $150, a box of printer paper for $60, and some ink cartridges for $12.50 each. What is the maximum number of ink cartridges that Jeph can buy? (**Note:** Tax is not included in the sales price.)

a. Set up the linear inequality. Use the variable c to represent the number of ink cartridges.

b. Solve the equation from Part **a.** for the variable.

c. What does the answer from Part **b.** mean? Write a complete sentence.

102. Sarah is participating in National Novel Writers Month where she has to write a rough draft of a novel with at least 50,000 words during the month of November. At the end of the day on November 20th, she has a total of 32,500 words. What is the minimum number of words that Sarah needs to write each day for the rest of the month to make the goal of 50,000 words?

a. Set up the linear inequality. Use the variable w to represent the number of words per day.

b. Solve the equation from Part **a.** for the variable.

c. What does the answer from Part **b.** mean? Write a complete sentence.

103. Andrew needs to earn at least a B in each class to keep his scholarship. The grade in his economics class is based on five exams that are equally weighted. On the first four exams, Andrew received the following scores: 92, 74, 80, 72. Andrew needs an average of at least 80 to earn a B for the class. What range of scores does he need on the fifth exam to keep his scholarship?

a. Set up the linear inequality. Use the variable E to represent the fifth exam score.

b. Solve the equation from Part **a.** for the variable.

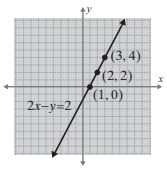
c. What does the answer from Part **b.** mean? Write a complete sentence.

Writing & Thinking

104. **a.** Write a list of three situations where inequalities might be used in daily life.

b. Illustrate theses situations with algebraic inequalities and appropriate numbers.

6.



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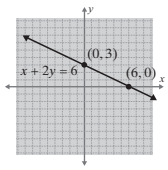
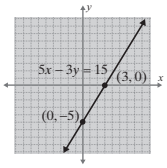


x-intercept $= (6, 0)$, y-intercept $= (0, 3)$

8.



x-intercept $= (3, 0)$, y-intercept $= (0, -5)$

3.1 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. In the Cartesian coordinate system, the plane is divided into _____ _____.
2. There is a one-to-one correspondence between _____ on a plane and _____ _____ of real numbers.
3. Scatter plots are used to determine whether there is any _____ to a set of data.
4. In an ordered pair (x, y), the variable x is called the _____ variable and the variable y is called the _____ variable.
5. Every line corresponds to some linear _____.
6. If a line passes through the point (___, ___) then the y-intercept and the x-intercept are the same point.

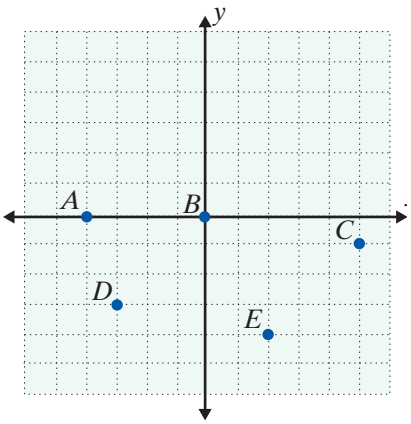
True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The y-intercept is the point where a line crosses the y-axis.
8. The terms ordered pair and point are used interchangeably.
9. A horizontal line does not have a y-intercept.
10. All x-intercepts correspond to an ordered pair of the form $(0, y)$.

Practice

List the sets of ordered pairs that correspond to each graph. Assume that the grid lines are marked one unit apart.

1.



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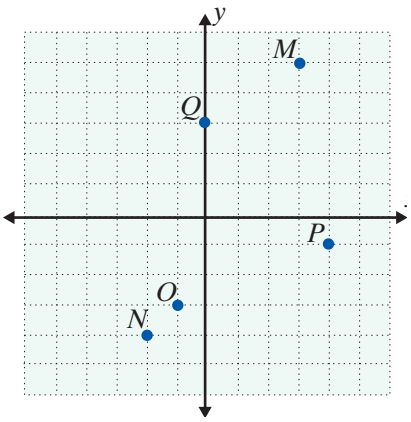

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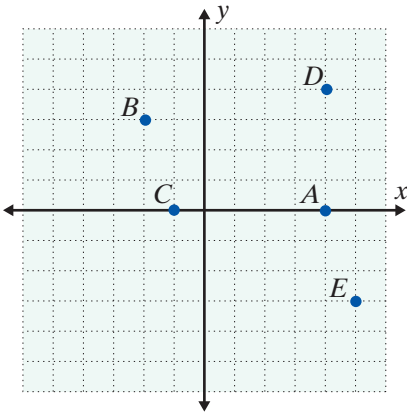

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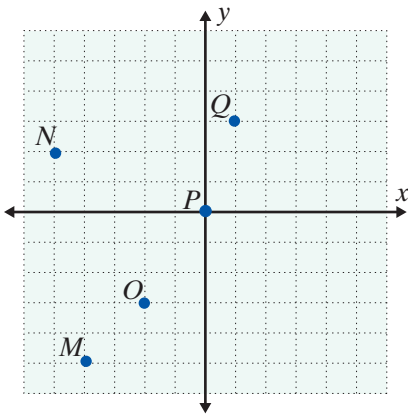

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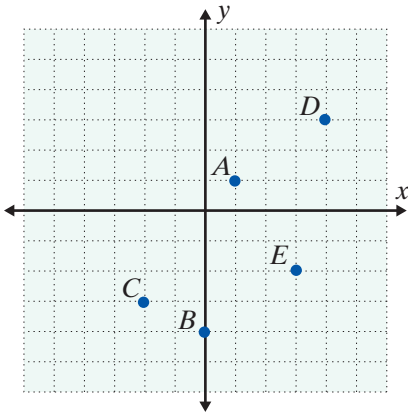

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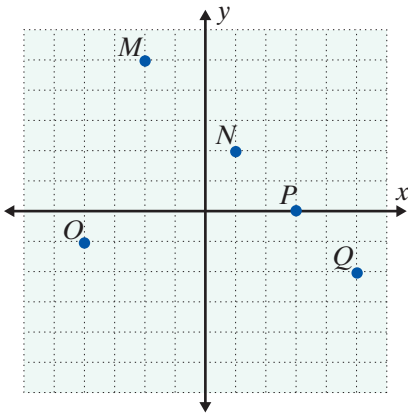


Graph the sets of ordered pairs and label the points. See Example 1.

7. $\{A(-5,1), B(-3,4), C(-1,1), D(2,2), E(2,-2)\}$

8. $\{M(-4,1), N(-2,5), O(0,3), P(1,6), Q(3,2)\}$

9. $\{P(-3,2), Q(-1,-1), R(1,5), S(3,-2), T(6,5)\}$

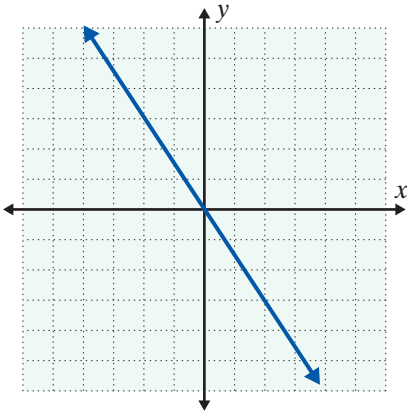
10. $\{C(-5,-5), D(-3,1), E(0,-2), F(3,1), G(5,0)\}$

11. $\{A(-3,4), B(-2,-1), C(-1,6), D(2,0), E(3,-3)\}$

12. $\{M(-7,2), N(-4,5), O(0,-4), P(1,-2), Q(4,-4)\}$

List any three points on each line. (There is more than one correct answer.) See Example 2.

13.



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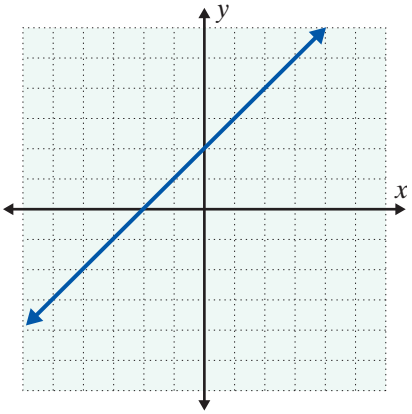

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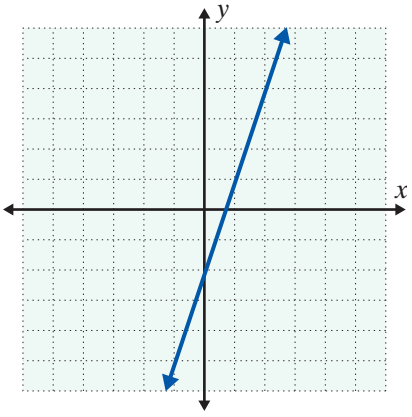

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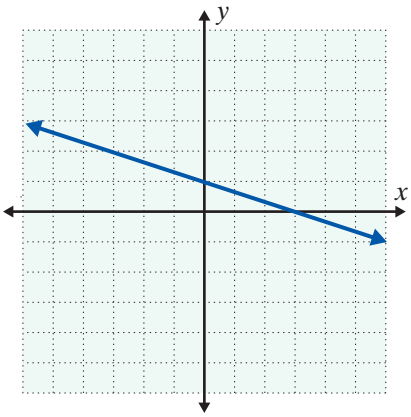

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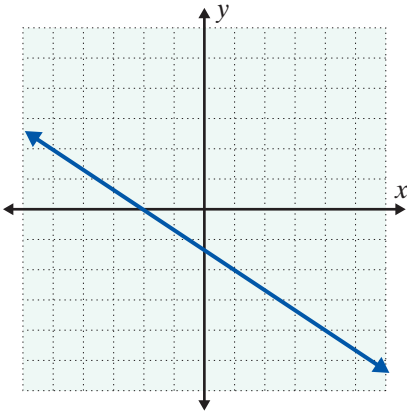

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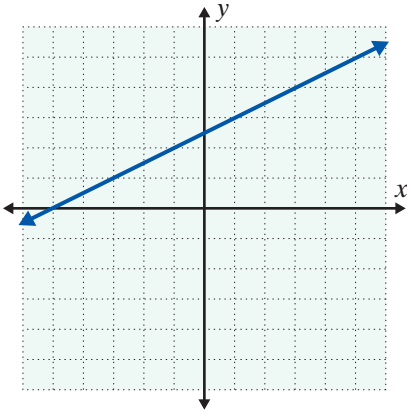


For each data set, **a.** rewrite the data in the table as ordered pairs and **b.** use the data to create a scatter plot. See Example 3.

19. The following table gives the independent expenditures for and against candidates running for the senate and house of representatives.

Independent Expenditures Towards Senate and House for the 2017-2018 Election Cycle

Senate Expenditures (in millions)	House Expenditures (in millions)
67.3	12.0
87.2	44.6
70.1	1.5
77.9	0.2
44.9	8.0
67.0	22.9
18.5	0.3
58.0	0.4
15.7	48.1
37.1	4.0

Source: FEC

20. The following table gives the height and weight of NBA players who scored the most points in 2018.

Height and Weight of NBA Players

Height (in inches)	Weight (in pounds)
77	220
82	253
80	250
75	195
83	242
81	240
75	200
75	193
83	260
76	210

Source: ESPN

Use following table for Exercises 21 and 22. The table contains the in-state total cost per year, the in-state total cost per year with need-based aid, and the average debt of a student after graduation of 10 top public colleges.

Total Cost per Year (Without and With Need-Based Aid) vs. Average Debt After Graduation

Total Cost per Year (in thousands)	Total Cost per Year after Need-Based Aid (in thousands)	Average Debt After Graduation (in thousands)
22	5	21
18	10	22
29	8	25
27	9	26
31	14	19
30	10	21
24	9	22
21	12	25
21	11	22
24	14	28

Source: www.kiplinger.com

21. Create a scatter plot using the cost per year vs. average debt.

22. Create a scatter plot using cost after aid vs. average debt.

Find the missing coordinate of each ordered pair so that the ordered pair belongs to the solution set of the given equation.

23. $2x + y = 5$

x	y
0	
	0
–2	
	3

24. $x + 2y = 6$

x	y
0	
	0
4	
	–2

25. $3x - y = 4$

x	y
0	
	0
2	
	5

26. $x - 3y = 9$

x	y
0	
	0
–3	
	–1

27. $y = 5 - 2x$

a. $(0, \ )$

b. $(\ , 0)$

c. $(2, \ )$

d. $(\ , 7)$

28. $y = 5x - 3$

a. $(0, \ )$

b. $(\ , 0)$

c. $(-1, \ )$

d. $(\ , 7)$

29. $3x - 2y = 6$

a. $(0, \ )$

b. $(\ , 0)$

c. $(-2, \ )$

d. $(\ , 3)$

30. $5x + 2y = 10$

a. $(0, \ )$

b. $(\ , 0)$

c. $(4, \ )$

d. $(\ , 10)$

Locate at least two ordered pairs of real numbers that satisfy each linear equation and graph the corresponding line in the Cartesian coordinate system. See Examples 4 through 6.

31. $x + y = 3$

32. $x + y = 4$

33. $y = x$

34. $2y = x$

35. $2x + y = 0$

36. $3x + 2y = 0$

37. $2x + 3y = 7$

38. $4x + 3y = 11$

39. $3x - 4y = 12$

40. $2x - 5y = 10$

41. $-4x + y = 4$

42. $-3x + 3y = 6$

43. $3y = 2x - 4$

44. $4x = 3y + 8$

45. $3x + 5y = 6$

46. $2x + 7y = -4$

47. $2x + 3y = 1$

48. $5x - 3y = -1$

49. $5x - 2y = 7$

50. $3x + 4y = 7$

51. $\frac{2}{3}x - y = 4$

52. $x + \frac{3}{4}y = 6$

53. $2x + \frac{1}{2}y = 3$

54. $\frac{2}{5}x - 3y = 5$

55. $5x = y + 2$

56. $4x = 3y - 5$

Graph each linear equation by locating the *y*-intercept and the *x*-intercept. See Examples 7 and 8.

57. $x - 2y = 8$

58. $x + y = 6$

59. $2x + 3y = 12$

60. $3x - 7y = -21$

61. $4x - y = 10$

62. $\frac{1}{2}x + 2y = 3$

63. $3x + 2y = 15$

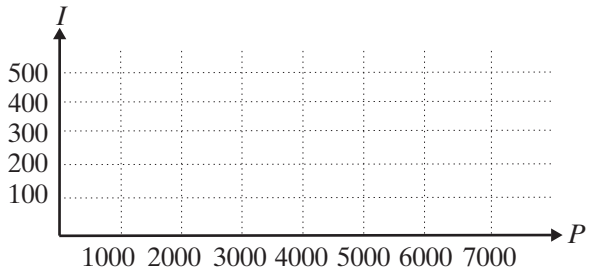
64. $x - 4y = -6$

65. Given the equation $I = 0.06P$, where I is the interest earned on a principal P at the rate of 6%:

a. Make a table of ordered pairs for the values of P and I if P has the values \$1000, \$2000, \$4000, \$5000, and \$7000.

b. Graph the points corresponding to the ordered pairs.

P	I
1000	
2000	
4000	
5000	
7000	

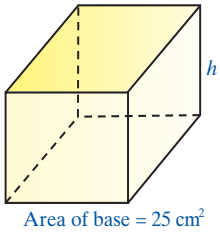


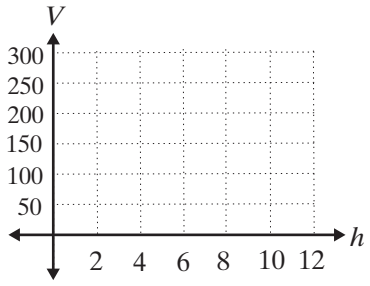
66. Given the equation $V = 25h$, where V is the volume (in cm^3) of a box with a variable height h in cm and a fixed base of area 25 cm^2:

a. Make a table of ordered pairs for the values of h and V with h as the values 2 cm, 3 cm, 8 cm, 10 cm, and 12 cm.

b. Graph the points corresponding to the ordered pairs.

P	I
2	
3	
8	
10	
12	





Writing & Thinking

67. The following table of values gives the number of push-ups and pull-ups completed by ten students in a physical education class.

a. Plot these points on a scatter diagram.

Person	1	2	3	4	5	6	7	8	9	10
Push-ups	20	15	25	23	35	30	42	40	25	35
Pull-ups	5	2	9	8	10	11	15	14	7	12

b. Does there seem to be a pattern in the relationship between push-ups and pull-ups? If so, what is this pattern?

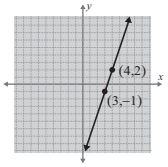
c. Using the scatter diagram in Part a, predict the number of pull-ups that a student might be able to do if he or she has just done each of the following numbers of push-ups: 22, 32, 35, and 45. (**Note:** In each case, there is no one correct answer. The answers are only estimates based on the diagram.)

68. Explain in your own words why it is sufficient to find the x-intercept and y-intercept to graph a line (assuming that they are not the same point).

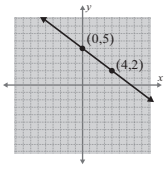
69. Explain in your own words how you can determine if an ordered pair is a solution to an equation.

Margin Exercise Answers

1. slope = 3



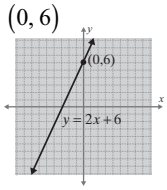
2. slope = $-\frac{3}{4}$



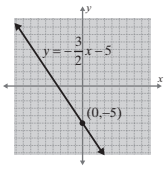
3. $y = -2$; slope is 0

4. $x = 2$; slope is undefined

5. $m = 2$; y-intercept = $(0, 6)$



6. $m = -\frac{3}{2}$; y-intercept = $(0, -5)$



7. $y = \frac{2}{3}x - 3$

3.2 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The slope of a line is the ratio of rise to _____.

2. Another name for slope is the rate of _____.

3. A line that rises (increases) from left to right has a/an _____ slope.

4. The slope of every vertical line is _____.

5. The slope of every horizontal line is _____.

6. In the equation $y = mx + b$, m represents the _____ and $(0, b)$ represents the _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If the y-intercept and the slope of a line are given, there is enough information to write the equation of the line.

8. When using the slope formula, the slope of a line changes if the order of the points is reversed.

9. A line that falls (decreases) from left to right has a negative slope.

10. The line that represents the equation $y = 2x + 4$ has a y-intercept of $(0, 4)$.

Practice

Find the slope of the line determined by each pair of points. See Examples 1 and 2.

1. $(2, 4); (1, -1)$

2. $(1, -2); (1, 4)$

3. $(-6, 3); (1, 2)$

4. $(-3, 7); (4, -1)$

5. $(-5, 8); (3, 8)$

6. $(-2, 3); (-2, -1)$

7. $(5, 1); (3, 0)$

8. $(0, 0); (-2, -3)$

9. $\left(\frac{3}{4}, \frac{3}{2}\right); (1, 2)$

10. $\left(4, \frac{1}{2}\right); (-1, 2)$

11. $\left(\frac{3}{2}, \frac{4}{5}\right); \left(-2, \frac{1}{10}\right)$

12. $\left(\frac{7}{2}, \frac{3}{4}\right); \left(\frac{1}{2}, -3\right)$

Determine whether each equation represents a horizontal line or vertical line and give its slope. Graph the line. See Examples 3 and 4.

13. $y = 5$

14. $y = -2$

15. $x = -3$

16. $x = 1.7$

17. $3y = -18$

18. $4x = 2.4$

19. $-3x + 21 = 0$

20. $2y + 5 = 0$

Write each equation in slope-intercept form. Find the slope and *y*-intercept, and then use them to draw the graph. See Examples 5 and 6.

21. $y = 2x - 1$

22. $y = 3x - 4$

23. $y = 5 - 4x$

24. $y = 4 - x$

25. $y = \frac{2}{3}x - 3$

26. $y = \frac{2}{5}x + 2$

27. $x + y = 5$

28. $x - 2y = 6$

29. $x + 5y = 10$

30. $4x + y = 0$

31. $4x + y + 3 = 0$

32. $2x + 7y + 7 = 0$

33. $2y - 8 = 0$

34. $3y - 9 = 0$

35. $2x = 3y$

36. $4x = y$

37. $3x + 9 = 0$

38. $4x + 7 = 0$

39. $5x - 6y = 18$

40. $3x + 6 = 6y$

41. $5 - 3x = 4y$

42. $5x = 11 - 2y$

43. $6x + 4y = -8$

44. $7x + 2y = 4$

45. $6y = -6 + 3x$

46. $4x = 3y - 7$

47. $5x - 2y + 5 = 0$

48. $6x + 5y = -15$

In reference to the equation $y = mx + b$, sketch the graphs of three lines for each of the two characteristics listed below.

49. $m > 0$ and $b > 0$

50. $m < 0$ and $b > 0$

51. $m > 0$ and $b < 0$

52. $m < 0$ and $b < 0$

Find an equation in slope-intercept form for the line passing through the given point with the given slope. See Example 7.

53. $(0, 3); m = -\frac{1}{2}$

54. $(0, 2); m = \frac{1}{3}$

55. $(0, -3); m = \frac{2}{5}$

56. $(0, -6); m = \frac{4}{3}$

57. $(0, -5); m = 4$

58. $(0, 9); m = -1$

59. $(0, -4); m = 1$

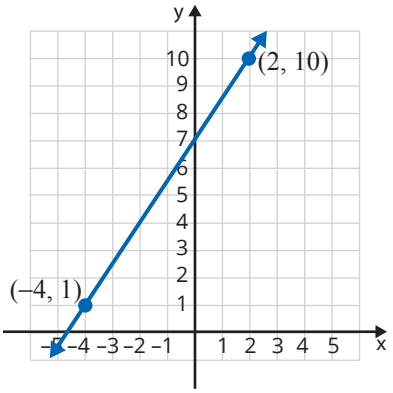
60. $(0, 6); m = -5$

61. $(0, -3); m = -\frac{5}{6}$

62. $(0, -1); m = -\frac{3}{2}$

The graph of a line is shown with two points labeled. Find **a.** the slope, **b.** the *y*-intercept (if there is one), and **c.** the equation of the line in slope-intercept form.

63.



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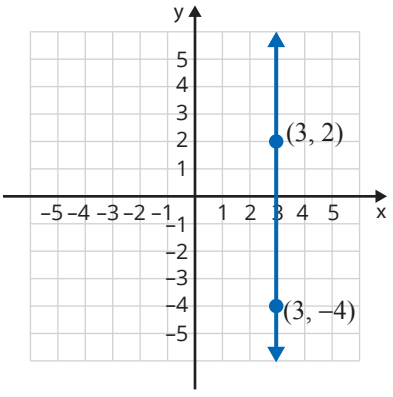

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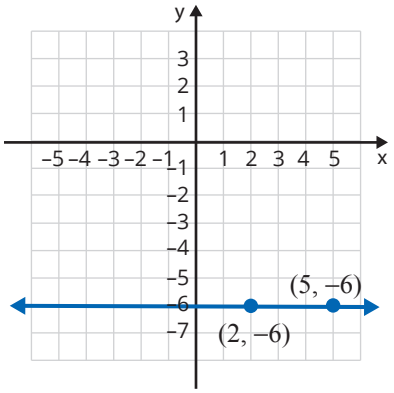

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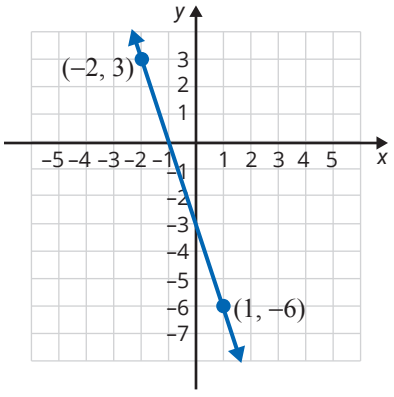

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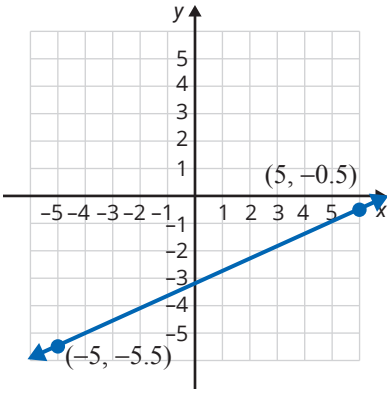

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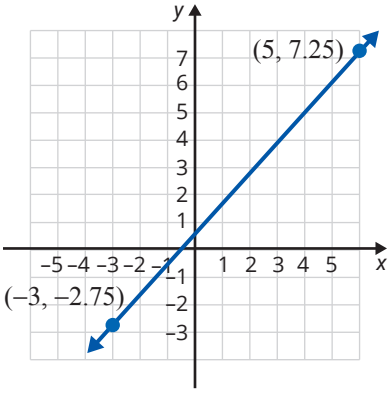

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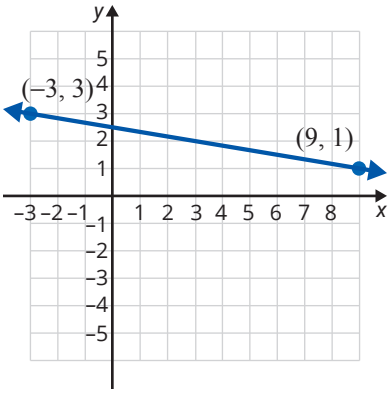

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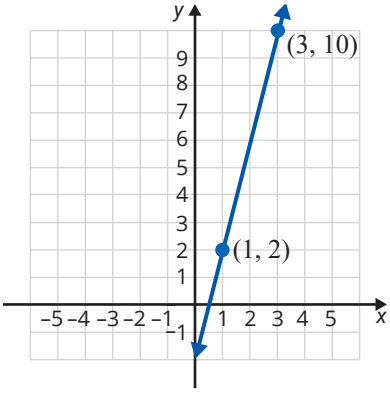


Points are said to be **collinear** if they lie on a straight line. If points are collinear, then the slope of the line through any two of them must be the same (because the line is the same line). Use this idea to determine whether or not the three points in each of the sets are collinear.

71. $\{(-1,3),(0,1),(5,-9)\}$

72. $\{(-2,-4),(0,2),(3,11)\}$

73. $\{(-2,0),(0,30),(1.5,5.25)\}$

74. $\{(-1,-7),(1,1),(2.5,7)\}$

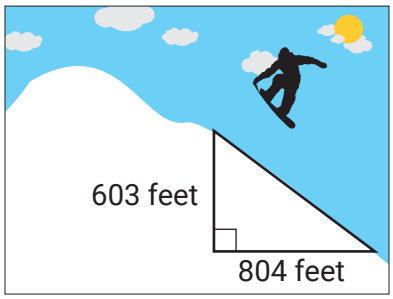
75. $\left\{\left(\frac{2}{3},\frac{1}{2}\right),\left(0,\frac{5}{6}\right),\left(-\frac{3}{4},\frac{29}{24}\right)\right\}$

76. $\left\{\left(\frac{3}{2},-\frac{1}{3}\right),\left(0,\frac{1}{6}\right),\left(-\frac{1}{2},\frac{3}{4}\right)\right\}$

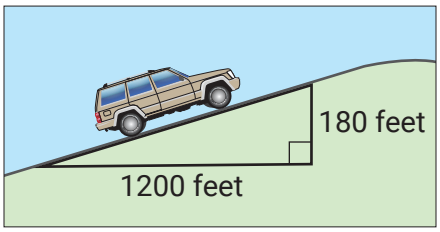
Applications

Solve.

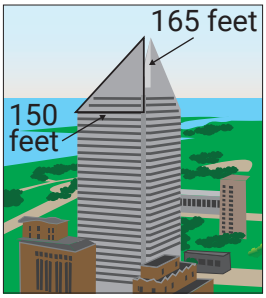
77. Find the slope of the ski slope.



78. Find the slope of the road.

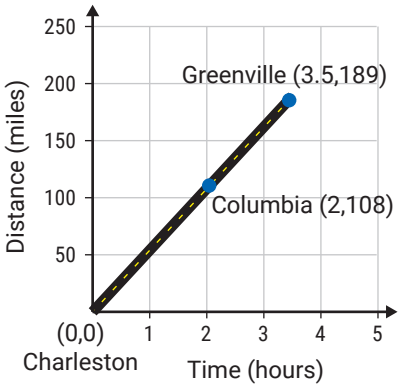


79. Find the slope of the roof of the skyscraper.

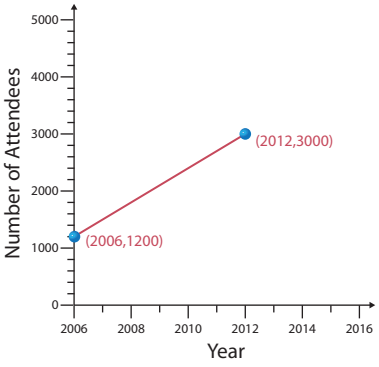


80. Find the slope of the larger sail on the sailboat.

81. A car travels from Charleston to Greenville. Its distance related to time traveled is given on the following graph. Find the average speed of the car in miles per hour from Columbia to Greenville.



82. The attendance at Smithville's Spring Festival has been increasing steadily as shown in the graph. Find the average increase in attendees per year. How many people do you predict will attend the festival in 2016?

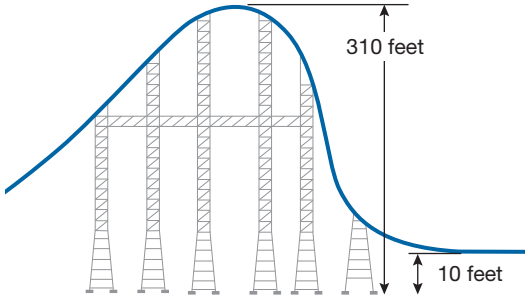


83. John bought his new car for $35,000 in the year 2014. He knows that the value of his car has depreciated linearly. If the value of the car in 2017 was $23,000, what was the annual rate of depreciation of his car? Show this information on a graph. (When graphing, use years as the x-coordinates and the corresponding values of the car as the y-coordinates.)

84. The number of people in the United States with mobile cellular phones was about 198 million in 2011 and about 232 million in 2016. If the growth in the usage of mobile cellular phones was linear, what was the approximate rate of growth per year from 2011 to 2016. Show this information on a graph. (When graphing, use years as the x-coordinates and the corresponding numbers of users as the y-coordinates.) [1]

1 Source: https://www.statista.com/statistics/231612/number-of-cell-phone-users-usa/

85. The Millennium Force roller coaster at Cedar Point in Sandusky, Ohio, has been voted the Best Steel Coaster by Golden Ticket seven times since 2001. The Millennium Force is known for its steep slope on the first hill and speeds up to 93 miles per hour. The descent from the first hill starts at 310 feet above the ground and ends 10 feet above the ground. The descent runs approximately 53 feet. Find the slope of this hill to the nearest hundredth. (**Hint:** The slope is running downhill. Consider how that affects the slope.)



86. The grade, or slope, of a road is commonly given as a percentage. The grade can be determined by multiplying the slope by 100. Calculate the slope of each road or track and then determine its grade. Round each percent to the nearest hundredth if necessary.

a. A road increases in height 5 feet for every 120 feet of run.

b. The railway line with the steepest grade that does not run on a track system is the Lisbon tramway network in Portugal which has a section that increases 5 feet in height for every 37 feet of run.

c. A road on the Route des Crêtes (Route of the Ridges) in France has an elevation that increases in height 450 feet over 1500 feet.

87. Jared sells paintings at an open-air market. He starts his work day with $30 and sells each painting for $15. Jared wants to create a linear equation to model this situation where y is the amount of money Jared has at the end of the work day and x is the number of paintings sold.

a. The slope, or rate of change, is the increase in the amount of money Jared makes when he sells a painting. Determine the value of the slope and list the units for both variables.

b. The y-coordinate of the y-intercept of this equation is the amount of money Jared has before he sells any paintings. What is the y-intercept?

c. Write a linear equation in slope-intercept form to model this situation using the answers from Parts **a.** and **b.**

d. Graph the equation from Part **c.**

e. Are there any solutions to the equation which do not make sense in the context of the problem? Explain why.

f. Use the graph to determine the amount of money Jared will have after selling 4 paintings.

88. The given table shows the estimated number of internet users from 2010 to 2014. The number of users for each year is shown in millions.

a. Plot these points on a graph.

b. Connect the points with line segments.

c. Find the slope of each line segment.

d. Interpret each slope as a rate of change.

Year	Internet Users (in millions)
2010	222
2011	218
2012	250
2013	267
2014	279

Source: https://www.statista.com/statistics/276445/number-of-internet-users-in-the-united-states/

89. The following table shows the urban growth from 1850 to 2000 in New York, NY.

a. Plot these points on a graph.

b. Connect the points with line segments.

c. Find the slope of each line segment.

d. Interpret each slope as a rate of change.

Year	Population
1850	515,547
1900	3,437,202
1950	7,891,957
2000	8,008,278

Source: U.S. Census Bureau

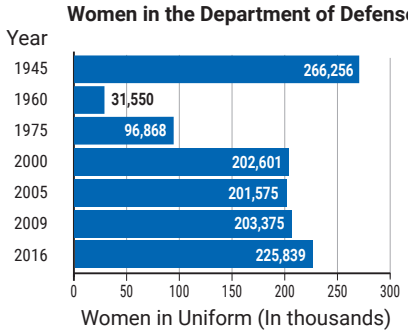
90. The following graph shows the number of female active duty military personnel over a span from 1945 to 2016. The number of women listed includes both officers and enlisted personnel from the Army, the Navy, the Marine Corps, and the Air Force.

a. Plot these points on a graph.

b. Connect the points with line segments.

c. Find the slope of each line segment.

d. Interpret each slope as a rate of change.



Source: U.S. Dept. of Defense

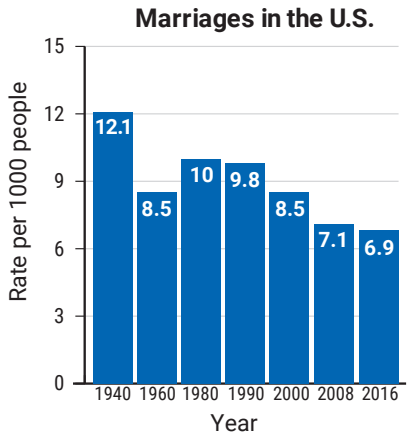
91. The following graph shows the rates of marriage per 1000 people in the U.S. over a span from 1940 to 2016.

a. Plot these points on a graph.

b. Connect the points with line segments.

c. Find the slope of each line segment.

d. Interpret each slope as a rate of change.



Source: U.S. National Center for Health Statistics

Writing & Thinking

92. **a.** Explain in your own words why the slope of a horizontal line must be 0.

b. Explain in your own words why the slope of a vertical line must be undefined.

93. The slope of a road is called a **grade**. A steep grade is cause for truck drivers to have slow speed limits in mountains. What do you think that a "grade of 12%" means? Draw a picture of a right triangle that would indicate a grade of 12%.

Collaborative Learning

94. The class should be divided into teams of 2 or 3 students. Each team will need access to a digital camera, a printer, and a ruler.

a. Take pictures of 8 things with a defined slope. (**Suggestions:** A roof, a stair railing, a beach umbrella, a crooked tree, etc. Be creative!)

b. Print each picture.

c. Use a ruler to draw a coordinate system on top of each picture. You will probably want to use increments of in. or cm, depending on the size of your picture.

d. Identify the line in each picture whose slope you are calculating and then use the coordinate systems you created to identify the coordinates of two points on each line.

e. Use the points you just found to calculate the slope of the line in each picture.

f. Share your findings with the class.

3.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. Perpendicular lines have slopes that are _____ _____ of each other.
2. The point-slope form for an equation is _____.
3. Parallel lines have the _____ slope.
4. Lines represented by the equation $Ax + By = C$ are in _____ form.
5. In the equation $y - y_1 = m(x - x_1)$, m represents the _____.
6. _____ lines are of the form $x = a$.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Given two perpendicular lines (neither of which have slope 0), we know that one has a positive slope and the other has a negative slope.
8. If Line 2 is parallel to Line 3, then the slope of Line 2 equals the slope of Line 3.
9. A line perpendicular to a horizontal line has a slope that is undefined.
10. All pairs of lines are either parallel or perpendicular.

Practice

Find **a.** the slope, **b.** a point on the line, and **c.** the graph of the line for the following equations in point-slope form.

1. $y - 1 = 2(x - 3)$
2. $y - 4 = \frac{1}{2}(x - 1)$
3. $y + 2 = -5(x)$
4. $y = -(x + 8)$
5. $y - 3 = -\frac{1}{4}(x + 2)$
6. $y + 6 = \frac{1}{3}(x - 7)$

Find an equation in standard form for the line passing through the given point with the given slope. Graph the line. See Examples 1 and 2.

7. $(-2,1);\ m = -2$
8. $(3,4);\ m = 3$
9. $(5,-2);\ m = 0$
10. $(0,0);\ m = -3$
11. $(-3,6);\ m = \frac{1}{2}$
12. $(-3,-1);\ m$ is undefined
13. $(7,10);\ m = \frac{3}{5}$
14. $(-1,-1);\ m = -\frac{1}{4}$
15. $\left(-2,\frac{1}{3}\right);\ m = \frac{2}{3}$
16. $\left(\frac{5}{2},\frac{1}{2}\right);\ m = -\frac{4}{3}$

Find an equation in slope-intercept form for the line passing through the two given points. See Example 3.

17. $(-5,2);(3,6)$

18. $(-3,4);(2,1)$

19. $(-5,1);(2,0)$

20. $(-4,-4);(3,1)$

21. $(0,2);\left(1,\frac{3}{4}\right)$

22. $\left(\frac{5}{2},0\right);\left(2,-\frac{1}{3}\right)$

23. $(2,-5);(4,-5)$

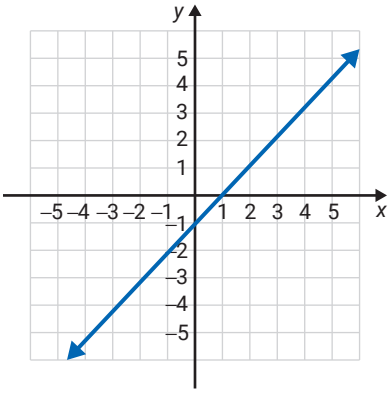
24. $(0,4);\left(1,\frac{1}{2}\right)$

25. $(-2,6);(3,1)$

26. $(8,2);(0,0)$

Find an equation in standard form for each line shown. See Example 4.

27.



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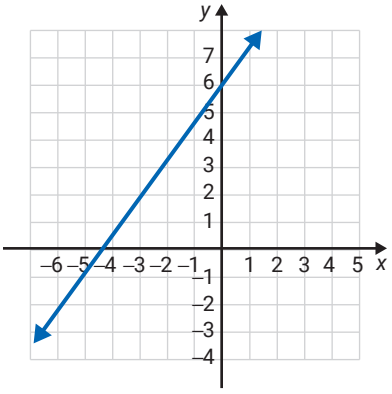

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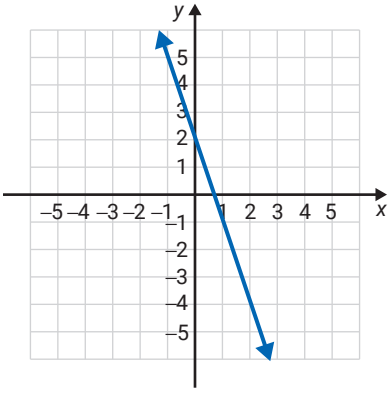

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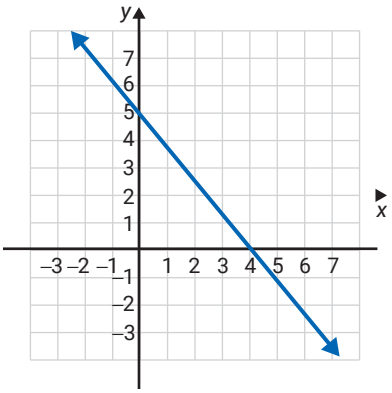

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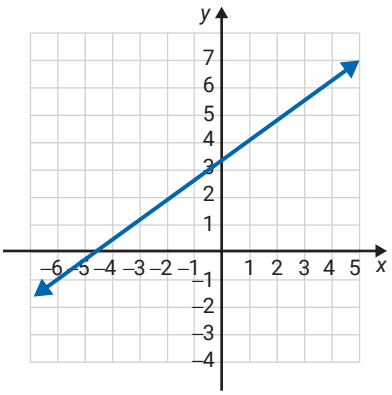

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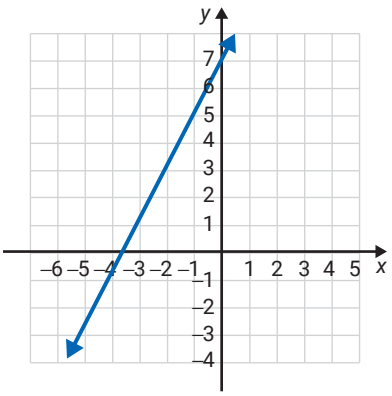

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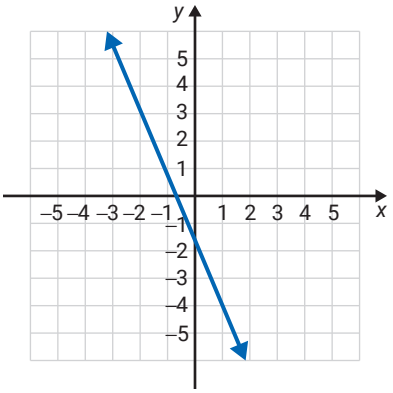

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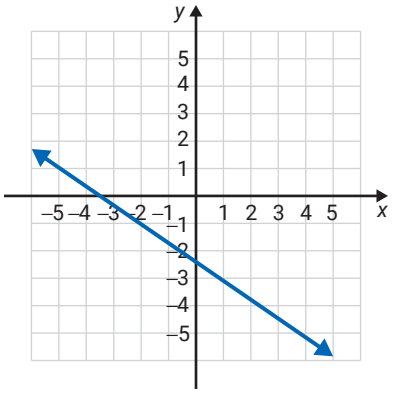


Find an equation in slope-intercept form that satisfies each set of conditions. See Examples 5 and 6.

35. Find an equation for the horizontal line through the point $(-2, 6)$.

36. Find an equation for the vertical line through the point $(-1, -4)$.

37. Write an equation for the line parallel to the x-axis and containing the point $(2, 7)$.

38. Find an equation for the line parallel to the y-axis and containing the point $(2, -4)$.

39. Find an equation for the line perpendicular to $x = 4$ and that passes through $(-1, 7)$.

40. Find an equation for the line parallel to the line $-6y = 1$ and containing the point $(-3, 2)$.

41. Write an equation for the line parallel to the line $2x - y = 4$ and containing the origin. Graph both lines.

42. Find an equation for the line parallel to $7x - 3y = 1$ and containing the point $(1, 0)$. Graph both lines.

43. Write an equation for the line parallel to $5x = 7 + y$ and through the point $(-1, -3)$. Graph both lines.

44. Write an equation for the line that contains the point $(2, 2)$ and is perpendicular to the line $4x + 3y = 4$. Graph both lines.

45. Find an equation for the line that passes through the point $(4, -1)$ and is perpendicular to the line $5x - 3y + 4 = 0$. Graph both lines.

46. Write an equation for the line that is perpendicular to $8 - 3x - 2y = 0$ and passes through the point $(-4, -2)$.

47. Write an equation for the line through the origin that is perpendicular to $3x - y = 4$.

48. Find an equation for the line that is perpendicular to $2x + y = 5$ and that passes through $(6, -1)$.

49. Write an equation for the line that is perpendicular to $2x - y = 7$ and has the same y-intercept as $x - 3y = 6$.

50. Find an equation for the line with the same y-intercept as $5x + 4y = 12$ and that is perpendicular to $3x - 2y = 4$.

51. Show that the points $A(-2, 4)$, $B(0, 0)$, $C(6, 3)$, and $D(4, 7)$ are the vertices of a rectangle. (Plot the points and show that opposite sides are parallel and that adjacent sides are perpendicular.)

52. Show that the points $A(0, -1)$, $B(3, -4)$, $C(6, 3)$, and $D(9, 0)$ are the vertices of a parallelogram. (Plot the points and show that opposite sides are parallel.)

Determine whether each pair of lines is **a.** parallel, **b.** perpendicular, or **c.** neither. Graph both lines. (**Hint:** Write the equations in slope-intercept form and then compare slopes.)

53. $\begin{cases} y = -2x + 3 \\ y = -2x - 1 \end{cases}$

54. $\begin{cases} y = 3x + 2 \\ y = -\frac{1}{3}x + 6 \end{cases}$

55. $\begin{cases} 4x + y = 4 \\ x - 4y = 8 \end{cases}$

56. $\begin{cases} 2x + 3y = 5 \\ 3x + 2y = 10 \end{cases}$

57. $\begin{cases} 2x + 2y = 9 \\ 2x - y = 6 \end{cases}$

58. $\begin{cases} 3x - 4y = 16 \\ 4x + 3y = 15 \end{cases}$

Applications

Solve.

59. The cost for an airline to fly from Raleigh, NC, to Nashville, TN, is $5000. The airline charges $100 for the one-way ticket from Raleigh to Nashville.

a. Find an equation for the profit P made by the airline on this one-way flight if they sell t tickets.

b. Use the equation found in part **a.** to determine the number of tickets that must be sold for the airline to "break even;" that is, for the profit to be equal to 0?

60. United Cellular offers a basic text plan that is $9.95 per month for the first 100 texts, and then charges $0.10 for each additional text over 100.

a. Write an equation for the total bill, b, in a month in which you used t texts (where t is at least 100).

b. Jenny's bill last month was $27.85. How many texts did she use?

61. Betsy earns 10 hours of paid time off (PTO) per month and the accumulated hours rollover each month. Two months into the current year, she has accumulated a total of 80 hours of PTO. Let y be the number of PTO hours accumulated and x be the number of months.

a. Graph the line that represents her projected PTO accumulation for the current calendar year if she does not use any PTO. (Graph should specify that x-axis is months and y-axis is hours of PTO.)

b. Write the equation in point-slope form that represents Betsy's projected PTO accumulation.

62. The price p of a college textbook increases as the number of pages n increases. In fact, the price increases \$20 for every 100 pages that are added to the textbook.

a. Assuming there are no "fixed" costs, find an equation for the price of a textbook in terms of the number of pages.

b. Use the equation found in Part **a.** to approximate the price of a 560-page textbook.

63. A NYC Taxi charges a fare of \$5.00 plus \$0.25 per eighth of a mile for a ride.

a. Find an equation for the fare f in terms of the number of miles m.

b. Use the equation found in Part **a.** to determine the cost for a 15-mile ride to JFK Airport?

64. Natalie invested some money in a simple interest savings fund. After 2 years, she earned \$120 in interest. After 5 years, she earned \$300 in interest.

a. Write two ordered pairs from the information given where x represents the time in years and y represents the amount of interest earned.

b. Find the slope of the line which contains the two ordered pairs from Part **a.**

c. Write the point-slope equation that models the situation.

d. Rewrite this equation in $y = mx + b$ form.

65. An archaeology crew finds the foundation of a house during a dig. The corners of the foundations are plotted on their grid map at the following points: $(1, 7)$, $(3, 2)$, $(9, 4)$, and $(7, 9)$.

a. Plot the points on the coordinate plane.

b. Find the slope of each side of the foundation.

c. Are any of the sides parallel? If so, which sides?

d. Are any of the sides perpendicular to each other? If so, which sides?

e. Is the foundation in the form of a geometric shape? If so, which shape

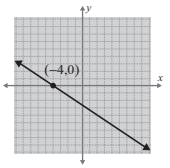
Writing & Thinking

66. Ramps for persons in wheelchairs or otherwise handicapped are now built into most buildings and walkways. (If ramps are not present in a building, then there must be elevators.) What do you think that the slope of a ramp should be for handicapped access? Look in your library or contact your local building permit office to find the recommended slope for such ramps.

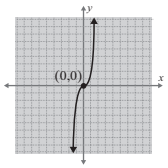
b. function; $D = (-\infty, \infty)$ $R = [-2, \infty)$ **c.** not a function; $D = [-5, 7]$ $R = [-1, 5]$ **5.** $D = (-\infty, -3) \cup (-3, \infty)$ or $x \neq -3$

6. a. 7 **b.** −8 **c.** −2 **7. a.** 4 **b.** 5 **c.** −16 **8. a.** −5 **b.** −2 **c.** −1

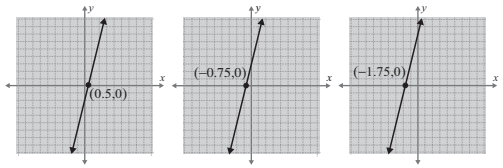
9. a.



b.



c.



3.4 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The equation $y = mx + b$ represents a linear function and $f(x) = mx + b$ is the same equation written in _____ notation.
2. The _____ line test can be used to determine if a relation is a function.
3. The set of all first coordinates in a relation is the _____, D.
4. The set of all second coordinates in a relation is the _____, R.
5. In the graph of a relation, the x-axis is called the _____ axis.
6. In the graph of a relation, the y-axis is called the _____ axis.

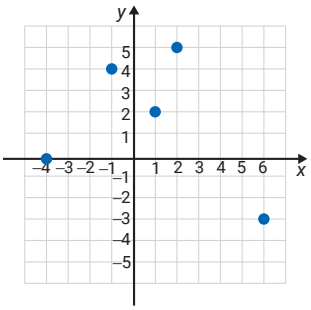
True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If the domain of a linear function is not explicitly stated, the implied domain is the set of all values of x that produce real values for y.
8. A relation is a function in which each domain element has exactly one corresponding range element.
9. In a function, the range elements can have more than one corresponding domain element.
10. If $s = \{(1, -6), (3, 5), (4, 0), (1, 2)\}$, then s is a function.

Practice

List the sets of ordered pairs that correspond to the points. State the domain and range and indicate which of the relations are also functions. See Examples 1 and 2.

1.



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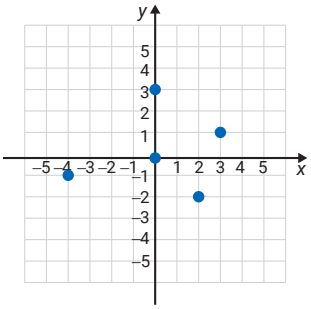

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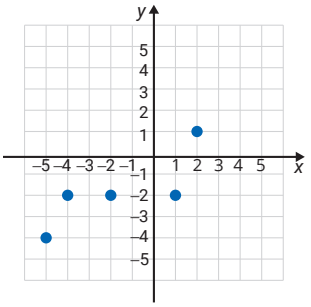

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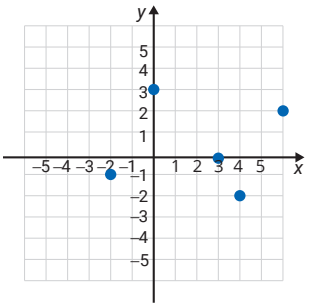

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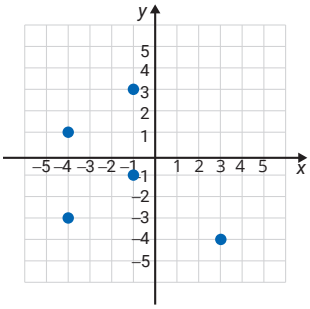

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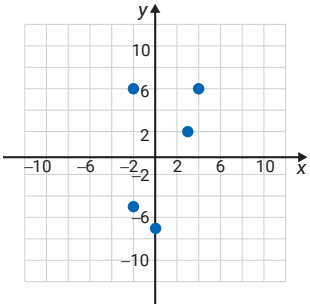

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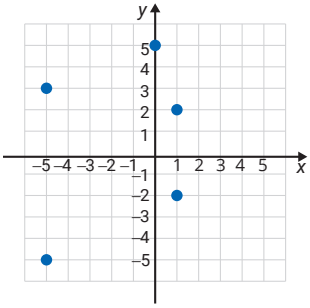

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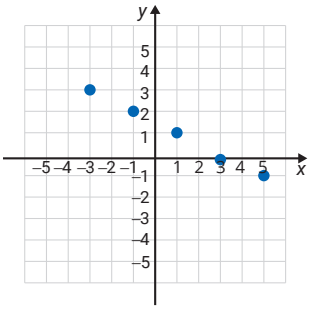


Graph the relations. State the domain and range and indicate which of the relations are functions. See Example 3.

9. $f = \{(0, 0), (1, 6), (4, -2), (-3, 5), (2, -1)\}$

10. $h = \{(1, -5), (2, -3), (-1, -3), (0, 2), (4, 3)\}$

11. $g = \{(-4, 4), (-3, 4), (1, 4), (2, 4), (3, 4)\}$

12. $f = \{(-3, -3), (0, 1), (-2, 1), (3, 1), (5, 1)\}$

13. $s = \{(0, 2), (-1, 1), (2, 4), (3, 5), (-3, 5)\}$

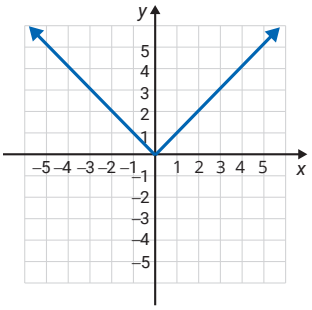
14. $t = \{(-1, -4), (0, -3), (2, -1), (4, 1), (1, 1)\}$

15. $f = \{(-1, 4), (-1, 2), (-1, 0), (-1, 6), (-1, -2)\}$

16. $g = \{(0, 0), (-2, -5), (2, 0), (4, -6), (5, 2)\}$

Use the vertical line test to determine whether or not each graph represents a function. State the domain and range using interval notation. See Example 4.

17.



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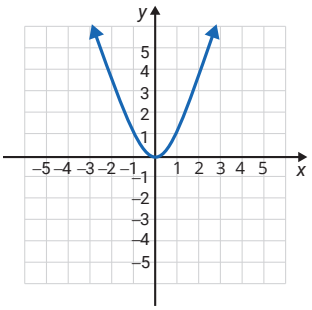

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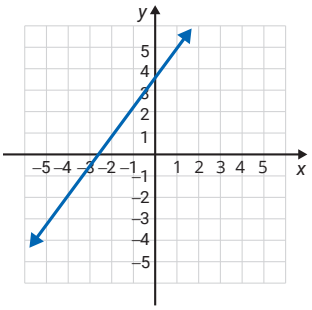

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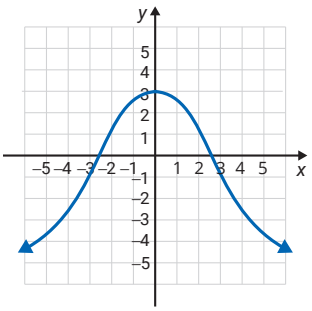

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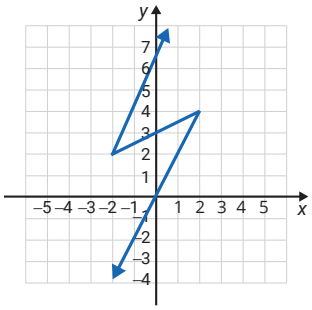

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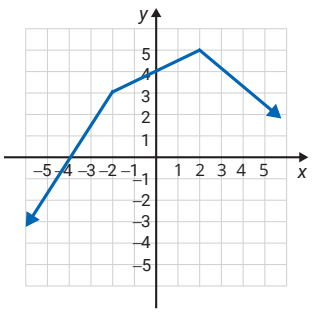

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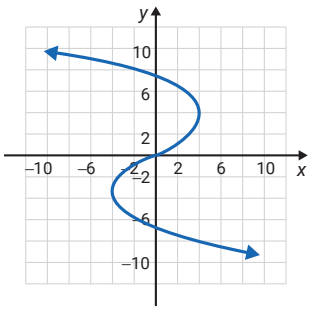

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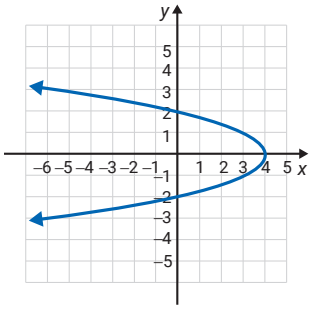

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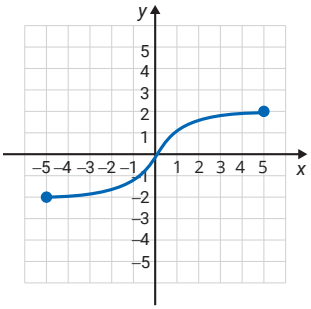

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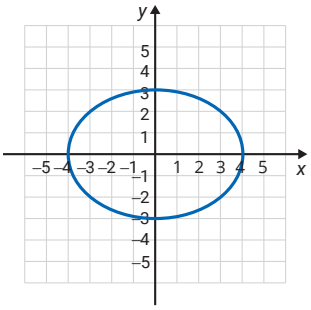

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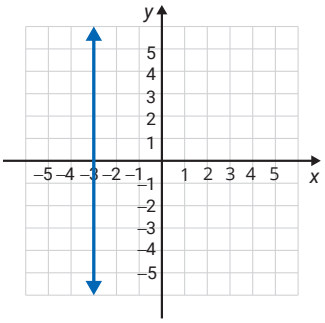

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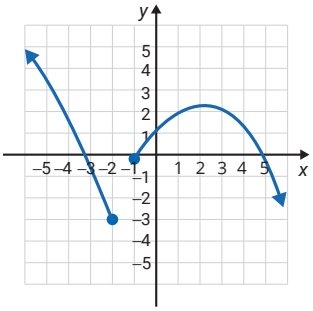


Express the function as a set of ordered pairs for the given equation and given domain. (**Hint:** Substitute each domain element for x and find the corresponding y-coordinate.)

29. $y = 3x + 1; D = \left\{-9, -\frac{1}{3}, 0, \frac{4}{3}, 2\right\}$

30. $y = -\frac{3}{4}x + 2; D = \{-4, -2, 0, 3, 4\}$

31. $y = 1 - 3x^2; D = \{-2, -1, 0, 1, 2\}$

32. $y = x^3 - 4x; D = \left\{-1, 0, \frac{1}{2}, 1, 2\right\}$

State the domains of the functions. See Example 5.

33. $y = -5x + 10$

34. $2x + y = 14$

35. $g(x) = \frac{8}{x}$

36. $h(x) = \frac{7}{3x}$

37. $y = \frac{13x^2 - 5x + 8}{x - 3}$

38. $f(x) = \frac{35}{x - 6}$

Find the values of the functions as indicated. See Examples 6 and 7.

39. $f(x) = 3x - 10$

a. $f(2)$

b. $f(-2)$

c. $f(0)$

40. $g(x) = -4x + 7$

a. $g(-3)$

b. $g(6)$

c. $g(0)$

41. $G(x) = x^2 + 5x + 6$

a. $G(-2)$

b. $G(1)$

c. $G(5)$

42. $F(x) = 6x^2 - 10$

a. $F(0)$

b. $F(-4)$

c. $F(4)$

43. $h(x) = x^3 - 8x$

a. $h(-3)$

b. $h(0)$

c. $h(3)$

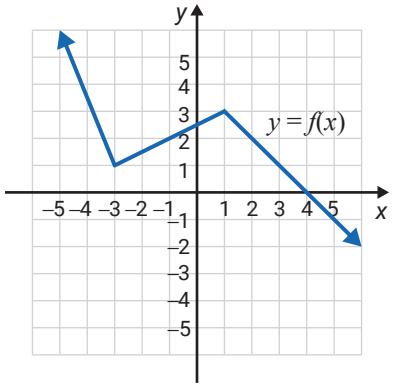
44. $P(x) = x^2 + 4x + 4$

a. $P(-2)$

b. $P(10)$

c. $P(-5)$

Using the graph of $f(x)$, find each value. See Example 8.



45. $f(1)$

46. $f(-3)$

47. $f(4)$

48. $f(-1)$

Use a graphing calculator to graph the functions. Use the CALC features to find *x*-intercepts, if any. (The value of *y* will be 0 at those points.) For absolute value functions, select the `MATH` menu, then the `NUM` menu, and then `1: abs(`. Remember to press) after entering the absolute value. See Example 9.

49. $y = 6$

50. $y = 4x$

51. $y = x + 5$

52. $y = -2x + 3$

53. $y = x^2 - 4x$

54. $y = 1 + 2x - x^2$

55. $y = -|3x|$

56. $y = |x + 2|$

57. $y = |x^2 - 3x|$

58. $y = 2x^3 - 5x^2 + 1$

59. $y = -x^3 + 3x - 1$

60. $y = x^4 - 10x^2 + 9$

Use the CALC features of the calculator to find the coordinates of any points of intersection of the graphs. (**Hint:** Item 5 on the CALC menu `5: intersect` will help in finding the point (or points) of intersection of two functions, if there is one.) In the Y = menu use both Y1 = and Y2 = to be able to graph both functions at the same time.

61. $y = 3x + 2$
$y = 4 - x$

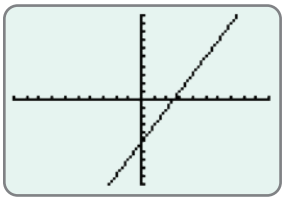
62. $y = 2 - x$
$y = x$

63. $y = 2x - 1$
$y = x^2$

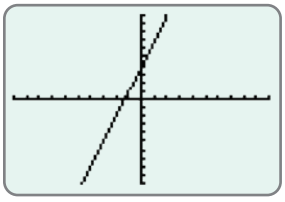
64. $y = x + 3$
$y = -x^2 + x + 7$

The calculator display shows an incorrect graph for the corresponding equation. Explain how you know, by just looking at the graph, that a mistake has been made.

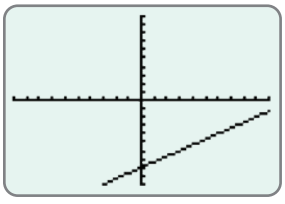
65. $y = 2x + 5$



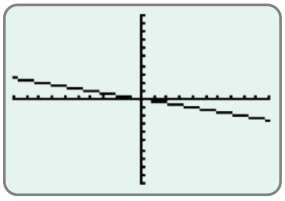
66. $y = -3x + 4$



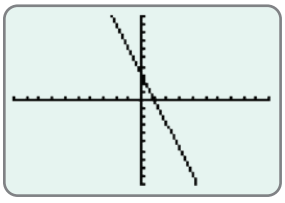
67. $y = \frac{2}{3}x - 2$



68. $y = -4x$



69. $y = -\frac{1}{3}x$



Applications

Solve.

70. A nurse hangs a 1000-milliliter IV bag which is set to drip at 120 milliliters per hour. Create a model of this situation to represent the amount of IV solution left in the bag after x hours.

a. The y-intercept is the amount of IV solution in the bag initially (time = 0). What is the y-intercept?

b. The slope is equal to the rate that the IV solution is dispensed per hour. What is the slope? (**Hint:** Consider whether the amount of IV solution in the bag is increasing or decreasing and how this would affect the slope.)

c. Write an equation in slope-intercept form to model this situation.

d. Write the equation from Part **c.** using function notation.

e. State the domain and range of the function.

f. State any additional restrictions that should be made on the domain for it to make sense in the context of this problem.

g. How much IV solution is left in the bag after 5 hours?

71. Ariella is a full-time sales associate at a clothing store. She earns a weekly salary of $250 and earns 15% commission on all of her sales. Create a model of this situation to represent the amount of money Ariella makes after x dollars in sales.

a. What is the y-intercept and what does the y-coordinate of the y-intercept represent?

b. What is the slope and what does this value represent?

c. Write an equation in slope-intercept form to model this situation using the answers from Parts **a.** and **b.**

d. Write the equation from Part **c.** using function notation.

e. State the domain and range of the function.

f. State any additional restrictions that should be made on the domain for it to make sense in the context of this problem.

g. How much will Ariella make if she sells $5000 worth of merchandise?

Writing & Thinking

72. Enter a variety of functions in your calculator, investigate your findings, and report these to your class. Certainly, interesting discussions will follow!

Notice that there are two different data points with 70 minutes spent studying. One of those corresponds to a grade of 75 and the other to a grade of 90. Therefore, it seems reasonable that substituting 70 for x in the equation of the regression line would result in a number between those two grades.

b. $f(110) = 0.44(110) + 47.97$
$= 96.37$

Notice that the data point for studying 110 minutes resulted in a grade of 94, which is pretty close to the grade predicted by the regression line.

c. $f(95) = 0.44(95) + 47.97$
$= 89.77$

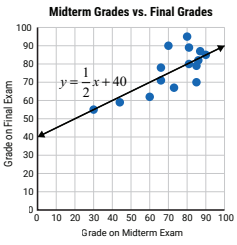
Notice that the data points for studying 90 minutes resulted in grades of 80 and 85, and the data point for studying 100 minutes resulted in a grade of 90. While there's not a data point for studying 95 minutes, there is a positive linear correlation between the amount of time spent studying and the grade earned. Therefore, we would expect that studying 95 minutes would result in a grade that is somewhere in the range of 80 to 90.

Now work margin exercise 5.

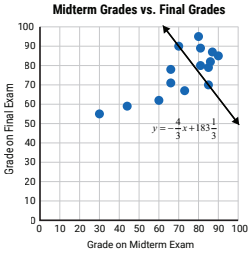
Margin Exercise Answers

1. negative correlation **2. a.** weak negative correlation **b.** strong negative correlation **c.** weak positive correlation

3. a. $y = \frac{1}{2}x + 40$



b. $y = -\frac{4}{3}x + 183\frac{1}{3}$



4. $y = 0.54x + 37.93$ **5. a.** 59.53 **b.** 70.33 **c.** 83.83

3.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The relationship between variables is called _____.
2. A regression line with a correlation coefficient of 1 is a called _____ _____ correlation.
3. If there is no linear relationship on a scatter plot, the correlation will be _____.

4. The process of modeling the _____ _____ between two variables is called linear regression.

5. If there is a strong linear relationship between two variables in a data set, the linear regression can be used to _____ values in the data set.

6. The linear equation that best fits the data is called the _____ line.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement is true. (**Note:** There may be more than one acceptable change.)

7. When a data set is presented in a scatter plot and there appears to be an upward trend in the data, we say there is a positive correlation.

8. If the correlation coefficient r of a data set is close to zero, the two variables have a strong correlation.

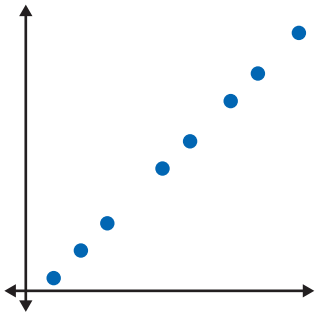
9. If the correlation coefficient r of a data set is zero, the variables are perfectly correlated.

10. Linear regression can be used to predict some values under certain conditions.

Practice

Determine if the data set appears to have a positive correlation, a negative correlation, or no correlation. See Example 1.

1.



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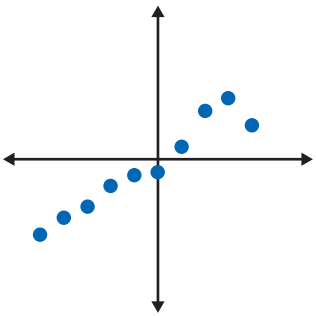

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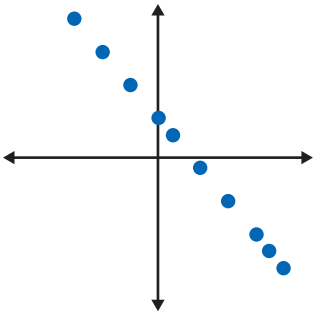

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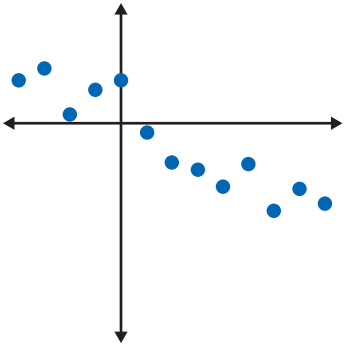

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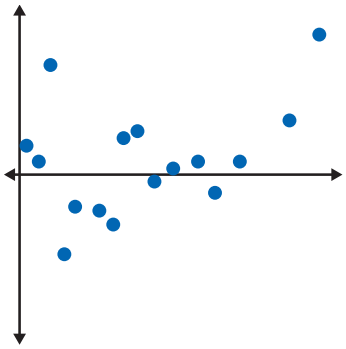

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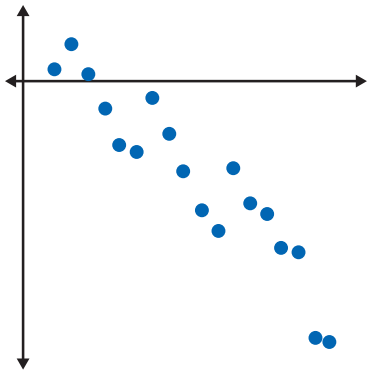

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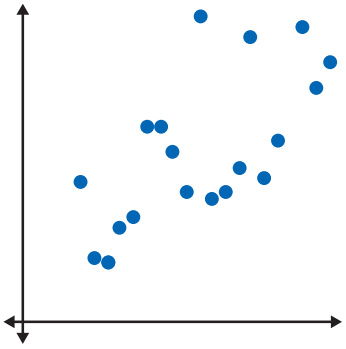

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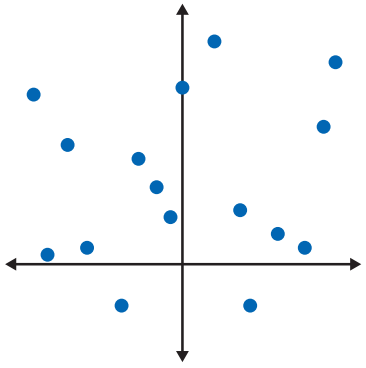


Determine whether a data set with the given correlation coefficient has a positive correlation, a negative correlation, or no correlation. If the data set has a positive or negative correlation, indicate whether it is strong or weak. See Example 2.

9. $r = 1$

10. $r = 0.9$

11. $r = -0.95$

12. $r = 0$

13. $r = -1$

14. $r = 0.1$

15. $r = -0.15$

16. $r = -0.8$

17. $r = 0.35$

18. $r = -0.2$

Solve. See Example 3.

19. Use the given points to determine the point-slope form of the equation that represents the data set for the discount in a clothing store versus the number of customers in this store per day. Graph both lines on the scatter plot of the data set.

Discount vs. Number of Customers

Discount (%)	Number of Customers
5	51
10	59
15	68
20	78
25	101

a. (5, 51) and (10, 59)

b. (15, 68) and (25, 101)

20. Use the given points to determine the point-slope form of the equation that represents the data set for outside temperature versus the number of scoops sold at an ice-cream stand during the morning. Graph both lines on the scatter plot of the data set.

Temperature vs. Number of Ice-Cream Scoops Sold

Temperature (in °F)	Number of Scoops Sold
78	24
80	22
64	12
67	11
59	9
69	13
75	19

a. (67, 11) and (59, 9)

b. (80, 22) and (64, 12)

21. Use the given points to determine the point-slope form of the equation that represents the data set for the price of a used car versus the number of calls the seller receives from potential buyers. Graph both lines on the scatter plot of the data set.

Price vs. Number of Calls Received

Price (in thousand dollars)	Number of Calls Received
14.7	10
15.6	10
19	7
20.6	8
23.1	6
24.1	4
27.5	4
29.6	3

a. (15.6, 10) and (29.6, 3)

b. (23.1, 6) and (24.1, 4)

22. Use the given points to determine the point-slope form of the equation that represents the data set for the number of cars of a particular brand sold in several cities in 2017 and 2018. Graph both lines on the scatter plot of the data set.

Number of Cars Sold in 2017 vs. 2018

Number of Cars Sold in 2017	Number of Cars Sold in 2018
1248	1589
1529	1712
1188	1390
1000	1130
485	606
863	1036
609	676
528	671
750	1050
642	796

a. (1248, 1589) and (528, 671)

b. (1000, 1130) and (750, 1050)

23. Use the given points to determine the point-slope form of the equation that represents the data set for the monthly average high temperatures in Miami, Florida. Graph both lines on the scatter plot of the data set.

Month vs. Average High Temperature

Month Number	Temperature (in °C)
1	23.3
2	23.9
3	25
4	26.7
5	28.3
6	30
7	30.6
8	31.1
9	30
10	28.3
11	26.1
12	24.4

a. (4, 26.7) and (6, 30.0)

b. (7, 30.6) and (11, 26.1)

24. Use the given points to determine the point-slope form of the equation that represents the data set for the ratings of several movies on a critic website versus their positions in the list of best movies. Graph both lines on the scatter plot of the data set.

Movie Rating vs. Position in the List of Best Movies

Rating	Position in List
9.2	1
9.1	2
9	3
8.9	5
8.8	9
8.7	13
8.6	17
8.5	28
8.4	48
8.3	70
8.2	99
8.1	144
8	218

a. (8.8, 9) and (8.2, 99)

b. (9.0, 3) and (8.5, 28)

25. Use the given points to determine the point-slope form of the equation that represents the data set for the number of months that marathon runners train versus their overall place in a marathon. Graph both lines on the scatter plot of the data set.

Number of Months Training vs. Place in the Marathon

Number of Months	Overall Place
2	30
4	27
4	21
5	25
8	15
9	23
12	12
14	7
18	10
24	5
30	2
36	9
36	1
42	4
48	3

a. (4, 27) and (8, 15)

b. (36, 9) and (48, 3)

26. Use the given points to determine the point-slope form of the equation that represents the data set for the number of games played versus the number of goals scored for several soccer forwards. Graph both lines on the scatter plot of the data set.

Number of Games Played vs. Number of Goals Scored

Games	Goals
33	21
36	22
28	17
29	7
34	18
27	10
15	3
31	12
24	6
32	12
30	14
37	16
16	5
25	7
27	7
35	13

a. (28, 17) and (29, 7)

b. (24, 6) and (32, 12)

Use the following data sets and a TI-84 graphing calculator to find the linear regression equation. Round all values to two decimal places. See Example 4.

27. Number of Cars Sold in 2017 vs. 2018

Number of Cars Sold in 2017	Number of Cars Sold in 2018
1248	1589
1529	1712
1188	1390
1000	1130
485	606
863	1036
609	676
528	671
750	1050
642	796

28. Month vs. Average High Temperature

Month Number	Temperature (in °C)
1	23.3
2	23.9
3	25
4	26.7
5	28.3
6	30
7	30.6
8	31.1
9	30
10	28.3
11	26.1
12	24.4

29. Movie Rating vs. Position in the List of Best Movies

Rating	Position
9.2	1
9.1	2
9	3
8.9	5
8.8	9
8.7	13
8.6	17
8.5	28
8.4	48
8.3	70
8.2	99
8.1	144
8	218

30. Number of Months Training vs. Place in the Marathon

Number of Months	Overall Place
2	30
4	27
4	21
5	25
8	15
9	23
12	12
14	7
18	10
24	5
30	2
36	9
36	1
42	4
48	3

Solve. See Example 5.

31. Use the provided regression line, $y = 2.38x + 35.7$, to predict the number of customers in the clothing shop for the given discount (in percent form).

a. 0%

b. 20%

c. 40%

32. Use the provided regression line, $y = 0.73x - 35.42$, to predict the number of scoops sold at the ice-cream stand for the given temperature.

a. 50 °F

b. 60 °F

c. 75 °F

33. Use the provided regression line, $y = -0.49x + 17.22$, to predict the number of calls the seller will receive for the given car price.

a. 25 thousand dollars

b. 30 thousand dollars

c. 35 thousand dollars

34. Use the provided regression line, $y = 1.11x + 84.66$, to predict the number of cars sold in a city in 2018 for the given number of cars sold in 2017.

a. 800 cars

b. 1200 cars

c. 1500 cars

35. Use the provided regression line, $y = -0.54x + 23.47$, to predict the place of a marathon runner for the given number of months spent training.

a. 10 months

b. 15 months

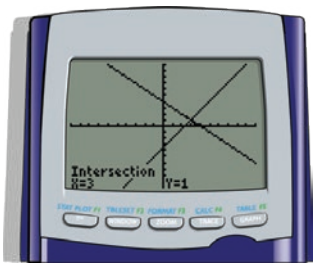
c. 25 months

36. Use the provided regression line, $y = 0.73x - 9.10$, to predict the number of goals scored by a striker for the given number of games played.

a. 20 games

b. 30 games

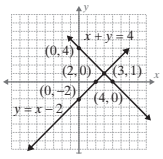
c. 40 games

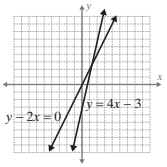


We see that the solution is $x = 3$ and $y = 1$.

(**Note:** In this case the solution shown is exact. In many cases the solution shown will be only an estimate. Thus, even with a calculator, the graphing method is limited. Also, **if the lines are parallel (an inconsistent system), the calculator will give an error message when you try to find the intersection point**.)

Margin exercise answers

1. 

2. 

3. $\left(\frac{3}{2}, 3\right)$ **4.** No solution **5.** $(2, -3)$

6. Infinite number of solutions: $(x, -2x + 5)$ or $\left(\frac{-1}{2}y + \frac{5}{2}, y\right)$ **7.** 30 kg of the 20% alloy and 10 kg of the 60% alloy **8.** \$35,000 in the savings account at 6% and \$15,000 in the stock at 10%.

9. 6 hours at job A and 14 hours at job B.

3.6 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. Two or more linear equations considered at one time are said to form a _____ of equations.
2. A system of linear equations is _____ if it has one or more solutions.
3. A system of linear equations is _____ if it has no solutions.
4. Two equations are _____ if the graphs of the equations are the same line.
5. Two equations are _____ if the graphs of the equations are different lines.
6. When a system of linear equations by substitution, substituting the value of a variable into one of the original equations to find the value of the other variable is called _____ substitution.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A system of equations with graphs that are parallel lines has exactly one solution.

8. A system of equations with graphs that intersect at one point has exactly one solution.

9. The method of substitution reduces the problem from one of solving two equations in two variables to solving one equation in one variable.

10. When using the method of addition, the solution only needs to be checked in one of the original equations.

Practice

Solve by graphing. See Examples 1 and 2.

1. $\begin{cases} x+y=5 \\ x-4y=5 \end{cases}$

2. $\begin{cases} 3x-y=6 \\ 2x+y=-1 \end{cases}$

3. $\begin{cases} 2x-y=8 \\ y=2x \end{cases}$

4. $\begin{cases} y=\frac{5}{6}x+1 \\ x-2y=2 \end{cases}$

5. $\begin{cases} 5x+2y=21 \\ x=y \end{cases}$

6. $\begin{cases} 2x+3y=4 \\ 4x-y=1 \end{cases}$

7. $\begin{cases} 2x+y+1=0 \\ 3x+4y-1=0 \end{cases}$

8. $\begin{cases} 4x-2y=10 \\ -6x+3y=-15 \end{cases}$

9. $\begin{cases} x-2y=11 \\ 2x-3y=18 \end{cases}$

10. $\begin{cases} 4x+3y+7=0 \\ 5x=2y-3 \end{cases}$

Use the substitution method or the addition method to solve the systems of linear equations. State whether each system is consistent, inconsistent, or dependent.

11. $\begin{cases} x+4y=6 \\ 2x+y=5 \end{cases}$

12. $\begin{cases} 2x+y=0 \\ x-2y=-10 \end{cases}$

13. $\begin{cases} 5x-y=-2 \\ x+2y=-7 \end{cases}$

14. $\begin{cases} 7x-y=18 \\ x+2y=9 \end{cases}$

15. $\begin{cases} x+2y=3 \\ 4x+8y=8 \end{cases}$

16. $\begin{cases} 2x+3y=3 \\ x+4y=4 \end{cases}$

17. $\begin{cases} 6x+2y=16 \\ 3x+y=8 \end{cases}$

18. $\begin{cases} 4x-y=18 \\ 3x+5y=2 \end{cases}$

19. $\begin{cases} y=3x+3 \\ y=-2x+8 \end{cases}$

20. $\begin{cases} x=-7+4y \\ 2x=8y-14 \end{cases}$

21. $\begin{cases} 2x+y=4 \\ 4x+5y=11 \end{cases}$

22. $\begin{cases} 2x-3y=18 \\ 5x+4y=-1 \end{cases}$

23. $\begin{cases} 3x+4y=6 \\ x-8y=9 \end{cases}$

24. $\begin{cases} 3x+5y=3 \\ 9x-y=-7 \end{cases}$

25. $\begin{cases} 2x=5y-1 \\ 4x-10y=0 \end{cases}$

26. $\begin{cases} 6x+2y=5 \\ 2x+y=1 \end{cases}$

27. $\begin{cases} 4x+12y=5 \\ 5x-6y=1 \end{cases}$

28. $\begin{cases} 3x+y=4 \\ 9x+3y=12 \end{cases}$

29. $\begin{cases} x+y=7 \\ 2x+3y=16 \end{cases}$

30. $\begin{cases} 5x-7y=8 \\ 3x+11y=-12 \end{cases}$

31. $\begin{cases} 6x-y=15 \\ 1.2x-0.2y=3 \end{cases}$

32. $\begin{cases} 3x+y=14 \\ 0.1x-0.2y=1.4 \end{cases}$

33. $\begin{cases} x+y=12 \\ 0.05x+0.25y=1.6 \end{cases}$

34. $\begin{cases} x+y=20 \\ 0.1x+2.5y=3.8 \end{cases}$

35. $\begin{cases} 0.6x+0.5y=5.9 \\ 0.8x+0.4y=6 \end{cases}$

36. $\begin{cases} 0.5x+0.2y=7 \\ 1.5x+0.6y=2 \end{cases}$

Applications

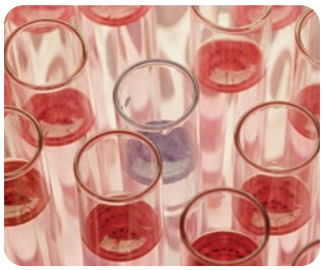
Solve.

37. How many liters each of a 12% iodine solution and a 30% iodine solution must be used to produce a total mixture of 90 liters of a 22% iodine solution?

38. A meat market has ground beef that is 40% fat and extra lean ground beef that is only 15% fat. How many pounds of each (ground beef and extra lean) must be ground together to get a total of 50 pounds of "lean" ground beef that is 25% fat?

39. A dairy needs 360 gallons of milk containing 4% butterfat. How many gallons each of milk containing 5% butterfat and milk containing 2% butterfat must be used to obtain the desired 360 gallons?

40. A druggist has two solutions of alcohol. One is 25% alcohol. The other is 45% alcohol. He wants to mix these two solutions to get 36 ounces that will be 30% alcohol. How many ounces of each of these two solutions should he mix together?



41. Investing: Pam inherited $124,000 from her Uncle Harold. She invested a portion in bonds and the remainder in a long-term certificate account. The amount invested in bonds was $24,000 less than 3 times the amount invested in certificates. How much was invested in bonds and how much in certificates?

42. Sang has invested $48,000, part at 6% and the rest in a higher risk investment at 10%. How much did she invest at each rate to receive $4000 in interest after one year?

43. An investor bought 500 shares of stock, some at $3.50 per share and some at $6.00 per share. If the total cost was $2187.50, how many shares of each stock did the investor buy?

44. Inez has 20 coins consisting of dimes and quarters. How many of each type does she have if all together she has $4.10?

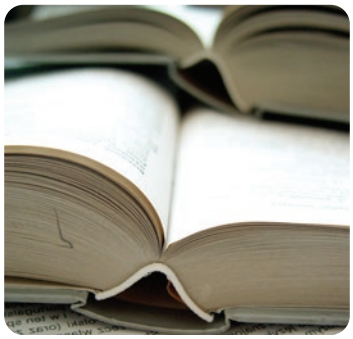
45. A confectioner is going to mix candy worth $3.90 per pound with candy worth $2.50 per pound to obtain 70 pounds of candy worth $3.30 per pound. How many pounds of each kind should she use?

46. The postal service charges 42¢ for letters that weigh 1 ounce or less and 17¢ more for letters that weigh between 1 and 2 ounces. Jeff, testing his father's math skills, gave his father $42.10 and asked him to purchase 80 stamps for his stamp collection, some 42¢ stamps and some 59¢ stamps. How many of each type of stamp did his dad buy?

47. Mike wants to mix two kinds of nuts to be eaten at a party he and his fraternity brothers are hosting tonight. One kind sells for 70 cents per pound, and the other sells for $1.30 per pound. He wants to mix a total of 20 pounds and pay a total of 82 cents per pound. How many pounds of each kind should he use in the mix?

48. A manufacturing plant is going to use two different stamping machines to complete an order of 975 units. One produces 100 units per hour, while the other produces 75 units per hour. How long must each machine operate to complete the order if, during the process, the faster machine is shut down for 2.5 hours for repairs?

49. The bookstore can buy a popular book as a paperback or a hardback. A hardback book costs $3.50 more than the paperback book. What is the cost of each if 90 paperback books cost the same as 55 hardback books?



50. In an election, the winner received 430 votes more than twice as many votes as the loser. If there was a total of 2290 votes cast, how many did each candidate receive?

51. A bill was defeated in the house of representatives after 25 more people voted against it than voted in favor of it. If one-tenth of those voting against the bill had voted in favor of it, then 21 more people would have voted in favor of it than against it. How many legislators voted in favor of the bill?

52. A private jet flies the same distance in 6 hours that a commercial jet flies in 2.5 hours. If the speed of the commercial jet was 75 mph less than three times the speed of the private jet, find the speed of each jet.

53. A manufacturer produces two new action figures, Ferocious Frank and Mighty Marcel. Ferocious Frank takes 4 hours to produce and costs $8 each. Mighty Marcel takes 3 hours to produce and costs $7 each. If the manufacturer allots a total of 5800 hours and $12,600 for production each week, how many of each model will be produced?

54. A car parts company has begun manufacturing two new products. One requires 2.5 hours of labor, 3 pounds of raw materials, and costs \$42.40 each to produce. The second product requires 4 hours of labor, 4 pounds of raw materials, and costs \$64 each to produce. Find the cost of labor per hour and the cost of raw materials per pound.

55. A furniture shop refinishes chairs. Employees use two methods to refinish a chair. Method I takes 1 hour, and the material costs \$6. Method II takes an hour and a half, and the material costs \$3. Next week, they plan to spend 144 hours in labor and \$600 in material refinishing chairs. How many chairs should they plan to refinish by each method?

56. A large feed lot uses two feed supplements, Ration I and Ration II. Each pound of Ration I contains 4 units of protein and 2 units of carbohydrates. Each pound of Ration II contains 3 units of protein and 6 units of carbohydrates. If the dietary requirement calls for 42 units of protein and 30 units of carbohydrates, how many pounds of each ration should be used to satisfy the requirements?

57. Two years ago, Sue was half as old as Pat. Eight years from now, Sue will be two-thirds as old as Pat. How old is each of them now?

58. George is 8 years older than his brother Kurt. Four years from now, he will be twice as old as Kurt. How old is each brother at this time?

In Exercises 59–66, use a graphing calculator and the `CALC` and `5:intersect` commands to find the solutions to the given systems of linear equations. If necessary, round values to four decimal places. (Remember to solve each equation for y. Use both `Y1` and `Y2` in the Y= menu.)

59. $\begin{cases} 2x+y=3 \\ x-y=5 \end{cases}$

60. $\begin{cases} 3x+y=6 \\ 2x+y=-1 \end{cases}$

61. $\begin{cases} 8x-2y=8 \\ y=-2x \end{cases}$

62. $\begin{cases} x+y=-5 \\ 4x-y=5 \end{cases}$

63. $\begin{cases} x-3y=6 \\ -2x+y=-1 \end{cases}$

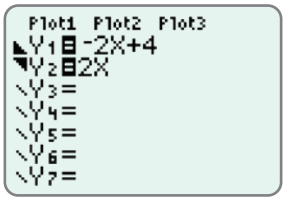
64. $\begin{cases} x+\frac{1}{2}y=0 \\ 6x-y=3 \end{cases}$

65. $\begin{cases} 2x+3y=2 \\ x+2y=-3 \end{cases}$

66. $\begin{cases} x-3y=5 \\ 2x+3y=4 \end{cases}$

Writing & Thinking

67. Explain, in your own words, why the answer to a consistent system of linear equations is written as an ordered pair.



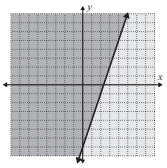
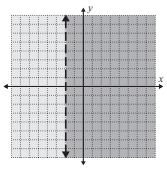
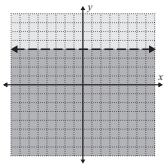
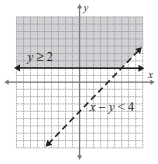
Steps 2 and 3: Press the Y= key and enter both functions and the corresponding symbols as they appear here.

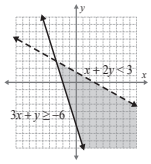
Remember: To shade your graphs, position the cursor over the slash next to Y1 (or Y2) and hit GRAPH repeatedly until the appropriate shading is displayed.

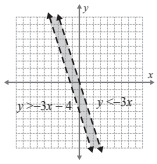
Step 4: Press ENTER. The display should appear as follows. The solution is the cross-hatched region and the points on the line $2x - y = 0$.

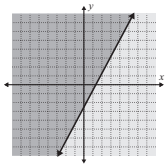
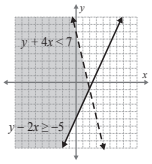
Now work margin exercise 8.

Margin Exercise Answers

1. 
2. 
3. 
4. 

5. 

6. 

7. 
8. 

3.7 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. To determine which half-plane is a solution of a linear inequality (and therefore should be shaded), _____ any point clearly on one side of the boundary line.

2. If a point is tested on one side of the boundary line and it _____ the inequality, shade that side of the boundary line. The shaded region is the solution set.

3. If a boundary line is not included in the solution set, the solution is a/an _____ half-plane.

4. A straight line that separates two half-planes is called a _____ line.

5. If the boundary lines are parallel, there are ___ possible types of solutions.

6. The solution set of a system of two linear inequalities consists of the points in the _____ of the two half-planes and portions of the boundary lines.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A solid boundary line indicates that the points on that line are included in the solution.

8. If the solution set is an open half-plane, then the boundary line is included in the solution.

9. When boundary lines are parallel, the system of linear inequalities has no solution.

10. Half-planes are the graphs of linear inequalities.

Practice

Graph the solution set of each of the linear inequalities. See Examples 1 through 4.

1. $x + y \le 7$

2. $x - y > -2$

3. $x - y > 4$

4. $x + y \le 6$

5. $y < 4x$

6. $y < -2x$

7. $y \ge -3x$

8. $y > x$

9. $x - 2y > 8$

10. $x + 3y \le 3$

11. $4x + y \ge 2$

12. $5x - y < 4$

13. $y \le 5 - 3x$

14. $y \ge 8 - 2x$

15. $2y - x \le 0$

16. $x + y > 0$

17. $x + 4 \ge 0$

18. $x - 5 \le 0$

19. $y \ge -2$

20. $y + 3 < 0$

21. $4x < -3y + 9$

22. $3x < 2y - 4$

23. $3y > 4x + 6$

24. $5x < 2y - 6$

25. $x + 3y < 7$

26. $3x + 4y > 11$

27. $\frac{1}{2}x - y > 1$

28. $\frac{1}{3}x + y \ge 3$

29. $\frac{2}{3}x + y \ge 4$

30. $2x - \frac{4}{3}y > 8$

Solve the system of two linear inequalities graphically. See Examples 4 through 6.

31. $\begin{cases} y > 2 \\ x \ge -3 \end{cases}$

32. $\begin{cases} 2x + 5 < 0 \\ y \ge 2 \end{cases}$

33. $\begin{cases} x < 3 \\ y > -x + 2 \end{cases}$

34. $\begin{cases} y \le -5 \\ y \ge x - 5 \end{cases}$

35. $\begin{cases} x \le 3 \\ 2x + y > 7 \end{cases}$

36. $\begin{cases} 2x - y > 4 \\ y < -1 \end{cases}$

37. $\begin{cases} x - 3y \le 3 \\ x < 5 \end{cases}$

38. $\begin{cases} 3x - 2y \ge 8 \\ y \ge 0 \end{cases}$

39. $\begin{cases} x - y \ge 0 \\ 3x - 2y \ge 4 \end{cases}$

40. $\begin{cases} y \geq x-2 \\ x+y \geq -2 \end{cases}$

41. $\begin{cases} 3x+y \leq 10 \\ 5x-y \geq 6 \end{cases}$

42. $\begin{cases} y > 3x+1 \\ -3x+y < -1 \end{cases}$

43. $\begin{cases} 3x+4y \geq -7 \\ y < 2x+1 \end{cases}$

44. $\begin{cases} 2x-3y \geq 0 \\ 8x-3y < 36 \end{cases}$

45. $\begin{cases} x+y < 4 \\ 2x-3y < 3 \end{cases}$

46. $\begin{cases} 2x+3y < 12 \\ 3x+2y > 13 \end{cases}$

47. $\begin{cases} x+y \geq 0 \\ x-2y \geq 6 \end{cases}$

48. $\begin{cases} y \geq 2x+3 \\ y \leq x-2 \end{cases}$

49. $\begin{cases} x+3y \leq 9 \\ x-y \geq 5 \end{cases}$

50. $\begin{cases} x-y \geq -2 \\ x+2y < -1 \end{cases}$

51. $\begin{cases} y \leq x+3 \\ x-y \leq -5 \end{cases}$

52. $\begin{cases} y \geq 2x-5 \\ 3x+2y > -3 \end{cases}$

53. $\begin{cases} y \leq -2x \\ y > -2x-6 \end{cases}$

54. $\begin{cases} y > x-4 \\ y < x+2 \end{cases}$

Use a graphing calculator to graph each of the linear inequalities. See Example 7.

55. $y > \frac{1}{2}x$

56. $2x \geq -6y$

57. $x-y \leq 5$

58. $x+2y > 8$

59. $y \geq -3$

60. $y \leq -4$

61. $2x+y \leq 6$

62. $x-3y \geq 9$

63. $3x+2y \geq 12$

Use a graphing calculator to solve the systems of linear inequalities. See Example 8.

64. $\begin{cases} y \geq 0 \\ 3x-5y \leq 10 \end{cases}$

65. $\begin{cases} y \leq 0 \\ 3x+y \leq 11 \end{cases}$

66. $\begin{cases} 4x-3y \geq 6 \\ 3x-y \leq 3 \end{cases}$

67. $\begin{cases} 3x+2y \leq 15 \\ 2x+5y \geq 10 \end{cases}$

68. $\begin{cases} 3x-4y \geq -6 \\ 3x+2y \leq 12 \end{cases}$

69. $\begin{cases} 3y \leq 2x+2 \\ x+2y \leq 11 \end{cases}$

70. $\begin{cases} x+y \leq 8 \\ 3x-2y \geq -6 \end{cases}$

71. $\begin{cases} x+y \leq 7 \\ 2x-y \leq 8 \end{cases}$

72. $\begin{cases} y \leq x \\ y < 2x+1 \end{cases}$

Applications

Solve.

73. The grade for a 1-credit-hour survey class is based on an exam and a project, which are worth a maximum of 50 points each. The sum of the two scores must be at least 75 points for a student to earn a passing grade.

a. Let the amount of points earned on the exam be represented by the variable x and the amount of points earned on the project be represented by the variable y. Create a linear inequality to describe the solution set for a passing grade.

b. Graph the linear inequality from part **a.**

c. A student earns 45 points on their final exam and 22 points on their project. Plot this point on the graph. Did this student earn a passing grade?

d. Are there any points in the solution set which do not make sense for this situation?

74. A fail-safe is installed on a device with two electrical inputs. If the sum of the inputs is greater than 250 kilowatts, the fail-safe will activate and cause the machine to switch off.

a. If one electrical input is represented by the variable x and the other is represented by the variable y, create a linear inequality to describe the values that will activate the fail-safe.

b. Graph the linear inequality.

c. The device has electrical inputs of 95 kilowatts and 145 kilowatts. Plot this point on the graph. Will the fail-safe activate and switch off the device? Explain why.

75. Barbara's Bombtastic Bakery sells cookie bouquets where the price depends on the arrangement. Each completed bouquet arrangement needs to weigh less than 5 pounds for shipping purposes. The small cookies weigh 0.1 pounds and the large cookies weigh 0.3 pounds. The flower pot and Styrofoam weigh 1.2 pounds. The cost of each arrangement needs to be less than \$30. The small cookies cost \$1 each and the large cookies cost \$2 each. (The cost of the flower pot and foam are included in the cookie prices.)

a. Write two linear inequalities to describe the situation. Use the variable x to represent the number of small cookies and the variable y to represent the number of large cookies in a bouquet.

b. Graph the two linear inequalities on the same coordinate plane.

c. Describe the solution set for the situation.

d. Do any of the values in the solution set not make sense in the context of the problem? Explain why or why not.

76. Robin is planning a charity ball to raise money for her favorite charity. There are two different ticket options. The VIP option includes dinner, dancing, and cocktails for \$150 per ticket. The regular option includes dancing and cocktails for \$75 per ticket. Robin wants to make at least \$14,000 in ticket sales. The ballroom that is being used for the charity event has a maximum capacity of 150 people.

a. Write two linear inequalities to describe the situation. Let the variable x represent the number of VIP tickets sold and let the variable y represent the number of regular tickets sold.

b. Graph the two linear inequalities on the same coordinate plane.

c. Describe the solution set for the situation.

d. Can Robin reach her sales goal if she only sells tickets for the regular option? Explain why or why not.

Writing & Thinking

77. Explain in your own words how to test to determine which side of the graph of an inequality should be shaded.

78. Describe the difference between a closed and an open half-plane.

Remember that the choice of steps in dealing with exponents is yours. As long as you correctly apply the properties of exponents, the answer will be the same regardless of the order of the steps.

Summary of Properties and Rules for Exponents

If a and b are nonzero real numbers and m and n are integers:

1. The exponent 1: $a^1 = a$
2. The exponent 0: $a^0 = 1$
3. Product rule: $a^m \cdot a^n = a^{m+n}$
4. Quotient rule: $\dfrac{a^m}{a^n} = a^{m-n}$
5. Negative exponents: $a^{-n} = \dfrac{1}{a^n}$

The Power Rules

6. Power rule: $(a^m)^n = a^{mn}$
7. Power of a product: $(ab)^n = a^n b^n$
8. Power of a quotient: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

DEFINITION

Margin Exercise Answers

1. a. 32 **b.** 125 **c.** $-10x^7$ **d.** $-35a^2b^7$ **2. a.** 1 **b.** 1 **c.** 1 **d.** 1 **3. a.** x^2 **b.** 1 **c.** $3x^8$ **d.** $7x^6y^4$ **4.** a. $\dfrac{1}{x^7}$ **b.** $\dfrac{x^3}{y^3}$ **c.** $\dfrac{1}{x^4}$ **d.** $\dfrac{4}{x^3}$ **5. a.** $\dfrac{1}{x^{15}}$ **b.** $\dfrac{-27x^3}{y^9}$ **c.** $\dfrac{16b^2}{a^2}$ **6.** $\dfrac{y^9}{8x^{18}}$ **7.** $\dfrac{64}{225x^{18}y^2}$

4.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Exponents can be used to indicate repeated ____ by the same number.
2. In the expression x^5, x is the ____ and 5 is the ____.
3. To use the ____ rule, multiply powers with the same base, keep the base, and add the exponents.
4. If a is a nonzero real number, then a^0 = ____.

5. An expression is considered simplified if each base appears ____ and each base has only ____ exponents.

6. To raise a power to a power, ____ the exponents.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If a constant does not have an exponent written, it is assumed that the exponent is 0.

8. If a is a nonzero real number an n is an integer, then $a^{-n} = -a^n$.

9. Since the product rule is stated for integer exponents, the rule is also valid for 0 and negative exponents.

10. When using the quotient rule, you should subtract the smaller exponent from the larger exponent.

Practice

Use the properties of exponents to simplify the given expressions. Answers should contain only positive exponents.

1. $\left(7^2\right)\left(7^0\right)$
2. 7^{-2}
3. $3 \cdot 2^2$
4. -5^{-2}
5. $(-8)^{-2}$
6. $x^3 \cdot x^5$
7. $x^2 \cdot x^{-1}$
8. $x^{-2} \cdot x^3 \cdot x^5$
9. $y^{-3} \cdot y^{-2} \cdot y^0$
10. $\dfrac{x^{12}}{x^4}$
11. $\dfrac{x^2}{x^{-1}}$
12. $\dfrac{y^2}{y^{-5}}$
13. $\dfrac{x^3 x^5}{x^4}$
14. $\dfrac{x^0 x^3}{x^6}$
15. $\dfrac{x \cdot x^3}{x^5}$
16. $\dfrac{x^{-1} x^3}{x^{-4}}$
17. $\dfrac{x \cdot x^{-2}}{x^2 x^{-3}}$
18. $\dfrac{x^{16}}{x^{-2} x^{-8}}$
19. $\left(x^4\right)^2$
20. $\left(x^2\right)^{-2}$
21. $\left(x^0\right)^{-1}$
22. $\left(-x^3\right)^0$
23. $\left(y^0 y^{-1}\right)^5$
24. $\left(x^3 x^{-3}\right)^0$
25. $\dfrac{y^2 y^4}{y}$
26. $\dfrac{y \cdot y^4}{y}$
27. $\dfrac{x^5 x^2}{\left(x^2\right)^2}$
28. $\dfrac{x^{10} x^{-3}}{x^3 x^{-1}}$
29. $\dfrac{x^8 x^{-2}}{\left(x^2\right)^3}$
30. $\dfrac{\left(x^{-2}\right)^3}{x \cdot x^{-3}}$
31. $\dfrac{\left(y^2\right)^4}{y^{-2} y^{-1}}$
32. $\left(\dfrac{y^2 y^{-1}}{y^5 y^2}\right)^{-2}$
33. $\left(\dfrac{x^2 x^0}{x^4 x^{-1}}\right)^{-3}$
34. $\left(\dfrac{x^{-3} x^0}{x^2 x}\right)^3$
35. $\left(\dfrac{x^5 x^{-2}}{x \cdot x^{-3}}\right)^2$
36. $x^k \cdot x$

37. $x^k \cdot x^3$

38. $x^k \cdot x^{2k}$

39. $x^{3k} \cdot x^4$

40. $\dfrac{x^k}{x^2}$

41. $\dfrac{x^{2k}}{x^k}$

42. $\dfrac{x^{k+1}}{x^3}$

43. $\left(x^k\right)^2$

44. $\left(x^5\right)^k$

45. $x\left(x^2\right)^k$

46. $\dfrac{x^2x^k}{\left(x^2\right)^k}$

47. $\dfrac{x^{k+1}x^{-2}}{x^4}$

48. $\dfrac{x^{k+3}x}{x^{-2}}$

49. $\left(\dfrac{-3x^{-2}}{y^3}\right)^{-1}$

50. $\left(\dfrac{2ab^4}{3b^2}\right)^{-3}$

51. $\left(\dfrac{x^2y^{-3}}{3x^{-1}y}\right)^{-1}$

52. $\left(x^k y^m\right)^2$

53. $\left(x^{4n}y^3\right)\left(x^n y^{-k}\right)$

54. $\left(x^{k+1}y^{3k}\right)\left(x^2 y^{-k}\right)$

55. $\left(\dfrac{a^2b}{ab^{-2}}\right)\left(\dfrac{a^{-3}b}{b^{-3}}\right)$

56. $\left(\dfrac{x^2y^{-3}}{y^{-1}}\right)^2\left(\dfrac{xy^2}{2y}\right)^{-1}$

57. $\left(\dfrac{x^4y}{2}\right)^2\left(\dfrac{y^3}{x^2}\right)^{-1}$

58. $\left(\dfrac{3x}{2x^2y^2}\right)^{-1}\left(\dfrac{y^3}{2x}\right)^2$

59. $\left(\dfrac{5x^3y}{x^{-2}y^3}\right)^{-1}\left(\dfrac{4x^{-2}y^{-1}}{15xy^4}\right)^{-1}$

60. $\dfrac{\left(7x^3y^4\right)^0}{\left(2x^2y\right)\left(xy^{-3}\right)^{-1}}$

61. $\dfrac{\left(4^{-2}x^{-3}y\right)^{-1}}{\left(x^{-2}y^2\right)^3\left(5xy^{-2}\right)^{-1}}$

62. $\dfrac{\left(6x^2y\right)\left(x^{-1}y^3\right)^2}{\left(x^{-1}y\right)^2\left(3x^2y\right)^3}$

63. $\dfrac{\left(x^{-3}y^{-5}\right)^{-2}\left(x^2y^{-3}\right)^3}{\left(x^3y^{-4}\right)^2\left(x^{-1}y^{-2}\right)^{-2}}$

64. $\dfrac{(4xy)^2\left(x^{-2}y^2\right)^{-1}}{\left(3x^3y\right)^{-2}\left(2x^2y^{-2}\right)^3}$

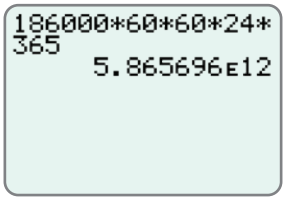
Writing & Thinking

65. Without looking at the text, show that $\left(\dfrac{x}{y}\right)^{-n} = \left(\dfrac{y}{x}\right)^n$ by using the Power Rules and the Rule for Negative Exponents.

Solution

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
365 days = 1 year

Multiplication gives the following display on your calculator.



Thus, a light-year is 5.865696×10^{12}, or 5,865,696,000,000 miles (5 trillion, 865 billion, 696 million miles).

Now work margin exercise 5.

Margin Exercise Answers

1. a. 6.39×10^{7} **b.** 2.45×10^{-6} **2. a.** 1.8×10^{-5} **b.** 1.2×10^{8} **3.** 4.816×10^{24} particles
4. a. 7E4 **b.** 6E12 **5.** 2.592E13 meters

4.2 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. In scientific notation, decimal numbers are written as a product of a number greater than or equal to _____ and less than _____, and an integer power of 10.

2. In scientific notation, there is/are _____ digit(s) to the left of the decimal point.

3. The exponent of a number written in scientific notation tells how many places the _____ _____ is to be moved and in what direction.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

4. The exponent in the number 1.4×10^{4} indicates that the decimal point should be moved 4 places to the right.

5. The exponent in the number 2.5×10^{-3} indicates that the decimal point should be moved 3 places to the right.

6. The number 3.53×10^{5} is less than 8.72×10^{-4}.

7. The number 4000 written in scientific notation is 0.4×10^{4}.

Practice

Write the following numbers in scientific notation. See Example 1.

1. 86,000

2. 927,000

3. 0.0362

4. 0.0061

5. 18,300,000

6. 376,000,000

7. 0.0000000002368

8. 1,030,000,000

9. 0.0000009

10. 0.0000000571

11. 0.0000000000328

12. 845,300,000

Write the following numbers in decimal form.

13. 4.2×10^{-2}

14. 8.35×10^{-3}

15. 7.56×10^{6}

16. 1.002×10^{-7}

17. 6.132×10^{-5}

18. 8.515×10^{8}

19. 3.067×10^{10}

20. 9.374×10^{7}

21. 7.205×10^{9}

22. 4×10^{11}

23. 6.91×10^{-6}

24. 7.408×10^{-9}

First write each of the numbers in scientific notation. Then perform the indicated operations and leave your answer in scientific notation.

25. $300 \cdot 0.00015$

26. $0.000024 \cdot 40{,}000$

27. $0.0003 \cdot 0.0000025$

28. $0.00005 \cdot 0.00013$

29. $23{,}400{,}000{,}000 \cdot 5{,}500{,}000{,}000$

30. $7{,}800{,}000{,}000 \cdot 0.00000081$

31. $\dfrac{3900}{0.003}$

32. $\dfrac{4800}{12{,}000}$

33. $\dfrac{125}{50{,}000}$

34. $\dfrac{0.0046}{230}$

35. $\dfrac{0.0000000000013}{0.000000026}$

36. $\dfrac{0.02\cdot3900}{0.013}$

37. $\dfrac{0.0084\cdot0.003}{0.21\cdot60}$

38. $\dfrac{0.005\cdot650\cdot3.3}{0.0011\cdot2500}$

39. $\dfrac{5.4\cdot0.003\cdot50}{15\cdot0.0027\cdot200}$

40. $\dfrac{0.00000000039\cdot15{,}000{,}000{,}000}{8{,}000{,}000\cdot0.0000000013}$

41. $\dfrac{\left(1.4\times10^{-2}\right)(922)}{\left(3.5\times10^{3}\right)\left(2.0\times10^{6}\right)}$

42. $\dfrac{(4300)\left(3.0\times10^{2}\right)}{\left(1.5\times10^{-3}\right)\left(860\times10^{-2}\right)}$

43. $\dfrac{(25)\left(3.75\times10^{-5}\right)}{\left(0.4\times10^{11}\right)\left(75\times10^{-7}\right)}$

44. $\dfrac{(1.1\times10^{4})(342\times10)}{(17.1\times10^{-11})(5.5\times10^{-14})}$

45. $\dfrac{\left(1.4\times10^{-7}\right)\left(7\times10^{13}\right)}{40}$

46. $\dfrac{\left(1.16\times10^{8}\right)\left(7.2\times10^{12}\right)}{\left(58\times10^{13}\right)\left(2.4\times10^{-15}\right)}$

47. $\dfrac{\left(1.95\times10^{-5}\right)(2650)\left(7.56\times10^{10}\right)}{\left(15\times10^{6}\right)\left(1.3\times10^{-13}\right)}$

48. $\dfrac{(88)\left(1.048\times10^{-5}\right)}{\left(3.2\times10^{7}\right)\left(13.75\times10^{4}\right)(2\times10)}$

Applications

Solve.

49. The mass of a hydrogen atom is approximately 0.00000000000000000000000167 grams. Write this number in scientific notation.

50. The circumference of the earth is approximately 1,580,000,000 inches. Express this circumference in scientific notation.

51. There are approximately 6×10^{13} cells in an adult human body. Express this number in decimal form.

52. The world population is approximately 7.5×10^{9} people. Write this number in decimal form.

53. Earth has a mass of about 5,980,000,000,000,000,000,000,000,000 grams. Write this number in scientific notation.

54. One year is approximately 31,500,000 seconds. Express this time in scientific notation.

55. One light-year is approximately 9.46×10^{15} meters. The distance to a certain star is 4.3 light-years. How many meters is this?

56. Light travels approximately 3×10^{10} centimeters per second. How many centimeters would this be per minute? Per hour? Express your answers in scientific notation.

57. The mass of an atom of gold is approximately 3.25×10^{-22} grams. What would be the mass of 2000 atoms of gold? Express your answer in scientific notation.

58. An ounce of gold contains 5×10^{22} atoms. All the gold ever taken out of the earth is estimated to be 3.0×10^{31} atoms. How many ounces of gold is this?

59. A scientist calculated that her experiment consumed 520,000 joules (J) of energy. She wrote the value as 52×10^{4} J on a report of the experiment. Did she write the value correctly in scientific notation? If not, what should it be?

60. A molecule of table salt weighs approximately 9.704×10^{-23} grams. What would be the weight of 4,000,000 molecules of table salt?

a. Write 4,000,000 in scientific notation.

b. Write an expression to find the weight of 4,000,000 molecules of table salt.

c. Simplify the expression from Part **b.**

d. What does the answer from Part **c.** mean? Write a complete sentence.

Use your calculator (set in scientific notation mode) to evaluate each expression. Leave the answer in scientific notation. See Example 4.

61. $90,000 \div 0.0003$

62. $0.0081 \div 9000$

63. $400 \times 175,000 + 5000 \times 3000$

64. $7000 \times 6000 + 200 \times 450,000$

65. $9.12 \times 10^{13} \div 3.04 \times 10^{-9}$

66. $1.989 \times 10^{-6} \div 6.12 \times 10^{5}$

67. $\left(4 \times 10^{6}\right)\left(1.75 \times 10^{7}\right) + \left(5.1 \times 10^{8}\right)\left(3.01 \times 10^{6}\right)$

68. $\left(2.37 \times 10^{-7}\right)\left(4 \times 10^{-9}\right) + \left(1.45 \times 10^{-8}\right)\left(5 \times 10^{-8}\right)$

69. $\dfrac{5.6 \cdot 0.003 \cdot 5000}{15 \cdot 0.0028 \cdot 20}$

70. $\dfrac{0.0006 \cdot 660 \cdot 40.4}{0.00011 \cdot 3600}$

71. $\dfrac{\left(5.6 \times 10^{7}\right)\left(3 \times 10^{13}\right)\left(5.1 \times 10^{-11}\right)}{\left(1.5 \times 10^{-10}\right)\left(2.8 \times 10^{-8}\right)\left(2 \times 10^{6}\right)}$

72. $\dfrac{\left(6 \times 10^{11}\right)\left(6.6 \times 10^{-6}\right)\left(4.04 \times 10^{7}\right)}{\left(11 \times 10^{-6}\right)\left(3.6 \times 10^{6}\right)}$

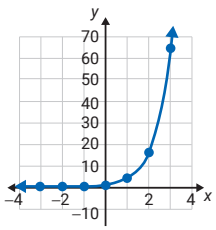
7. You may leave the `FreqList:` and the "`StoreREqEQ:`" fields blank.
8. Press ENTER or the down arrow until "`Calculate`" is highlighted.
9. Press ENTER.
10. The calculator will display the exponential regression in the form $y = ab^x$ and identify the values for a and b.

We can see that the exponential regression model computed for the number of downloads is $y = 3.99(1.25)^x$.

Now work margin exercise 6.

Margin Exercise Answers

1. a. exponential; $f(x) = 4^x$ **b.** $f(x) = 10x + 50$ **2.**



3. 12,150 people **4.** \$360.45 **5.** \$20,650.35

6. $y = 2.76(1.47)^x$

4.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. Equations that model a _____ rate of change are linear equations.
2. In an exponential function, the _____ can take on the value of any real number.
3. The _____ of an exponential function cannot equal one.
4. With exponential growth, the value of the function _____ as the value of the exponent _____.
5. With exponential decay, the value of the function _____ as the value of the exponent _____.
6. The value e is called the _____ base.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Exponential functions increase quickly at first and then grow very slowly.
8. For all exponential functions $f(x) = b^x, b < 0$.
9. The function $f(x) = 5^x$ is an examples of exponential growth.
10. Continuously compounding interest compounds once a month.

Practice

Determine whether the given situation is best described by a linear model or an exponential model. Create a function for the situation. See Example 1.

1. A petri dish contains 100 bacteria at the beginning of an experiment (hour zero). The population triples every hour.

2. A farmer has 50 sheep at the beginning of the year (month zero). Every month, she adds 3 more sheep to her flock.

3. An investor deposits 100 dollars in a bank account that pays 4 dollars in interest every month.

4. An investor puts 100 dollars in an account that pays 4% interest on the previous amount.

5. The number of people who signed for a social media platform is 100,000 and it doubles every year for 6 consecutive years.

6. The number of people signed up for a club on campus is 47 and it decreases by 3 member per year for 6 consecutive years.

Create a table of 6 values for each function and then sketch the graph. See Example 2.

7. $f(x)=3^x$

8. $f(x)=16^x$

9. $f(x)=\left(\frac{1}{2}\right)^x$

10. $f(x)=\left(\frac{1}{4}\right)^x$

11. $f(x)=2^{x-1}$

12. $f(x)=4^x+1$

13. $f(x)=\left(\frac{1}{2}\right)^x-1$

14. $f(x)=\left(\frac{1}{4}\right)^{x+1}$

15. $f(x)=0.5\left(16^{x-0.5}\right)$

16. $f(x)=0.5\left(4^x+2\right)$

For the following problems, use the first column as the independent variable *x* and the second column as the dependent variable *y*, use a TI-84 Plus graphing calculator to compute the exponential model of the data. See Example 6.

17. A biologist has measured the number of bacteria in a petri dish every hour and recorded the data in the table below.

Time (hours)	Number of Bacteria (in Thousands)
1	10.10
2.5	12.25
4	10.40
5	10.51

18. An accountant recorded the balance in an investment account.

Time (in Months)	Dollars
1	1515.07
2	1530.30
3	1545.68
4	1561.21

19. A maintenance manager recorded the volume of water in a pool that is being filled.

Time (in Hours)	Volume (in Gallons)
4	10406.04
5	10510.1
6	10615.2

20. Researcher estimated population of wild African elephants.

Years Since 1979	Number of Elephants (in Thousands)
0	1300
10	600
37	460

21. The number of hours a student spends working on math homework.

Time (in Weeks Since the Start of Semester)	Weekly Number of Study Hours.
1	10
2	12.5
3	15.625

22. The number of hours a student sleeps per night as finals week approach.

Number of weeks until final	Number of hours slept per nigh
1	5.6
2	6.25
3	7.00
4	7.85
5	8.85

Applications

Solve.

23. A biologist knows that in the laboratory, bacteria in a culture grow according to the function $y = y_0 \cdot 3^{2t}$, where y_0 is the initial number of bacteria present and t is time, measured in hours. How many bacteria will be present in a culture at the end of 5 hours if there were 250 bacteria present initially?

24. In Exercise 23, how many bacteria were present initially if at the end of 5 hours there were 2,500,000 bacteria present? Round to the nearest number of bacteria.

25. The balance (in dollars) of an investment account, t months after an initial deposit, is given by $P(t) = 1000(1.005)^t$. Determine the balance after 6 months. Round your answer to the nearest cent.

26. The population of pigeons at a city park, t weeks after the beginning of the summer, is given approximately by $P(t) = 55(1.02)^t$. Determine the number of pigeons at the park after 3 weeks. Round your answer to the nearest integer.

27. The population of bacteria in a petri dish, t minutes after the start of an experiment, is given by $P(t) = P_0(1.02)^t$. Knowing that the population after 3 minutes is equal to 1061, determine the value of the initial population to the nearest whole number. In other words, knowing that $P(3) = 1061$, find P_0.

28. A balloon has an initial volume of 1 liter. Air is slowly pumped into the balloon so that its volume increases by 5% every hour. Write a function that represents the volume of the balloon in liters, t hours after the beginning of the pumping process. Determine the volume of the balloon 5 hours into the process.

29. A car loses approximately 20% of its value every year after the initial purchase. The value of a 10,000-dollar car t years after its initial purchase is then given by $V(t) = 10{,}000(0.8)^t$. Determine the value of the car after 4 years.

30. A pool is full of water when a leak develops. The volume of water in the pool (in thousands of gallons) t minutes after the development of the leak is given by $V(t) = 10(0.99)^t$. Determine the volume of the pool 10 hours after the leak.

31. Consider that the amount of caffeine in the human body in milligrams is given by $V(t) = V_0(0.5)^{\frac{t}{6}}$ where V_0 is the initial ingested amount and t is the number of hours since it was ingested. Determine the amount of caffeine in the human body after 8 hours if the initial ingested amount is 95 milligrams (the equivalent to about one cup of coffee).

32. The blood concentration, in milligrams per liter (mg/L), of a particular antibiotic is given by $V(t) = V_0(0.5)^{\frac{t}{8}}$ where t is the number of hours since the patient ingested the initial dose of V_0 milligrams. Determine the blood concentration 8 hours after the patient has ingested 500 mg of the antibiotic.

33. The mass of radioactive Carbon-14 in a sample (measured in milligrams) t years after the beginning of an experiment is given by $M(t) = M_0 \cdot (0.5)^{\frac{t}{5730}}$ where M_0 is the initial mass. Determine the mass of Carbon-14 50,000 years for an initial amount on 200 milligrams.

34. The blood stream concentration, in milligrams per liter (mg/L), of a particular medicine is given by $V(t) = 250(0.5)^{\frac{t}{8}}$ where t is the number of hours since the initial dose. This particular medicine is only effective when the blood stream concentration is at least 150 mg/L. Determine if the medicine is being effective 4 hours after the initial dose has been administered.

35. Find the amount A repaid on a loan if \$2000 is borrowed at 6.25% compounded continuously for six years. Use $e = 2.71828$ or the e-button on your calculator. Round your answer to the nearest cent. Do not round until the final answer.

36. Find the amount A repaid on a loan if \$2675 is borrowed at 4.725% compounded continuously for three years. Use $e = 2.71828$ or the e-button on your calculator. Round your answer to the nearest cent. Do not round until the final answer.

37. Find the amount P initially invested if an account contains \$4500 at the end of four years at an interest rate of 6.25% compounded continuously. Use $e = 2.71828$ or the e-button on your calculator. Round your answer to the nearest cent. Do not round until the final answer.

38. Find the amount P initially invested if an account contains \$10,000 at the end of ten years at an interest rate of 1.07% compounded continuously. Use $e = 2.71828$ or the e-button on your calculator. Round your answer to the nearest cent. Do not round until the final answer.

39. Assume that the number of salmon caught in 2019 was equal to 2.1 million and that the number is supposed to decrease by 37% every year after that.

a. Write an exponential model $f(x) = ab^x$, where x is the number of years since 2019 and $f(x)$ is the number salmon caught in millions of units.

b. Use your model to predict how many salmon will be caught in 2022.

5. Evaluate the polynomial for the given values.

a. $P(x) = x^3 + 4x^2 - 7x - 9$, for $P(3)$.

b. $P(x,y) = 3xy^2 + 2xy - 5x + 6y + 13$, for $x = -4$ and $y = -1$

Example 5 Evaluating Polynomials

a. For the polynomial $P(x) = x^3 - 2x^2 + 3x + 5$, find $P(4)$.

b. Evaluate the polynomial $P(x, y) = 2x^2y - xy + 3x - 4y + 15$ for $x = -1$ and $y = -6$.

Solution

a. Substitute 4 for x throughout the polynomial.

$$\begin{aligned} P(4) &= 4^3 - 2(4)^2 + 3(4) + 5 = 64 - 32 + 12 + 5 \\ &= 32 + 12 + 5 \\ &= 49 \end{aligned}$$

b. Substitute -1 for x and -6 for y throughout the polynomial.

$$\begin{aligned} P(-1, -6) &= 2(-1)^2(-6) - (-1)(-6) + 3(-1) - 4(-6) + 15 \\ &= -12 - 6 - 3 + 24 + 15 \\ &= -21 + 39 \\ &= 18 \end{aligned}$$

Now work margin exercise 5.

Margin Exercise Answers

1. a. $x^4 + 2x^3 + 3x^2 + 5$ **b.** $4x^2 + 9x + 5$ **2.** $7x^3 - 8x^2 - 3x - 2$ **3.** $-2xy^2 + 7x - 5y$ **4. a.** $-3x^2 - 4x + 2$ **b.** $3x^4 - 9x^3 + 2x^2 + 12x - 2$ **5. a.** 33 **b.** 23

4.4 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. A monomial in x is an expression of the form kx^n, where n is a _____ number and k is the _____.

2. In general, expressions with _____ in the denominator are not polynomials.

3. A _____ is a monomial or the indicated sum and/or difference of monomials.

4. The coefficient of the term of the largest degree is the _____ coefficient.

5. The _____ of two or more polynomials can be found by combining like terms

6. In function notation, indicates that P is the ____ of the polynomial and x is the _____ used in the polynomial.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A nonzero constant is a monomial with no degree.

8. A monomial is a polynomial with one term.

9. When subtracting one polynomial from another polynomial, only the first term of the polynomial is subtracted.

Practice

State whether the expression is or is not a polynomial. If the expression is a polynomial, state its degree, its classification as a monomial, binomial, or trinomial, and its leading coefficient.

1. $x^3 - x^2$

2. 9

3. $-3x^{\frac{1}{2}} + x$

4. $x^4 + 8x^3 - y^2$

5. $\frac{1}{2}y^2 + \frac{5}{4}y^3 - \frac{7}{4}y$

6. $x^2 + y^2 - \frac{1}{y}$

7. 0

8. $-\sqrt{2}$

9. $\left(x^5 - y^3\right)^{\frac{1}{2}}$

10. $\frac{3}{2}x^2 - \sqrt{3}x - 7$

11. $7x^2 - 6x + 9x^{\frac{2}{3}}$

12. $\frac{x^3 - 3y^2}{x}$

Find the indicated sums and differences. Simplify each answer. See Examples 1 and 3.

13. $\left(3x^2 - 5x + 1\right) + \left(x^2 + 2x - 7\right)$

14. $\left(5x^2 + 8x - 3\right) + \left(-2x^2 + 6x - 4\right)$

15. $\left(x^2 - 9x + 2\right) + \left(-x^2 + 2x - 8\right)$

16. $\left(7x^2 - 4x + 6\right) + \left(4x^2 - 2x + 5\right)$

17. $\left(x^2 + y^2\right) + \left(2x^2 - 5y^2\right)$

18. $\left(x^2 - 3xy + y^2\right) + \left(2x^2 - 5xy - y^2\right)$

19. $\left(2x^2 + 3x + 8\right) - \left(x^2 + 4x - 2\right)$

20. $\left(6x^3 - 5x + 1\right) - \left(2x^3 + 3x - 4\right)$

21. $\left(2x^4 + 3x\right) - \left(5x^3 + 4x + 3\right)$

22. $\left(2x^3 - 3x^2 + 6\right) - \left(x^4 + x + 1\right)$

23. $\left(5x^2 + 6x - 1\right) + \left(x^4 - 3x^2 + 2x\right)$

24. $\left(7x^2 - 2xy + 3y^2\right) + \left(-3x^2 - 2xy + 5y^2\right)$

25. $\left(4x^3 - 7x^2 + 3x + 2\right) - \left(-2x^3 - 5x - 1\right)$

26. $\left(4x^2 - 8xy - 2y^2\right) + \left(-9x^2 + 5xy - 6y^2\right)$

27. $\left(3x^2 - 2y^2\right) + \left(7xy + 4y^2\right) - \left(-6x^2 - 6xy + 8y^2\right)$

28. $\left(9xy+8y^2\right)-\left(6x^2-8xy\right)+\left(5x^2-3xy+7y^2\right)$

29. $\left(5x^3-14x^2\right)-\left(5x^2+2x+1\right)-\left(-7x^3+2x^2-13\right)$

30. $\left(7x^3+4x^2-x\right)+\left(3x^3-4x+5\right)-\left(8x^3+x^2-x+3\right)$

31. $x^3-\left[3x^2-1-\left(x^3+4x^2+1\right)\right]+\left(3x^3-3x^2-2\right)$

32. $3x-4xy+\left[6y+\left(4x+3xy+2y\right)\right]-\left[-6x-\left(xy-4y\right)\right]$

33. $x^2-2xy+\left[y^2-\left(3xy+2y^2\right)-\left(3x^2-xy-2y^2\right)\right]$

34. $\left[\left(4x^2-3x\right)-\left(2x^2+5x\right)\right]+\left[\left(x^2-6x\right)+\left(-3x^2+x\right)\right]$

35. $\left[\left(2x+xy-y\right)+\left(x-2xy+4y\right)\right]-\left[\left(-3x+5xy+y\right)-\left(2x+3xy-2y\right)\right]$

Find each sum. See Example 2.

36.
$$\begin{array}{r} 2x^2-5x-6 \\ \underline{-3x^2+2x-1} \end{array}$$

37.
$$\begin{array}{r} x^3+2x^2+x-2 \\ \underline{x^3-2x^2-3x-1} \end{array}$$

38.
$$\begin{array}{r} 5x^3-4x^2\qquad -9 \\ \underline{2x^3-3x^2-6x+5} \end{array}$$

39.
$$\begin{array}{r} 3x^4+3x^3+x^2+x+2 \\ \underline{7x^4-x^3-5x^2+x-1} \end{array}$$

40.
$$\begin{array}{r} 14x^3+13x^2+10x-13 \\ \underline{20x^3\qquad\qquad -18x+25} \end{array}$$

Find each difference. See Example 4.

41.
$$\begin{array}{r} 9x^2-2x+3 \\ \underline{-\left(4x^2+5x-2\right)} \end{array}$$

42.
$$\begin{array}{r} -3x^2+7x-6 \\ \underline{-\left(2x^2-x+6\right)} \end{array}$$

43.
$$\begin{array}{r} 5x^3\qquad -10x+15 \\ \underline{-\left(x^3-4x^2-3x-9\right)} \end{array}$$

44.
$$\begin{array}{r} x^3-8x^2+12x+5 \\ \underline{-\left(-3x^3+8x^2+2x+5\right)} \end{array}$$

45.
$$\begin{array}{r} 2x^4-5x^3-6x^2+7x+7 \\ \underline{-\left(x^4\qquad +2x^2+4x+10\right)} \end{array}$$

Evaluate each polynomial in for the specified value(s) of the variable(s). See Example 5.

46. Given $P(x)=2x^2-x+3$; find $P(1)$.

47. Given $P(x)=3x^2-2x+5$; find $P(2)$.

48. Given $P(x)=3-x^2$; find $P(-2)$.

49. Given $P(x)=x^3-2x^2+x-1$; find $P(2)$.

50. Given $P(x)=x^3+x^2-4$; find $P(-3)$.

51. Given $P(x)=4x^3-2x^2-1$; find $P(-4)$.

52. Given $P(x, y) = 2x^2 - 3xy + y^2$; find $P(2, -2)$.

53. Given $P(x, y) = 4x - 2xy + 5y$; find $P(1, 1)$.

54. Given $P(x, y, z) = 3x + 4xy - 2yz + z$; find $P(1, 0, 2)$.

55. Given $P(x, y, z) = 2xyz - 3x + yz - xz$; find $P(2, -1, 2)$.

56. Use a graphing calculator to graph the following linear functions.

a. $P(x) = -2x + 5$

b. $P(x) = \frac{1}{4}x$

57. Use a graphing calculator to graph the following quadratic functions.

a. $P(x) = -x^2$

b. $P(x) = x^2 - 4x + 4$

58. Use a graphing calculator to graph the following cubic functions.

c. $P(x) = x^3$

d. $P(x) = x^3 - 4x$

Writing & Thinking

59. Write the definition of a polynomial.

60. Explain, in your own words, how to subtract one polynomial from another.

61. Describe what is meant by the degree of a polynomial in x.

62. Give two examples that show how the sum of two binomials might not be a binomial.

4.5 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. The distributive property can be used to find the _____ of a monomial and a polynomial.
2. When multiplying two polynomials together, the distributive property is applied by multiplying each _____ of one polynomial by each _____ of the other.
3. In the case of the product of two _____, the FOIL method is used.
4. The O in the FOIL method stands for multiplying the _____ terms together.
5. When binomials are in the form of the sum and difference of the same terms, their product is called the _____ of two squares.
6. The result of squaring a binomial is a _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The distributive property can only be used to multiply a monomial and a polynomial.
8. The FOIL method is a way to remember one specific order that the distributive property can be applied.
9. When two binomials are in the form of the sum and difference of the same term, the product will be a trinomial.
10. When the two binomials being multiplied together are the same, the product will be a trinomial.

Find the indicated products and simplify, if possible.

1. $5x\left(x^2-2x+3\right)$
2. $2x^2\left(3x^2+5x-1\right)$
3. $xy^2\left(x^2+4y\right)$
4. $x^2z(x-4y+z)$
5. $(x+3)(x-6)$
6. $(x-2)(x-5)$
7. $(x-8)(x-1)$
8. $(x+2)(x+4)$
9. $(2y+1)(y-6)$
10. $(y+5)(3y+2)$
11. $(3x-4)(x-5)$
12. $(2x-1)(x-2)$
13. $(2y+3)(3y+2)$
14. $(5y-2)(3y+1)$

15. $(8x+3)(x-5)$

16. $(7x+6)(2x-3)$

17. $(9x+1)(3x-2)$

18. $(5x-11)(3x+4)$

19. $(3x+1)^2$

20. $(4x-3)^2$

21. $(5x-2y)^2$

22. $(7x+4y)^2$

23. $(4x+7)(4x-7)$

24. $(3x+5)(3x-5)$

25. $(2x-3y)(2x+3y)$

26. $(6x-y)(6x+y)$

27. $x(3x^2-4)(3x^2+4)$

28. $3x(7x^2+8)(7x^2-8)$

29. $(x-1)(x^2+x+1)$

30. $(y+4)(y^2-4y+16)$

31. $(x+3)(x^2+6x+9)$

32. $(y-5)(y^2+3y+2)$

33. $(x^3+2)^2$

34. $(2x^3-3)^2$

35. $(2x^3-7)(2x^3+7)$

36. $(x+2y)(x^2-2xy+4y^2)$

37. $(x-3y)(x^2+3xy+9y^2)$

38. $(8y^2-7)(3y^2+2)$

39. $(x^2+6y^2)(x^2-6y^2)$

40. $(x^2-6y^2)(x^2+3y^2)$

41. $(5x^2+y^2)(2x^2-3y^2)$

42. $(x-2y)(x^2+2xy+4y^2)$

43. $[(x+y)+2][(x+y)-2]$

44. $[(x+1)+y][(x+1)-y]$

45. $[(5x-y)+3]^2$

46. $[(2x+1)-y]^2$

47. $[(x+4)-2y]^2$

48. $[(x-3y)+5]^2$

49. $x^2(x^k+3)$

50. $x^3(x^{2k}+x)$

51. $(x^k+3)(x^k-5)$

52. $(x^k+6)(x^k-6)$

53. $(x^k+1)(x^k+4)$

54. $(2x^k-3)(x^k+2)$

55. $(3x^k+2)(x^k+5)$

56. $\left(x+\frac{1}{4}\right)\left(x-\frac{1}{4}\right)$

57. $\left(x+\frac{5}{8}\right)\left(x-\frac{5}{8}\right)$

58. $\left(x+\frac{2}{3}\right)^2$

59. $\left(y-\frac{1}{5}\right)^2$

60. $\left(y+\frac{1}{4}\right)\left(y-\frac{3}{4}\right)$

61. $(x+2.5)(x-2.5)$

62. $(x+2.1)^2$

63.
$$\begin{array}{r} 2x^2 - 5x - 6 \\ \underline{\qquad\quad 3x + 1} \end{array}$$

64.
$$\begin{array}{r} x^2 + 2x + 1 \\ \underline{x^2 + 2x + 1} \end{array}$$

65.
$$\begin{array}{r} x^3 - 3x + 4 \\ \underline{\qquad\quad 2x - 3} \end{array}$$

66. $2x^3 + 6x^2 + 5$
$\underline{\qquad\qquad x^2 + 5}$

67. $x^3 - 7x - 4$
$\underline{\qquad\qquad 4x - 6}$

68. $x^3 - 5x + 14$
$\underline{\qquad\qquad 2x - 3}$

Applications

Solve.

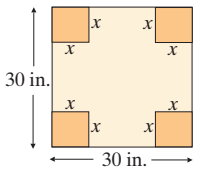
69. In the case of binomial probabilities, if x is the probability of success in one trial of an event, then the expression $P(x) = 10x^3(1-x)^2$ is the probability of 3 successes in 5 trials where $0 \le x \le 1$.

a. Represent the expression $P(x)$ as a single polynomial function.

b. If a fair coin is tossed, the probability of heads occurring is $\frac{1}{2}$. That is, $x = \frac{1}{2}$. Find the probability of exactly 3 heads occurring in 5 tosses.

c. A basketball player is known to make 80% of his free throws. What is the probability that he will make exactly 3 of his next 5 attempts?

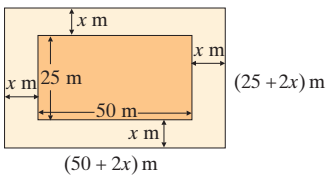
70. A square is 30 inches on each side. A small square, x inches on each side, is cut from each corner of the original square.



a. Represent the area of the remaining portion of the square in the form of a polynomial function $A(x)$.

b. Represent the perimeter of the remaining portion of the square in the form of a polynomial function $P(x)$.

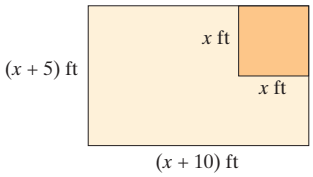
71. A swimming pool, 25 meters by 50 meters, is surrounded by a concrete deck that is x meters wide.



a. Represent the area covered by the deck and the pool in the form of a polynomial function.

b. Represent the area covered by the deck only in the form of a polynomial function.

72. A rectangle has sides $(x + 5)$ ft and $(x + 10)$ ft. A square x feet on each side is cut from one corner of the rectangle.

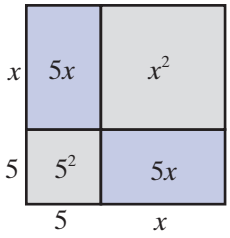


a. Represent the remaining area (light area in the figure shown) in the form of a polynomial function $A(x)$.

b. Represent the perimeter of the remaining figure (after the square in the corner has been removed) in the form of a polynomial function $P(x)$.

Writing & Thinking

73. A square with sides of length $(x + 5)$ can be broken up as shown in the diagram. The sums of the areas of the interior rectangles and squares is equal to the total area of the square: $(x+5)^2$. Show how this fits with the formula for the square of a sum.



5.1 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The result of multiplication is called the _____ and the numbers or expressions being multiplied are called _____ of the product.
2. The reverse of multiplication with polynomials is called _____.
3. GCF stands for _____ _____ _____. The GCF of a set of numbers is the _____ positive integer that is a factor of all numbers in the set.
4. Factoring polynomials with four or more terms can sometimes be accomplished by _____ terms and using the distributive property.
5. If the leading coefficient in a polynomial is a negative number, you may choose to factor out the _____ of the GCF.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. When finding the GCF of a polynomial, you need to consider only the coefficients.
7. An expression is factored completely if none of its factors can be factored.
8. One way to find the GCF of a set of numbers is to use the prime factorization of each number.
9. Binomials cannot be factored out of algebraic expressions.

Practice

Find the GCF for each set of terms. See Example 1.

1. $\{10, 15, 20\}$
2. $\{25, 30, 75\}$
3. $\{16, 40, 56\}$
4. $\{30, 42, 54\}$
5. $\{9, 14, 22\}$
6. $\{44, 66, 88\}$
7. $\{30x^3, 40x^5\}$
8. $\{15y^4, 25y\}$
9. $\{8a^3, 16a^4, 20a^2\}$
10. $\{36xy, 48xy, 60xy\}$
11. $\{26ab^2, 39a^2b, 52a^2b^2\}$
12. $\{28c^2d^3, 14c^3d^2, 42cd^2\}$
13. $\{45x^2y^2z^2, 75xy^2z^3\}$
14. $\{21a^5b^4c^3, 28a^3b^4c^3, 35a^3b^4c^2\}$

Simplify each expression. See Example 2.

15. $\frac{x^7}{x^3}$

16. $\frac{x^8}{x^3}$

17. $\frac{-8y^3}{2y^2}$

18. $\frac{12x^2}{2x}$

19. $\frac{9x^5}{3x^2}$

20. $\frac{-10x^5}{2x}$

21. $\frac{4x^3y^2}{2xy}$

22. $\frac{21x^4y^3}{-3xy^2}$

23. $\frac{8y^3-16y^2+24y}{8y}$

24. $\frac{18x^4+24x^3+36x^2}{6x^2}$

25. $\frac{34x^5-51x^4+17x^3}{17x^3}$

26. $\frac{14y^4+28y^3+12y^2}{2y^2}$

27. $\frac{110x^4-121x^3+11x^2}{11x}$

28. $\frac{15x^7+30x^6-45x^3}{15x^3}$

29. $\frac{-56x^4+98x^3-35x^2}{14x^2}$

30. $\frac{108x^6-72x^5+63x^4}{18x^4}$

31. $\frac{16y^6-56y^5-120y^4+64y^3}{16y^3}$

32. $\frac{20y^5-14y^4+21y^3+42y^2}{4y^2}$

Complete the factoring of the polynomial as indicated.

33. $3m+27=3(\quad)$

34. $2x+18=2(\quad)$

35. $5x^2-30x=5x(\quad)$

36. $6y^3-24y^2=6y^2(\quad)$

37. $13ab^2+13ab=13ab(\quad)$

38. $8x^2y-4xy=4xy(\quad)$

39. $-15xy^2-20x^2y-5xy=-5xy(\quad)$

40. $-9m^3-3m^2-6m=-3m(\quad)$

Factor each polynomial by finding the GCF (or –1 · GCF). See Examples 3 through 6.

41. $11x-121$

42. $14x+21$

43. $16y^3+12y$

44. $-3x^2+6x$

45. $-6ax+9ay$

46. $4ax-8ay$

47. $10x^2y-25xy$

48. $16x^4y-14x^2y$

49. $-18y^2z^2+2yz$

50. $-14x^2y^3-14x^2y$

51. $8y^2-32y+8$

52. $5x^2-15x-5$

53. $2xy^2-3xy-x$

54. $ad^2+10ad+25a$

55. $8m^2x^3 - 12m^2y + 4m^2z$

56. $36t^2x^4 - 45t^2x^3 + 24t^2x^2$

57. $-56x^4z^3 - 98x^3z^4 - 35x^2z^5$

58. $34x^4y^6 - 51x^3y^5 + 17x^5y^4$

59. $15x^4y^2 + 24x^6y^6 - 32x^7y^3$

60. $-3x^2y^4 - 6x^3y^4 - 9x^2y^3$

Factor each expression by factoring out the common binomial factor. See Example 7.

61. $7y^2(y+3) + 2(y+3)$

62. $6a(a-7) - 5(a-7)$

63. $3x(x-4) + (x-4)$

64. $2x^2(x+5) + (x+5)$

65. $4x^3(x-2) - (x-2)$

66. $9a(x+1) - (x+1)$

67. $10y(2y+3) - 7(2y+3)$

68. $a(x+5) + b(x+5)$

69. $a(x-2) - b(x-2)$

70. $3a(x-10) + 5b(x-10)$

Factor each of the polynomials by grouping. If a polynomial cannot be factored, write "not factorable." See Examples 8 through 12.

71. $bx + b + cx + c$

72. $3x + 3y + ax + ay$

73. $x^3 + 3x^2 + 6x + 18$

74. $2z^3 - 14z^2 + 3z - 21$

75. $10a^2 - 5az + 2a + z$

76. $x^2 - 4x + 6xy - 24y$

77. $3x + 3y - bx - by$

78. $ax + 5ay + 3x + 15y$

79. $5xy + yz - 20x - 4z$

80. $x - 3xy + 2z - 6zy$

81. $z^2 + 3 + az^2 + 3a$

82. $x^2 - 5 + x^2y + 5y$

83. $6ax + 12x + a + 2$

84. $4xy + 3x - 4y - 3$

85. $xy + x + y + 1$

86. $xy + x - y - 1$

87. $10xy - 2y^2 + 7yz - 35xz$

88. $7xy - 3y + 2x^2 - 3x$

89. $3xy - 4uy - 6vx + 8uv$

90. $xy + 5vy + 6ux + 30uv$

91. $3ab + 4ac + 2b + 6c$

92. $24y - 3yz + 2xz - 16x$

93. $6ac - 9ad + 2bc - 3bd$

94. $2ac - 3bc + 6ad - 9bd$

Applications

Solve.

95. Bonnie volunteers to bring bags of candy to her child's class for the Halloween party this year. She buys one bag of candy A containing 150 pieces of candy, one bag of candy B containing 180 pieces of candy, and one bag of candy C containing 330 pieces of candy. She needs to use all the candy to create identical treat bags. How many treat bags can Bonnie make so that each one has the same number and variety of candy? How many of each type of candy will be in each bag?

96. The area of a rectangular photo can be represented by the polynomial $15x^2 + 5x$.

a. If $x = 2$ inches, find the area of the photo.

b. Factor the polynomial to find a variable expression for the length and width of the photo.

c. If $x = 2$ inches, use the answer from Part **b.** to find the length and the width of the photo.

d. Find the area of the photo by multiplying the length and width values from Part **c.**

e. Are the answers from Parts **a.** and **d.** the same? Explain why or why not.

97. A circus performer is shot vertically into the air with an initial velocity of 48 feet per second. The height of the performer above the ground in feet can be described by the polynomial $48x - 16x^2$ after x seconds.

a. Find the height of the circus performer after 2 seconds.

b. Factor the polynomial $48x - 16x^2$.

c. Use the factored form of the polynomial from Part **b.** to find the height of the circus performer after 2 seconds.

d. Are the answers from Parts **a.** and **c.** the same? Explain why or why not.

Writing & Thinking

98. Explain why the GCF of $-3x^2 + 3$ is 3 and not -3.

Thus, for the original expression, we have the following.

$$18x^3 - 39x^2 + 18x = 3x\left(6x^2 - 13x + 6\right)$$
$$= 3x(2x - 3)(3x - 2)$$

Remember to include the original GCF in the final product

Now work margin exercise 7.

Tips to Keep in Mind while Factoring

1. When factoring polynomials, always look for a common factor first. Then, if there is one, remember to include this common factor as part of the answer.
2. **To factor completely** means to find factors of the polynomial such that none of the factors are themselves factorable.
3. Not all polynomials are factorable. (See $x^2 + 3x + 5$ in Example 4.) **Any polynomial that cannot be factored as the product of polynomials with integer coefficients is not factorable.**
4. Factoring can be checked by multiplying the factors. The product should be the original expression.

PROCEDURE

Margin Exercise Answers

1. $(x+3)(x+7)$ **2.** $-2(x+4)(x-5)$ **3.** $(4x-7)(2x+3)$ **4.** $3\left(x^2-2x+3\right)$
5. $(3x-2)(3x+1)$ **6.** $(x-7)(x+2)$ **7.** $3(x-2)(4x+5)$

5.2 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. When factoring a trinomial with leading coefficient 1, if two factors of the constant term do not exist, the trinomial is not _____.
2. The first step when factoring should be to factor out any common _____ factor.
3. To factor a trinomial with leading coefficient other than 1, the FOIL method is used with more of a _____ approach.
4. When factoring a trinomial, if the sigh of the constant term is _____, the signs of both factors will be the same.
5. The *ac*-method of factoring involves the method of factoring by _____.

6. To factor _____ means to find factors of the polynomial such that none of the factors themselves are factorable.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. In a trinomial such as $x^2 - 5x + 4$, one would need to find two factors of 4 whose sum is negative 5.

8. A trinomial is factorable if the middle term is the difference of the inner and outer products of two binomials.

9. The first step in the *ac*-method of factoring is to rewrite the middle term.

10. Factoring can be checked by multiplying the factors and verifying that the product matches the original polynomial.

Practice

Completely factor each polynomial. If a polynomial cannot be factored, write not factorable.

1. $x^2 + 9x + 18$

2. $y^2 - 7y - 30$

3. $x^2 - 6x - 27$

4. $y^2 - 5y - 14$

5. $x^2 - 27x + 50$

6. $x^2 - 15x + 36$

7. $2x^2 + 15x - 8$

8. $2x^2 + 9x - 35$

9. $6x^2 + 13x + 6$

10. $8y^2 + 10y - 25$

11. $2x^2 - 7x + 4$

12. $6x^2 - 35x - 5$

13. $2x^2 - 7x - 4$

14. $35y^2 + 9y - 18$

15. $18x^2 - 7x - 1$

16. $25x^2 + 5x - 6$

17. $12x^3 + 38x^2 + 20x$

18. $8x^3 - 6x^2 - 2x$

19. $2x^4 + 11x^2y - 15y^2$

20. $2x^4 + 11x^2y - 21y^2$

21. $2x^6 + 9x^3y^2 + 4y^4$

22. $5x^4 + 17x^2y^2 + 6y^4$

23. $-18x^2 + 72x - 8$

24. $-45y^2 + 30y + 120$

25. $-5y^2 + 40y - 60$

26. $-12x^2 + 22x + 4$

27. $21x^4 - 4x^3 - 32x^2$

28. $2x^4y^3 - 5x^3y^3 - 18x^2y^3$

29. $5x^4 + 17x^2y^2 + 6y^4$

30. $2x^6 + 9x^3y^2 + 4y^4$

31. $x^2(x-5) + 4x(x-5) - 21(x-5)$

32. $x^2(x+4) - 6x(x+4) - 15(x+4)$

33. $2x^2(2x+1)-9x(2x+1)-18(2x+1)$

34. $3x^2(4x-1)+19x(4x-1)+28(4x-1)$

35. $(2x+y)^2-9(2x+y)+20$

36. $(x-2y)^2+10(x-2y)+16$

37. $(x+3y)^2-14(x+3y)-32$

38. $(x+5y)^2+8(x+5y)+12$

39. $(6x-y)^2+8(6x-y)+7$

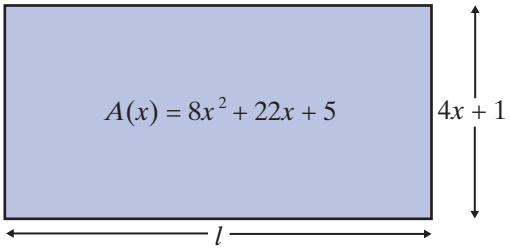
40. $5(3x-y)^2+15(3x-y)-20$

41. $4(2a+b)^2+4(2a+b)-24$

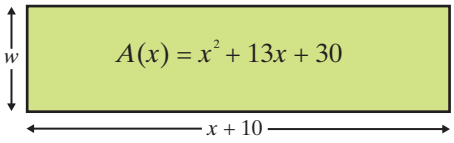
42. $-6(a-b)^2+6(a-b)+180$

Applications

43. The area (in square inches) of the rectangle shown is given by the polynomial function $A(x)=8x^2+22x+5$. If the width of the rectangle is $4x+1$ inches, what is the representation of the length?



44. The area (in square meters) of the rectangle shown is given by the polynomial function $A(x)=x^2+13x+30$. If the length of the rectangle is $(x+10)$ meters, what is the representation of the width?

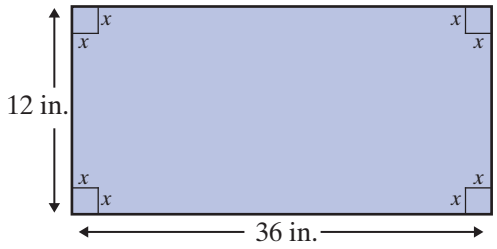


45. The volume of an open box is found by cutting equal squares (x units on a side) from a sheet of cardboard that is 12 inches by 36 inches. The function representing the volume is $V(x)=4x^3-96x^2+432x$, where $0<x<6$.

a. Factor this function in such a way that the factors represent the lengths of the sides of the box.

b. What is the value of $V(2)$?

c. What is the value of $V(4)$?



Writing & Thinking

46. The following statement is true:

$$4x^2 + 24x + 20 = (4x + 20)(x + 1) = (x + 5)(4x + 4) = (2x + 10)(2x + 2).$$

Explain how the trinomial can be factored in three ways. Is there some kind of error?

47. The following statement is true: $5x^2 + 5x - 60 = (5x + 20)(x - 3)$.
Explain why this is not the completely factored form of the trinomial.

48. Explain, in your own words, what is meant by factoring a polynomial.

Margin Exercise Answers

1. a. $7a(x-7)(x+7)$ **b.** $\left(y^3+10\right)\left(y^3-10\right)$ **2. a.** Not factorable **b.** $5\left(9x^2+4\right)$

3. a. $(z+20)^2$ **b.** $(y-7)^2$ **c.** $3z(x-3y)^2$ **d.** $(y+4-z)(y+4+z)$

4. a. $(y-3)\left(y^2+3y+9\right)$ **b.** $\left(2y-x^2\right)\left(4y^2+2x^2y+x^4\right)$ **c.** $6\left(2x^4-5\right)\left(4x^8+10x^4+25\right)$

5.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. Factoring a perfect square trinomial gives a square _____.
2. In a perfect square trinomial, both the first and last terms must be perfect _____.
3. If the first term of a perfect square trinomial is x^2, and the last term is of the form a^2, then the middle term must be of the form _____ or _____.
4. The formula for factoring the difference of cubes is $x^3 - a^3 =$ _____.
5. The formula for factoring the sum of two cubes is $x^3 + a^3 =$ _____.
6. The first 6 perfect cubes are 1, 8, ___, ___, ___, and ___.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The expression $x^2 + 20x + 100$ is a perfect square trinomial.
8. When factoring polynomials, always look for a common monomial factor first.
9. The sum of two squares, $\left(x^2 + a^2\right)$, is factorable.
10. Sixty-four is a perfect square and a perfect cube.

Practice

Completely factor each of the given polynomials. If a polynomial cannot be factored, write "not factorable." See Examples 1 through 4.

1. $x^2 - 25$
2. $y^2 - 121$
3. $81 - y^2$
4. $25 - z^2$
5. $2x^2 - 128$
6. $3x^2 - 147$
7. $4x^4 - 64$
8. $4x^2 + 49$
9. $y^2 - 16y + 64$
10. $z^2 + 18z + 81$

11. $-4x^2+100$

12. $-12x^4+3$

13. $9x^2-25$

14. $4x^2-49$

15. $y^2-10y+25$

16. $x^2+12x+36$

17. $4x^2-4x+1$

18. $49x^2-14x+1$

19. $25x^2+30x+9$

20. $9y^2+12y+4$

21. $16x^2-40x+25$

22. $9x^2-12x+4$

23. $2x^3y+32x^2y+128xy$

24. $3x^2y-30xy+75y$

25. $x^2-20x+100$

26. $25x^2-10x+1$

27. $x^4+10x^2y+25y^2$

28. $16x^4+8x^2y+y^2$

29. x^3-125

30. x^3-64

31. y^3+216

32. y^3+1

33. x^3+27y^3

34. $8x^3+1$

35. x^2+64y^2

36. $3x^3+81$

37. $4x^3-32$

38. $3x^4+375xy^3$

39. x^3y+y^4

40. x^4y^3-x

41. $x^2y^2-x^2y^5$

42. $2x^2-16x^2y^3$

43. $24x^4y+81xy^4$

44. x^6-64y^3

45. x^6-y^9

46. $64x^2+1$

47. $27x^3+y^6$

48. x^3+64z^3

49. $8x^3+y^3$

50. x^3+125y^3

51. $8y^3-8$

52. $36x^3+36$

53. $9x^2-y^2$

54. x^2-4y^2

55. x^4-16y^4

56. $81x^4-1$

57. m^2+7m+6

58. a^2-4a+3

59. $x^2+11x+18$

60. $y^2+8y+15$

61. $n^2-8n+12$

62. m^2-m-6

63. $a^2+2a+24$

64. $-x^2-12x-35$

65. $x^2+3x-10$

66. $x^2+9x-36$

67. $3a^2+12a-36$

68. $-2y^2+24y-70$

69. $-5x^2+70x-240$

70. $7t^2+14t-168$

71. $64+49t^2$

72. $3x^2-147$

73. x^3-4x^2-12x

74. $3n^3+15n^2+18n$

75. $112a-2a^2-2a^3$

76. $200x+20x^2-4x^3$

77. $-3x^2+17x-10$

78. $2x^2+7x+3$

79. $6x^2-11x+4$

80. $12x^2-32x+5$

81. $12m^2+m-6$

82. $6t^2+t-35$

83. $4x^2-14x+6$

84. $-4x^2+18x-20$

85. $8x^2+6x-35$

86. $12x^2+5x-3$

87. $20x^2-21x-54$

88. $21x^2-x-10$

89. $14+11x-15x^2$

90. $24+x-3x^2$

91. $20y^2+9y-20$

92. $35x^2-x-6$

93. $18x^2-15x+2$

94. $12x^2-47x+11$

95. $12n^2-60n-75$

96. $-12x^3-2x^2-70x$

97. $36x^3+21x^2-30x$

98. $63x-3x^2-30x^3$

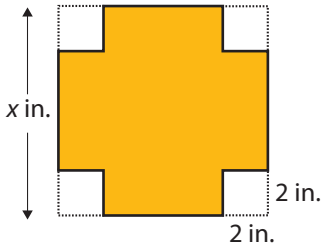
99. $16x^3-52x^2+22x$

100. $24y^3-4y^2-160y$

Solve.

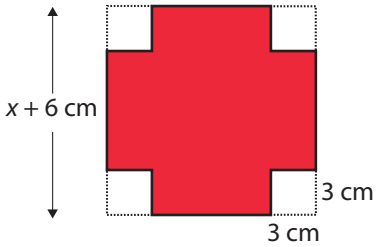
101. a. Represent the area of the shaded region of the square shown below as the difference of two squares.

b. Use the factors of the expression in Part **a.** to draw (and label the sides of) a rectangle that has the same area as the shaded region.



102. a. Use a polynomial function to represent the area of the shaded region of the square.

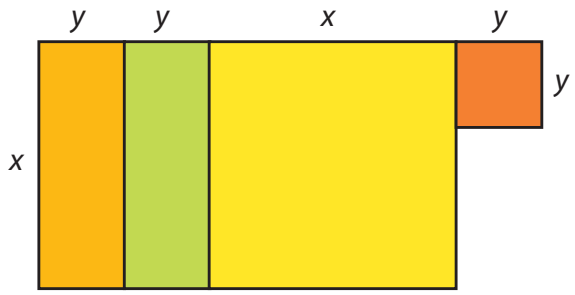
b. Use a polynomial function to represent the perimeter of the shaded figure.



Writing & Thinking

103. **a.** Show that the sum of the areas of the rectangles and squares in the figure is a perfect square trinomial.

b. Rearrange the rectangles and squares in the form of a square and represent its area as the square of a binomial.



104. Compound interest is interest earned on interest. If a principal P is invested and compounded annually (once a year) at a rate of r, then the amount, A_1 accumulated in one year is $A_1 = P + Pr$.
In factored form, we have $A_1 = P + Pr = P(1+r)$.
At the end of the second year the amount accumulated is
$A_2 = (P + Pr) + (P + Pr)r$.

a. Write the expression for A_2 in factored form similar to that for A_1.

b. Write an expression for the amount accumulated in three years, A_3, in factored form.

c. Write an expression for A_n the amount accumulated in n years.

d. Use the formula you developed in Part **c.** and your calculator to find the amount accumulated if $10,000 is invested at 6% and compounded annually for 20 years.

Common Error

A **common error** is to divide both sides of an equation by the variable x. This error can be illustrated by using the equation in Example 2.

Wrong Solution

$$3x^2 = 6x$$
$$\frac{3x^2}{x} = \frac{6x}{x}$$
$$3x = 6$$
$$x = 2$$

Do not divide by x, because you lose the solution $x = 0$.

Factoring is the method to use. By factoring, you will find all solutions as shown in the previous examples.

CAUTION

Completion Example Answers

8. $x^2 - 4x + 4 = 64$; $x^2 - 4x - 60 = 0$; $(x+6)(x-10) = 0$; $x + 6 = 0$ or $x - 10 = 0$; $x = -6$ or $x = 10$; The solutions are -6 and 10.

Margin Exercise Answers

1. $y = 7, \frac{5}{3}$ **2.** $v = 0, 2$ **3.** $x = 3$ (double root) **4.** $x = -1, 4$ **5.** $x = 4, 12$ **6.** $x = -\frac{7}{2}, \frac{4}{3}$

7. $x = -1, 3$ **8.** $x = -7, -1$ **9.** $x = -2, 0, 5$

5.4 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. When solving quadratic equations by factoring, it is necessary to have one side of the equation equal to _____.

2. The zero-factor property states that if the product of two or more factors equals zero, then at least one of the factors must be _____.

3. The factor theorem states that if $x = c$ is a root of a polynomial equation in the form $P(x) = 0$, then $x - c$ is a _____ of the polynomial $P(x)$.

4. In general, a quadratic equation has two solutions. If the two solutions are the same number, the equation is said to have a _____ solution or root.

5. Solutions can be checked by _____ them one at a time for x in the equation.

6. Second-degree polynomials are called _____ polynomials.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. When solving quadratic equations by factoring, it is important that all of the coefficients are integers.

8. The standard form for a quadratic equation is $ax^2 + bx = c$.

9. Not all quadratic equations can be solved by factoring.

10. All quadratic equations have two distinct solutions.

Practice

Solve each equation. See Example 1.

1. $(x-3)(x-2)=0$

2. $(x+5)(x-2)=0$

3. $(2x-9)(x+2)=0$

4. $(x+7)(3x-4)=0$

5. $0=(x+3)(x+3)$

6. $0=(x+10)(x-10)$

7. $(x+5)(x+5)=0$

8. $(x+5)(x-5)=0$

9. $2x(x-2)=0$

10. $3x(x+3)=0$

11. $(x+6)^2=0$

12. $5(x-9)^2=0$

Solve each equation by factoring.

13. $x^2-3x-4=0$

14. $x^2+7x+12=0$

15. $x^2-x-12=0$

16. $x^2-11x+18=0$

17. $0=x^2+3x$

18. $0=x^2-3x$

19. $x^2+8=6x$

20. $x^2=x+30$

21. $2x^2+2x-24=0$

22. $9x^2+63x+90=0$

23. $0=2x^2-5x-3$

24. $0=2x^2-x-3$

25. $3x^2-4x-4=0$

26. $3x^2-8x+5=0$

27. $2x^2-7x=4$

28. $4x^2+8x=-3$

29. $-2x=3x^2-8$

30. $6x^2+2=-7x$

31. $4x^2-12x+9=0$

32. $25x^2-60x+36=0$

33. $8x=5x^2$

34. $15x=3x^2$

35. $9x^2 - 36 = 0$

36. $4x^2 - 16 = 0$

37. $5x^2 = 10x - 5$

38. $2x^2 = 4x + 6$

39. $8x^2 + 32 = 32x$

40. $6x^2 = 18x + 24$

41. $\frac{x^2}{9} = 1$

42. $\frac{x^2}{2} = 8$

43. $\frac{x^2}{5} - x - 10 = 0$

44. $\frac{2}{3}x^2 + 2x - \frac{20}{3} = 0$

45. $\frac{x^2}{8} + x + \frac{3}{2} = 0$

46. $\frac{x^2}{6} - \frac{1}{2}x - 3 = 0$

47. $x^2 - x + \frac{1}{4} = 0$

48. $\frac{x^2}{3} - 2x + 3 = 0$

49. $x^3 + 8x = 6x^2$

50. $x^3 = x^2 + 30x$

51. $6x^3 + 7x^2 = -2x$

52. $3x^3 = 8x - 2x^2$

53. $0 = x^2 - 100$

54. $0 = x^2 - 121$

55. $3x^2 - 75 = 0$

56. $5x^2 - 45 = 0$

57. $x^2 + 8x + 16 = 0$

58. $x^2 + 14x + 49 = 0$

59. $3x^2 = 18x - 27$

60. $5x^2 = 10x - 5$

61. $(x-1)^2 = 4$

62. $(x-3)^2 = 1$

63. $(x+5)^2 = 9$

64. $(x+4)^2 = 16$

65. $(x+4)(x-1) = 6$

66. $(x-5)(x+3) = 9$

67. $27 = (x+2)(x-4)$

68. $-1 = (x+2)(x+4)$

69. $x(x+7) = 3(x+4)$

70. $x(x+9) = 6(x+3)$

71. $3x(x+1) = 2(x+1)$

72. $2x(x-1) = 3(x-1)$

73. $x(2x+1) = 6(x+2)$

74. $3x(x+3) = 2(2x-1)$

Applications

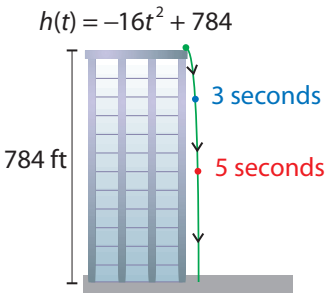
Solve.

75. A ball is dropped from the top of a building that is 784 feet high. The height of the ball above ground level is given by the polynomial function $h(t) = -16t^2 + 784$ where t is measured in seconds.

a. How high is the ball after 3 seconds? 5 seconds?

b. How far has the ball traveled in 3 seconds? 5 seconds?

c. When will the ball hit the ground? Explain your reasoning in terms of factors.



76. A tennis ball is dropped from a building. The position of the ball after t seconds is given by the polynomial function $s(t) = -4.9t^2 + 490$, where s is the height in meters of the ball.

a. Find $s(0)$. What does this value represent in the context of this problem?

b. How high is the tennis ball 2 seconds after it has been dropped?

c. How long before the tennis ball hits the ground?

77. A ball is thrown upward from an initial height of 96 feet with an initial velocity of 16 feet per second. After t seconds, the height of the ball can be described by the equation $h = -16t^2 + 16t + 96$.

a. What happens when $h = 0$?

b. Rewrite the equation with $h = 0$.

c. Solve the equation by factoring.

d. What does the answer to Part **c.** mean?

e. Do both solutions from Part **c.** make sense in the context of the problem? Explain why or why not.

78. Robin is putting the finishing touches on a quilt. The quilt is currently 80 inches long by 60 inches wide and she plans to add a border around the quilt. The width of the border on the sides will be twice the width of the border on the top and bottom of the quilt.

a. If x is the width of the border in inches that will be added to the top and bottom of the quilt, write an expression for the length and width of the quilt with the border added.

b. Write a simplified expression to find the area of the quilt with the border added.

c. Robin has a total of 5712 square inches of fabric to use for the back of the quilt. Use the expression from Part **b.** to write an equation to describe the total area of the back of the quilt.

d. Solve the quadratic equation from Part **c.** by factoring.

e. Do both of the solutions from Part **d.** make sense in the context of the problem?

f. What is the total length and width of the quilt?

79. We know that the area of a circle is proportional to the square of the radius. In fact, if the radius of a circle is r, then the area of the circle is $A = \pi r^2$. Let's determine how the area is changed when we double the radius.

a. Find the area of the circle for a radius of 1 in.

b. Find the area of the circle for a radius of 2 in.

c. Find the area of the circle for a radius of 4 in.

d. Find the area of the circle for a radius of 8 in.

e. Do you see the pattern? If you double the radius, by how many times does the area increase?

80. The St. Louis Arch is not quite in the shape of a parabola, but it can be closely modeled with the polynomial function $h(x) = -0.007x^2 + 0.003x + 625$, where x and $h(x)$ are both measured in feet and the center of the arch lies along the y-axis.

a. Find $h(0)$. What does this mean in this context?

b. Find $h(100)$. What does this mean in this context?

c. Find $h(300)$. Use this value to approximate the total distance between the two points at which the arch hits the ground.

d. Use your graphing calculator to sketch the graph and find the x-intercepts (to the nearest integer). What is the actual distance between the two points at which the arch hits the ground? (rounded to the nearest foot)

Writing & Thinking

81. When solving equations by factoring, one side of the equation must be 0. Explain why this is so.

82. In solving the equation $(x+5)(x-4) = 6$, why can't we just put one factor equal to 3 and the other equal to 2? Certainly $3 \cdot 2 = 6$.

5.5 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. When a number is multiplied by itself, the product is said to be that number's _____.
2. To reverse the process of squaring, find the _____ _____ of the number.
3. If a is a nonnegative real number, then $\sqrt{a}$ is the _____ square root of a.
4. In $\sqrt{102}$, the symbol $\sqrt{\ }$ is called the _____ sign and 102 is the _____.
5. When converted to decimal form, rational numbers either _____ or _____.
6. When rationalizing the denimonator, the objective is to find an _____ fraction that has a rational number for a denominator.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If a number is squared and the principal square root of the result is found, that square root is always equal to the original number.
8. There is no real number that can be a square root of a negative number.
9. The simplest form of a radical expression can be found by using prime factorization.
10. The fraction $\frac{\sqrt{2}}{3}$ is in simplest form.

Practice

Simplify the following square roots. See Examples 1 and 2.

1. $\sqrt{9}$
2. $\sqrt{49}$
3. $\sqrt{81}$
4. $\sqrt{36}$
5. $\sqrt{289}$
6. $\sqrt{121}$
7. $\sqrt{169}$
8. $\sqrt{361}$
9. $\sqrt{\frac{1}{4}}$
10. $\sqrt{\frac{9}{16}}$
11. $\sqrt{0.04}$
12. $\sqrt{0.0081}$
13. $-\sqrt{100}$
14. $-\sqrt{144}$
15. $-\sqrt{0.0016}$
16. $-\sqrt{0.000004}$
17. $\sqrt{\frac{9}{25}}$
18. $\sqrt{\frac{25}{81}}$

Estimates (rounded to the nearest ten-thousandth) of radicals are given. Show that these are reasonable estimates. See Example 3.

19. $\sqrt{74} \approx 8.6023$

20. $\sqrt{18} \approx 4.2426$

21. $\sqrt{32} \approx 5.6569$

22. $\sqrt{110} \approx 10.4881$

Use your knowledge of square roots to determine whether each number is rational, irrational, or not a real number.

23. $\sqrt{4}$

24. $\sqrt{17}$

25. $\sqrt{169}$

26. $\sqrt{8}$

27. $\sqrt{\frac{2}{9}}$

28. $-\sqrt{\frac{1}{4}}$

29. $\sqrt{-36}$

Simplify each of the following radical expressions. See Example 4.

30. $\sqrt{12}$

31. $-\sqrt{45}$

32. $\sqrt{288}$

33. $-\sqrt{63}$

34. $-\sqrt{72}$

35. $\sqrt{98}$

36. $-\sqrt{56}$

37. $\sqrt{162}$

38. $-\sqrt{125}$

39. $-\sqrt{121}$

40. $\sqrt{\frac{1}{4}}$

41. $\sqrt{\frac{32}{49}}$

42. $-\sqrt{\frac{11}{64}}$

43. $-\sqrt{\frac{125}{100}}$

44. $\sqrt{\frac{28}{25}}$

45. $\sqrt{\frac{147}{100}}$

Simplify the following radical expressions. Assume that all variables represent positive real numbers. See Examples 5 and 6.

46. $3\sqrt{2} + 5\sqrt{2}$

47. $7\sqrt{3} - 2\sqrt{3}$

48. $4\sqrt{11} - 3\sqrt{11}$

49. $6\sqrt{5} + \sqrt{5}$

50. $8\sqrt{10} - 11\sqrt{10}$

51. $\sqrt{7} + 6\sqrt{7} - 2\sqrt{7}$

52. $3\sqrt{2} + 5\sqrt{3} - 2\sqrt{3} + \sqrt{2}$

53. $\sqrt{5} + \sqrt{4} - 2\sqrt{5} + 6$

54. $\sqrt{12} + \sqrt{27}$

55. $\sqrt{32} - \sqrt{18}$

56. $3\sqrt{5} - \sqrt{45}$

57. $2\sqrt{7} + 5\sqrt{28}$

58. $\sqrt{50} - \sqrt{18} - 3\sqrt{12}$

59. $2\sqrt{48} - \sqrt{54} + \sqrt{27}$

60. $2\sqrt{20} - \sqrt{45} + \sqrt{36}$

61. $\sqrt{18} - 2\sqrt{12} + 5\sqrt{2}$

62. $\sqrt{8} - 2\sqrt{3} + \sqrt{27} - \sqrt{72}$

63. $\sqrt{80} + \sqrt{8} - \sqrt{45} + \sqrt{50}$

Multiply the following radical expressions and then simplify the results. Assume that all variables represent positive real numbers. See Examples 7 through 9.

64. $\sqrt{2}\left(3 - 4\sqrt{2}\right)$

65. $2\sqrt{7}\left(\sqrt{7} + 3\sqrt{2}\right)$

66. $3\sqrt{18} \cdot \sqrt{2}$

67. $2\sqrt{10} \cdot \sqrt{5}$

68. $-2\sqrt{6} \cdot \sqrt{8}$

69. $2\sqrt{15} \cdot 5\sqrt{6}$

70. $\sqrt{3}\left(\sqrt{2} + 2\sqrt{12}\right)$

71. $\sqrt{2}\left(\sqrt{3} - \sqrt{6}\right)$

72. $\left(3 + \sqrt{2}\right)\left(5 - \sqrt{2}\right)$

73. $\left(\sqrt{6} + 2\right)\left(\sqrt{6} - 2\right)$

74. $\left(2\sqrt{7} + 4\right)\left(\sqrt{7} - 3\right)$

75. $\left(5\sqrt{3} - 2\right)\left(2\sqrt{3} - 7\right)$

76. $\left(\sqrt{5} + 2\sqrt{2}\right)^2$

77. $\left(2\sqrt{5} + 3\sqrt{2}\right)^2$

78. $\left(\sqrt{2} + \sqrt{3}\right)\left(\sqrt{5} - \sqrt{3}\right)$

79. $\left(\sqrt{6} + \sqrt{5}\right)\left(\sqrt{6} - \sqrt{2}\right)$

80. $\left(\sqrt{11} + \sqrt{3}\right)\left(\sqrt{11} - 2\sqrt{3}\right)$

81. $\left(3\sqrt{7} + \sqrt{5}\right)\left(3\sqrt{7} - \sqrt{5}\right)$

Rationalize the denominator and simplify, if possible. Assume that all variables represent positive real numbers. See Examples 10 and 11.

82. $\dfrac{5}{\sqrt{2}}$

83. $\dfrac{7}{\sqrt{5}}$

84. $\dfrac{-3}{\sqrt{7}}$

85. $\dfrac{-10}{\sqrt{2}}$

86. $\dfrac{6}{\sqrt{3}}$

87. $\dfrac{8}{\sqrt{2}}$

88. $\dfrac{\sqrt{18}}{\sqrt{2}}$

89. $\dfrac{\sqrt{25}}{\sqrt{3}}$

90. $\dfrac{\sqrt{5}}{\sqrt{12}}$

91. $\dfrac{\sqrt{4}}{\sqrt{3}}$

92. $\sqrt{\dfrac{3}{8}}$

93. $\sqrt{\dfrac{9}{2}}$

94. $\sqrt{\dfrac{3}{5}}$

95. $\dfrac{-10}{3\sqrt{5}}$

96. $\dfrac{21}{5\sqrt{7}}$

Applications

Solve.

97. Isaac Newton fell asleep under an apple tree thinking about math. While he was sleeping, a squirrel knocked an apple off of a branch of the tree. The equation $t = \sqrt{\dfrac{2d}{9.8}}$ can be used to find the time t in seconds it takes for the apple to drop a certain distance d, where d is in meters. Round all answers to the nearest hundredth.

a. If the apple was connected to a branch 2 m above Newton's head, how long would it take before the apple hit Newton's head?

b. If the squirrel knocked a second apple off a branch that was 5 m above Newton's head, how long would it take before the apple hit Newton's head?

c. Suppose the second apple missed Newton's head and landed on the ground instead. If Newton's head was 0.8 m above the ground, how long would it take for the apple to hit the ground?

Use the following two formulas associated with electricity to answer Exercises 61-64.

$I = \sqrt{\dfrac{P}{R}}$

$E = \sqrt{PR}$

P = power (in watts)
I = current (in amperes)
E = voltage (in volts)
R = resistance (in ohms, Ω)

98. What is the current in amperes of a light bulb that produces 150 watts of power and has a 25 Ω resistance?

99. If a light bulb has a resistance of 30 Ω and produces 90 watts of power, what is its current in amperes?

100. How many volts of electricity would Meghan need to produce 48 Ω of resistance from a 300 watt lamp?

101. A 5000 Ω resistor is rated at 2.5 watts. What is the maximum voltage of electricity that should be connected across it?

102. A nut company is determining how to package their new type of party mix. The marketing department is experimenting with different-sized cans for the party mix packaging. The designers use the equation $r = \sqrt{\frac{V}{h\pi}}$ to determine the radius of the can for a certain height h and volume V. The company decides they want the can to have a volume of 1200π cm^3. Keep your answers in simplified radical form.

a. Find the radius of the can if the height is 12 cm.

b. Find the radius of the can if the height is 10 cm.

c. Find the radius of the can if the height is 8 cm.

103. For a complete radio circuit, $d = \sqrt{2g} + \sqrt{2h}$, where d equals the visual horizon distance and g and h are the heights of the radio antennas at the respective stations. What is d when $g = 75$ ft and $h = 85$ ft?

Writing & Thinking

104. Under what conditions is the expression $\sqrt{a}$ not a real number?

105. Discuss, in your own words, why the square root of a negative number is not a real number.

106. In your own words, explain how to rationalize the denominator of a fraction containing the sum or difference of square roots in the denominator. Why does this work?

Quadratic equations should be solved by factoring whenever possible. Factoring is generally the easiest method. This fact was illustrated in Examples 11 and 12. However, the quadratic formula will always give the solutions whether they are rational, irrational, or nonreal (as shown in Example 13). Thus, the quadratic formula is a very useful tool. **In future courses in mathematics, every text and instructor will assume that you know or can easily remember the quadratic formula.**

13. Use the quadratic formula to solve the following quadratic equation.

$x^2 + 3x + 5 = 0$

Example 13 The Quadratic Formula and Nonreal Solutions

Use the quadratic formula to solve the following quadratic equation.

$x^2 + x + 1 = 0$

Solution

$a = 1,\ b = 1,\ c = 1$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

There is no real solution. This example illustrates the fact that not every equation has real solutions. Such solutions are called **nonreal complex numbers** and are discussed in more depth in higher-level math courses.

Now work margin exercise 13.

Completion Example Answers

6. 20; $\pm 2\sqrt{5}$; $-2 \pm 2\sqrt{5}$ **10.** $a = 2, b = -5, c = -3$; $x = \dfrac{-(-5) + \sqrt{(-5)^2 - 4(2)(-3)}}{2 \cdot 2} = \dfrac{5 \pm 7}{4}$ $x = 3$ and $x = -\dfrac{1}{2}$

Margin Exercise Answers

1. $x = \pm 2\sqrt{3}$ **2.** $x = -5 \pm 2\sqrt{10}$ **3.** $x = 1 \pm 5\sqrt{3}$ **4.** $x = 2 \pm \sqrt{2}$ **5.** no solution
6. $x = 4 \pm 2\sqrt{3}$ **7.** $x = -2 \pm \sqrt{5}$ **8.** $x = \dfrac{-1 \pm \sqrt{13}}{4}$ **9.** $x = \pm\dfrac{2\sqrt{30}}{3}$ **10.** $x = \dfrac{-3 \pm \sqrt{5}}{4}$
11. $x = \dfrac{3 \pm \sqrt{13}}{2}$ **12.** $x = -\dfrac{5}{2}$ **13.** no real solution

5.6 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A direct way of solving an equation of the form $x^2 = c$ is to use the _____ _____ property.

2. When using the square root property, we do not set the polynomial expression equal to ____.

3. When solving a quadratic equation, the quadratic formula will _____ work.

4. The expression $b^2 - 4ac$ is called the _____.

5. If $b^2 - 4ac > 0$, then there are ____ real solutions.

6. Quadratic equations should be solved by _____ whenever possible.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Quadratic equations that are not easily solved using factoring might be solved by the square root method.

8. The quadratic formula will always work when solving quadratic equations.

9. When using the quadratic formula, if the discriminant is greater than zero, there are infinite solutions.

10. If the discriminant is less than zero, there is no real solution.

Practice

Solve the following quadratic equations by using the square root method. Write each radical in simplest form. See Examples 1 through 6.

1. $x^2 = 121$

2. $x^2 = 81$

3. $3x^2 = 108$

4. $5x^2 = 245$

5. $x^2 = 35$

6. $x^2 = 42$

7. $x^2 - 62 = 0$

8. $x^2 - 75 = 0$

9. $x^2 - 45 = 0$

10. $x^2 - 98 = 0$

11. $3x^2 = 54$

12. $5x^2 = 60$

13. $9x^2 = 4$

14. $4x^2 = 25$

15. $(x-1)^2 = 4$

16. $(x+3)^2 = 9$

17. $(x+2)^2 = -25$

18. $(x-5)^2 = 36$

19. $(x+1)^2 = \frac{1}{4}$

20. $(x-9)^2 = -\frac{9}{25}$

21. $(x-3)^2 = \frac{4}{9}$

22. $(x-2)^2 = \frac{1}{16}$

23. $(x-6)^2 = 18$

24. $(x+8)^2 = 75$

25. $2(x-7)^2 = 24$

26. $3(x+11)^2 = 60$

27. $(3x+4)^2 = 27$

28. $(2x+1)^2 = 48$

29. $(5x-2)^2 = 63$

30. $(4x-3)^2 = 125$

31. $3(x-5)^2 + 5 = -25$

32. $2(x-6)^2 - 11 = 25$

Rewrite each of the quadratic equations in the form $ax^2 + bx + c = 0$ with $\boldsymbol{a} > 0$; then identify the constants $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$.

33. $x^2 - 3x = 2$

34. $x^2 + 2 = 5x$

35. $x = 2x^2 + 6$

36. $5x^2 = 3x - 1$

37. $4x + 3 = 7x^2$

38. $x = 4 - 3x^2$

39. $4 = 3x^2 - 9x$

40. $6x + 4 = 3x^2$

41. $x^2 + 5x = 3 - x^2$

42. $x^2 + 4x - 1 = 2x + 3x^2$

Solve each of the quadratic equations by using the quadratic formula. See Examples 7 through 13.

43. $x^2 - 4x - 1 = 0$

44. $x^2 - 3x + 1 = 0$

45. $x^2 - 3x = 4$

46. $x^2 + 5x = 2$

47. $-2x^2 + x = -1$

48. $-3x^2 + x = -1$

49. $5x^2 + 3x - 2 = 0$

50. $-2x^2 + 5x - 1 = 0$

51. $9x^2 = 3x$

52. $4x^2 - 81 = 0$

53. $x^2 - 7 = 0$

54. $2x^2 + 5x - 3 = 0$

55. $x^2 + 4x = x - 2x^2$

56. $x^2 - 2x + 1 = 2 - 3x^2$

57. $3x^2 + 4x = 0$

58. $4x^2 - 10 = 0$

59. $\frac{2}{5}x^2 + x - 1 = 0$

60. $2x^2 + 3x - \frac{3}{4} = 0$

61. $-\frac{x^2}{2} - 2x + \frac{1}{3} = 0$

62. $\frac{x^2}{3} - x - \frac{1}{5} = 0$

63. $(2x + 1)(x + 3) = 2x + 6$

64. $(x + 5)(x - 1) = -3$

65. $\frac{5x + 2}{3x} = x - 1$

66. $-6x^2 = -3x - 1$

67. $4x^2 = 7x - 3$

68. $9x^2 - 6x - 1 = 0$

Solve each of the quadratic equations by using any method.

69. $2x^2 + 7x + 3 = 0$

70. $5x^2 - x - 4 = 0$

71. $3x^2 - 7x + 1 = 0$

72. $2x^2 - 2x - 1 = 0$

73. $10x^2 = x + 24$

74. $9x^2 + 12x = -2$

75. $3x^2 - 11x = 4$

76. $5x^2 = 7x + 5$

77. $-4x^2 + 11x - 5 = 0$

78. $-4x^2 + 12x - 9 = 0$

79. $10x^2 + 35x + 30 = 0$

80. $6x^2 + 2x = 20$

81. $25x^2 + 4 = 20$

82. $3x^2 - 4x + \frac{1}{3} = 0$

83. $\frac{3}{4}x^2 - 2x + \frac{1}{8} = 0$

84. $\frac{11}{2}x + 1 = 3x^2$

85. $\frac{3}{7}x^2 = \frac{1}{2}x + 1$

86. $\frac{35}{4x} = x - 1$

$$(x+7)^2 + x^2 = 13^2$$
$$x^2 + 14x + 49 + x^2 = 169$$
$$2x^2 + 14x + 49 - 169 = 0$$
$$2x^2 + 14x - 120 = 0$$
$$2(x^2 + 7x - 60) = 0$$
$$2(x-5)(x+12) = 0$$
$$x - 5 = 0 \quad \text{or} \quad x + 12 = 0$$
$$x = 5 \qquad \cancel{x = -12}$$

A negative number does not fit the conditions of the problem.

Thus, $x = 5$ and $x + 7 = 12$.

Check

$$5^2 + 12^2 \stackrel{?}{=} 13^2$$
$$25 + 144 \stackrel{?}{=} 169$$
$$169 = 169$$

True statement

The width is 5 feet and the length is 12 feet.

Now work margin exercise 8.

Margin Exercise Answers

1. 15, 10 or −15, −10 **2.** 21 rows **3.** 16 feet by 18 feet **4.** 12 feet by 14 feet or 7 feet by 24 feet **5.** 33.9 ft **6.** 127.3 ft **7.** 8 feet **8.** 6 ft and 8 ft

5.7 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. Once an application problem has been read and understood, you should assign a _____ to the _____ quantities.

2. After an application problem has been solved, it is important to _____ the solution with the problem to make sure the answer makes sense.

3. The formula (or equation) related to the Pythagorean Theorem is _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

4. The Pythagorean Theorem states that if the two legs of a right triangle are added, the sum will equal the hypotenuse.

5. The Pythagorean Theorem can be used with any triangle.

Applications

Write a quadratic equation for each of the following word problems. Then solve the word problem. Remember to check each solution with the wording of the original problem to make sure it is reasonable.

1. One number is eight more than another. Their product is −16. What are the numbers?

2. One number is 10 more than another. If their product is −25, find the numbers.

3. The square of an integer is equal to seven times the integer. Find the integer.

4. The square of an integer is equal to twice the integer. Find the integer.

5. If the square of a positive integer is added to three times the integer, the result is 28. Find the integer.

6. If the square of a positive integer is added to three times the integer, the result is 54. Find the integer.

7. One number is three more than another. Their product is 40. Find the numbers.

8. One positive number is three more than twice another. If the product is 27, find the numbers.

9. One positive number is five more than another. The sum of their squares is 53. What are the numbers?

10. One number is five less than another. The sum of their squares is 97. Find the numbers.

11. The difference between two positive integers is 4. If the smaller is added to the square of the larger, the sum is 38. Find the integers.

12. One positive number is 3 more than twice another. If the square of the smaller is added to the larger, the sum is 51. Find the numbers.

13. The product of a negative integer and 5 less than twice the integer equals the integer plus 56. Find the integer.

14. Find a positive integer such that the product of the integer with a number three less than the integer is equal to the integer increased by 32.

15. The length of a rectangle is twice the width. The area is 72 square inches. Find the length and width of the rectangle.

16. The length of a rectangle is three times the width. If the area is 147 square centimeters, find the length and width of the rectangle.

17. The length of a rectangle is four times the width. If the area is 64 square feet, find the length and width of the rectangle.

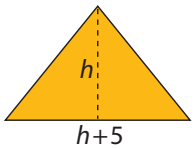
18. The length of a rectangle is five times the width. If the area is 180 square inches, find the length and width of the rectangle.

19. The width of a rectangle is 4 feet less than the length. The area is 45 square feet. Find the length and width of the rectangle.

20. The length of a rectangular yard is 3 meters greater than the width. If the area of the yard is 54 square meters, find the length and width of the yard.

21. The height of a triangle is 4 feet less than the base. The area of the triangle is 16 square feet. Find the length of the base and the height of the triangle.

22. The base of a triangle exceeds the height by 5 meters. If the area is 12 square meters, find the length of the base and the height of the triangle.



23. The base of a triangle is 6 inches greater than the height. If the area is 20 square inches, find the length of the base.

24. The base of a triangle is 3 feet less than the height. The area is 9 square feet. Find the height.

25. The perimeter of a rectangle is 32 inches. The area of the rectangle is 48 square inches. Find the dimensions of the rectangle.

26. The area of a rectangle is 24 square centimeters. If the perimeter is 20 centimeters, find the length and width of the rectangle.

27. A rectangle has a length 5 m less than twice its width. If the area is 63 m^2, find the dimensions of the rectangle.

28. The length of a rectangle is 2 cm less than 3 times its width. If the area of the rectangle is 225 cm^2, find the dimensions of the original rectangle.

29. Each side of a square is increased by 10 cm. The area of the resulting square is 9 times the area of the original square. Find the length of the sides of the original square.

30. If 5 m are added to each side of a square, the area of the resulting square is four times the area of the original square. Find the length of the sides of the original square.

31. The diagonal of a rectangle is 13 m. The length is 2 m more than twice the width. Find the dimensions of the rectangle.

32. The length of a rectangle is 4 m more than its width. If the diagonal is 20 m, what are the dimensions of the rectangle?

33. An orchard has 140 apple trees. The number of rows exceeds the number of trees per row by 13. How many trees are there in each row?

34. One formation for a drill team is rectangular. The number of members in each row exceeds the number of rows by 3. If there is a total of 108 members in the formation, how many rows are there?

35. A theater can seat 144 people. The number of seats in each row is 7 more than the number of rows. How many rows of seats are there?

36. The parking garage at Baltimore-Washington International Airport contains 8400 parking spaces. The number of cars that can be parked on each floor exceeds the number of floors by 1675. How many floors are there in the parking garage?

37. One bookshelf in the public library can hold 175 books. The number of books on each shelf exceeds the number of shelves by 18. How many books are on each shelf?

38. The length of a rectangle is 7 centimeters greater than the width. If 4 centimeters are added to both the length and width, the new area would be 98 square centimeters. Find the dimensions of the original rectangle.

39. Susan is going to fence a rectangular flower garden in her back yard. She has 50 feet of fencing and she plans to use the house as the fence on one side of the garden. If the area is 300 square feet, what are the dimensions of the flower garden?

40. A rancher is going to build a corral with 52 yards of fencing. He is planning to use the barn as one side of the corral. If the area is 320 square yards, what are the dimensions?

41. A telephone pole is to have a guy wire attached to its top and anchored to the ground at a point that is at a distance 34 feet less than the height of the pole from the base. If the wire is to be 2 feet longer than the height of the pole, what is the height of the pole?

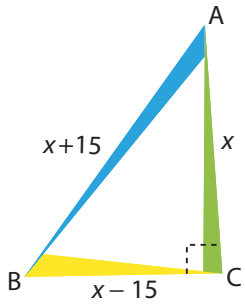
42. Lucy is standing next to the General Sherman tree in Sequoia National Park, home of some of the largest trees in the world. The distance from Lucy to the base of the tree is 71 m less than the height of the tree. If the distance from Lucy to the top of the tree is 1 m more than the height of the tree, how tall is the General Sherman?

43. A Christmas tree is supported by a wire that is 1 foot longer than the height of the tree. The wire is anchored at a point whose distance from the base of the tree is 49 feet shorter than the height of the tree. What is the height of the tree?

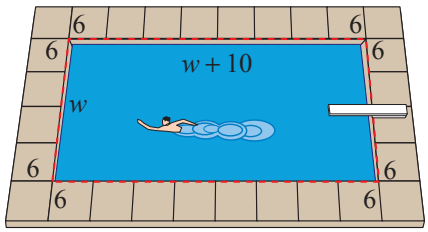
44. An architect wants to draw a rectangle with a diagonal of 13 inches. The length of the rectangle is to be 2 inches more than twice the width. What dimensions should she make the rectangle?

45. Incline mats, or triangle mats, are offered with different levels of incline to help gymnasts learn basic moves. As the name may suggest, two sides of the mat are right triangles. If the height of the mat is 28 inches shorter than the length of the mat and the hypotenuse is 8 inches longer than the length of the mat, what is the length of the mat?

46. Bill uses mirrors to augment the "laser experience" at a laser show. At one show, he places three mirrors, A, B, C, in a right triangular form. If the distance between A and B is 15 m more than the distance between A and C, and the distance between B and C is 15 m less than the distance between A and C, what is the distance between mirror A and mirror C?



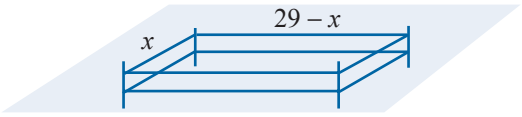
47. The Wilsons have a rectangular swimming pool that is 10 ft longer than it is wide. The pool is completely surrounded by a concrete deck that is 6 ft wide. The total area of the pool and the deck is 1344 ft^2. Find the dimensions of the pool.



48. An apartment building has the same number of units on each floor. The building has five times as many units per floor as number of floors, and there are 405 units total. How many floors does the building have?

49. A large U-Haul truck is 8 ft tall. The length of the truck is 4 ft longer than three times the width. What are the dimensions of the truck if the volume is 1590 ft^3?

50. A farmer fenced in a 198-square-meter portion of his field with 58 meters of fencing. What are the length and width of the field? (**Hint:** The length plus the width is equal to half of the perimeter.)

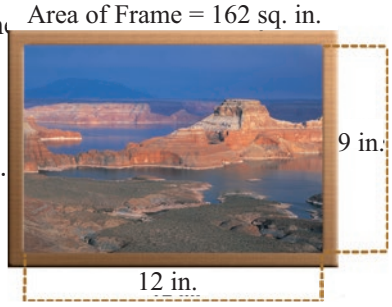


a. Write an equation to express the area of the field.

b. Solve the equation from Part **a.** for the variable.

c. Use the answer from Part **b.** to determin the field.

51. A photograph 9 in. wide and 12 in. long is surrounded by a frame of uniform thickness. The area of the frame itself, not including the center, is 162 in.^{2} Find the thickness of the frame.



52. The Mona Lisa is a famous painting by Leonardo da Vinci. The painting is 30 in. by 21 in. It is surrounded by a frame of uniform thickness whose area (not including the center) is 756 in.^{2} Find the thickness of the frame.

53. A 40-volt generator with a resistance of 4 ohms delivers power externally of $40I - 4I^2$ watts, where I is the current measured in amperes. Find the current needed for the generator to deliver 100 watts of power.

54. Find the current needed for the 40-volt generator in Exercise 17 to deliver 64 watts of power.

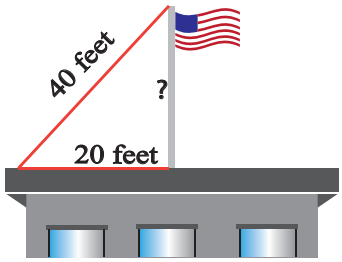
55. Raymond operates a small sign-making business. He finds that if he charges x dollars for each sign, he sells $40 - x$ signs per week. What is the least number of signs he can sell to have an income of \$336 in one week?

56. It costs Mrs. Snow \$3 to build a picture frame. She estimates that if she charges x dollars each, she can sell $60 - x$ frames per week. What is the lowest price necessary to make a profit of \$432 each week?

57. The Ski Club is planning to charter a bus to a ski resort. The cost will be \$900 and each member will share the cost equally. If the club had 15 more members, the cost per person would be \$10 less. How many are in the club now? (**Hint:** If x = number in club now, $\frac{900}{x}$ = cost per person.)

58. A ladder is 30 ft long and you want to place the base of the ladder 10 ft from the base of a building. About how far up the building (to the nearest tenth of a foot) will the ladder reach?

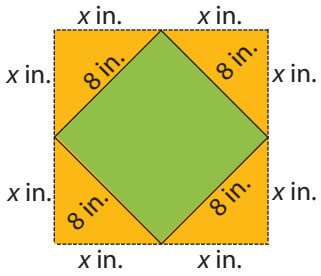
59. A flag pole is on top of a building and is held in place by steel cables attached to the top of the pole. If one such cable is 40 ft long and is attached at a point on the roof of the building 20 ft from the base of the flag pole, what is the length of the flag pole (to the nearest tenth of a foot)?



60. The approximate horizontal distance d, in yards, that a football travels after it leaves a player's hands at a speed of v miles per hour is given by $d = 0.022v^2$. Determine the exit velocity of a football that travelled 50 yards.

Writing & Thinking

61. The pattern in Kara's linoleum flooring is in the shape of a square 8 inches on a side with right triangles (with legs whose lengths are x inches) placed on each side of the original square so that a new larger square is formed. What is the area of the new square? Explain why you do not need to find the value of x.



62. Suppose that you are to solve an applied problem and the solution leads to a quadratic equation. You decide to use the quadratic formula to solve the equation. Explain what restrictions you must be aware of when you use the formula.

5.8 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. When a linear function is written in the form $f(x) = mx + b$, m is the _____ and b is the _____.
2. The turning point of a parabola is called the _____.
3. A parabola is a mirror image of itself on either side of the axis of _____.
4. For a quadratic function $y = ax^2 + bx + c$, if $a < 0$, then the parabola opens _____.
5. If a parabola opens upward, and (h, k) is the lowest point, the value k is called the _____ value of the function.
6. If a parabola opens downward, and $(h. k)$ is the highest point, the value k is called the _____ value of the function.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The vertex of a vertical parabola is the lowest point on the parabola.
8. The maximum or minimum value of a quadratic function written in general form can be found by letting $x = -\frac{b}{2a}$ and solving for y.
9. Quadratic functions of the form $y = ax^2 + bx + c$, have a line of symmetry at $x = \frac{b}{2a}$.

Practice

For each quadratic function, **a.** find its vertex, **b.** find its line of symmetry, **c.** locate (or estimate) its x-intercepts, and **d.** graph the function. (**Note:** If solving the quadratic equation results in nonreal solutions, then the graph does not cross the x-axis.) See Examples 1 through 3.

1. $y = x^2 + 4$
2. $y = x^2 - 6$
3. $y = 8 - x^2$
4. $y = -2 - x^2$
5. $y = x^2 - 2x - 3$
6. $y = x^2 - 4x + 5$
7. $y = x^2 + 6x$
8. $y = x^2 - 4x$
9. $y = -x^2 - 4x + 2$
10. $y = -x^2 - 5x + 4$
11. $y = 2x^2 - 10x + 3$
12. $y = 2x^2 - 12x - 5$
13. $y = x^2 + 7x - 4$
14. $y = x^2 + 5x + 4$

15. $y = -x^2 + x - 3$

16. $y = -x^2 - 7x + 3$

17. $y = 2x^2 + 7x - 4$

18. $y = 2x^2 - x - 3$

19. $y = 3x^2 + 5x + 2$

20. $y = 3x^2 - 9x + 5$

Applications

Solve the following word problems. See Example 4.

21. The perimeter of a rectangle is 60 yd. What are the dimensions of the rectangle with maximum area?

22. The perimeter of a rectangle is 56 ft. What are the dimensions of the rectangle with maximum area?

In the following exercises use the formula $h = at^2 + v_0t + h_0$, where $\boldsymbol{h}$ is the height (in feet) of a projectile after time, $\boldsymbol{t}$ (in seconds). The initial velocity is $\boldsymbol{v}_0$ and the initial height is $\boldsymbol{h}_0$.

23. A ball is thrown vertically upward from the ground with an initial velocity of 112 ft per sec.

a. When will the ball reach its maximum height?

b. What will be the maximum height?

24. A ball is thrown vertically upward from the ground with an initial velocity of 104 ft per sec.

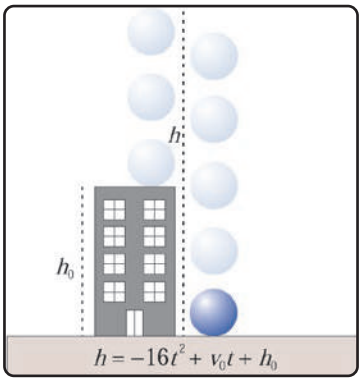
c. When will the ball reach its maximum height?

d. What will be the maximum height?

25. A stone is projected vertically upward from a platform that is 32 ft high at a rate of 128 ft per sec.

a. When will the stone reach its maximum height?

b. What will be the maximum height?



26. A stone is projected vertically upward from a platform that is 20 ft high at a rate of 160 ft per sec.

c. When will the stone reach its maximum height?

d. What will be the maximum height?

In business, the term revenue represents income. The revenue (income) is found by multiplying the number of units sold times the price per unit.
[Revenue = (price) · (units sold)]

27. A store owner estimates that by charging x dollars each for a certain lamp, he can sell $40 - x$ lamps each week. What price will yield maximum revenue?

a. What is the revenue function $R(x)$?

b. What price will yield a maximum revenue?

28. When fishing reels are priced at x dollars each, local consumers will buy $36 - x$ fishing reels.

a. What is the revenue function $R(x)$?

b. What price will yield a maximum revenue?

29. A manufacturer produces calculators. She estimates that by selling them for x dollars each, she will be able to sell $80 - 2x$ calculators each week.

a. What is the revenue function $R(x)$?

b. What price will yield a maximum revenue?

c. What will be the maximum revenue?

30. A manufacturer produces radios. He estimates that by selling them for x dollars each, he will be able to sell $100 - x$ radios each month.

a. What is the revenue function $R(x)$?

b. What price will yield a maximum revenue?

c. What will be the maximum revenue?

Writing & Thinking

31. Use your graphing calculator to graph each of the following quadratic functions.

a. Graph each equation one at a time.

b. Graph all three parabolas at the same time (all three curves should appear on the display).

i. $y = x^2 + 1$ ii. $y = x^2 + 3$ iii. $y = x^2 - 4$

32. In your own words, state the effect on the graph of $y = x^2$ by changing the constant term k in the function $y = x^2 + k$.

33. Use your graphing calculator to graph each of the following quadratic functions

a. Graph each equation one at a time.

b. Graph all three parabolas at the same time (all three curves should appear on the display).

i. $y=(x-3)^2$ ii. $y=(x-5)^2$ iii. $y=(x+2)^2$

34. In your own words, state the effect on the graph of $y=x^2$ by changing the constant term h in the function $y=(x-h)^2$.

35. Discuss the general relationship of the graph of a function of the form $y=(x-h)^2+k$ to the graph of the function $y=x^2$.

5. The average movie run time is around 1.8 hours. How many minutes long is the average movie?

Example 5 Application: Converting US Units of Measure

A fully grown African elephant can weigh as much as 7.5 tons. How many pounds is this?

Solution

There are 2000 pounds in 1 ton. Using a unit fraction to convert from tons to pounds gives the following.

$$7.5\ \text{T} = 7.5\ \text{T} \cdot \frac{2000\ \text{lb}}{1\ \text{T}} = 7.5 \cdot 2000\ \text{lb} = 15{,}000\ \text{lb}$$

Thus, a fully grown African elephant can weigh as much as 15,000 pounds.

Now work margin exercise 5.

6. How many fluid ounces are in 8 gallons of apple juice?

Example 6 Application: Converting US Units of Measure

Determine how many seconds are in a 5-day work week assuming an 8 hr work day.

Solution

This number can be found as follows.

$$5\ \text{days} = 5\ \text{days} \cdot \frac{8\ \text{hr}}{1\ \text{day}} \cdot \frac{60\ \text{min}}{1\ \text{hr}} \cdot \frac{60\ \text{sec}}{1\ \text{min}} = 5 \cdot 8 \cdot 60 \cdot 60\ \text{sec} = 144{,}000\ \text{sec}$$

Thus, there are 144,000 seconds in a 5-day work week.

Now work margin exercise 6.

Margin Exercise Answers

1. a. 1 **b.** 2 **c.** 24 **d.** 5280 **2. a.** 108 **b.** 32 **c.** 4 **d.** $3\frac{3}{4}$ or 3.75 **3.** $4\frac{1}{2}$ or 4.5 pounds **4. a.** 5 **b.** 136 **c.** 16 **d.** 15,840 **5.** 108 minutes **6.** 1024 fluid ounces

6.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. There are two cups in 1 ____, which is equivalent to ___ fluid ounces.

2. Sixteen ounces equals 1 _____ and 2000 _____ equals 1 ton.

3. Both 3 feet and 36 inches equal 1 ____.

4. When converting from one unit of measure to another smaller unit, _____ is necessary.

5. When using a unit fraction for conversions, the numerator should have the same units as the result and the denominator should be in the units to be _____.

6. A fraction equivalent to 1 is called a ____ fraction.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Capacity can be measured using ounces, quarts, and gallons.

8. One mile is equivalent to 2000 feet.

9. To convert from smaller units to larger units, division will be required.

10. Multiplication by a unit fraction does not change the value of the expressions being converted.

Practice

Use the units you have memorized from Table 1 to convert each measurement. See Example 1.

1. 1 ft = ___ in.
2. 1 week = ___ days
3. 2000 lb = ___ T
4. 2 c = ___ pt
5. 4 qt = ___ gal
6. 36 in. = ___ yd
7. 1 mi = ___ ft
8. 1 lb = ___ oz
9. 24 hr = ___ day
10. 12 in. = ___ ft
11. 1 c = ___ fl oz
12. 1 qt = ___ pt

Use multiplication or division to convert each measurement. See Example 2.

13. 3 ft = ___ in.
14. 2 lb = ___ oz
15. 5 min = ___ sec
16. 3 yd = ___ ft
17. 10,560 ft = ___ mi
18. 6000 lb = ___ T
19. 3 weeks = ___ days
20. 5 yd = ___ ft
21. 72 in. = ___ yd
22. 48 in. = ___ ft
23. 90 min = ___ hr
24. 9 qt = ___ gal

Use unit fractions to convert each measurement. First write the unit fraction you are going to use, then perform the conversion. See Example 4.

25. 7 yd = ___ ft
26. 5 qt = ___ pt
27. 6 pt = ___ qt
28. 32 fl oz = ___ c

29. 13 qt = ___ gal

30. 3 pt = ___ fl oz

31. 18 in. = ___ ft

32. 24 oz = ___ lb

33. 3 mi = ___ ft

34. 5 T = ___ lb

35. 7920 ft = ____ mi

36. 7000 lb = ___ T

Convert each measurement.

37. 4 pt = ___ c

38. 3 hr = ___ min

39. 16 T = ___ lb

40. 10 mi = ___ ft

41. 96 hr = ___ days

42. 39 ft = ___ yd

43. 5.5 lb = ___ oz

44. 3.5 ft = ___ in.

45. 6 qt = ___ gal

46. 150 min = ___ hr

47. 2.5 min = ___ sec

48. 1.5 yd = ___ in.

Applications

Solve.

49. Find the area (in square feet) of a rectangle that is $1\frac{1}{2}$ ft by 7 in.

50. A small jar of peanut butter sells for \$0.08 per ounce. A large jar of peanut butter sells for \$1.20 per pound. Which is the better buy and by how much (in cents per ounce)?

51. Sheer fabric costs \$7.99 per yard. If it will take 35 feet of fabric to make drapes for the entire house, how much must you spend on fabric for the drapes, to the nearest cent?

52. While cleaning out the garage, Nelson discovers some containers of oil that need to be taken in for recycling. The containers hold 20 fluid ounces, 3 cups, and 1 quart, respectively. Find the total amount of oil ready to be recycled (in fluid ounces).

53. Joel runs track for Northside High School. He runs the 220-yard sprint, the $\frac{1}{4}$-mile hurdles and the $\frac{1}{2}$-mile relay (of which he runs one leg, which is $\frac{1}{8}$ of a mile). How far does Joel run in total (in miles)?

54. How many scoops will it take a 2-ton crane (it can scoop 2 tons of material at one time) to move 17,000 pounds of dirt?

55. A ball dropped from the top of a building hits the ground at a speed of 22 feet per second. How fast does the ball hit the ground in miles per hour?

56. The author of this textbook spent 1 year, 23 weeks, 5 days, and 14 hours writing it. How many seconds is this? (**Hint:** There are 52 weeks in a year.)

57. A small bag of cookies sells for \$0.12 per ounce. A large bag of cookies sells for \$2.00 per pound. Which is the better buy and by how much (in cents per ounce)?

58. Adrian exercised three days during the past week. He exercised 50 minutes and 15 seconds during the first day, 84 minutes and 25 seconds on the second day, and 45 minutes and 20 seconds on the third day. How many hours did he spend exercising during the week?

Writing & Thinking

59. Colby needs to find out how many yards are in one mile. What two sets of equivalent units would he need to make that determination?

60. In your own words, explain when you would multiply and when you would divide when converting between units.

61. Briefly describe a unit fraction and explain when and how it would be used.

62. Give at least two examples of when you might want to convert between units of measure (outside of a class).

6.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. In the metric system, commas are not used. Digits are instead grouped in _____ with spaces between the groups.
2. If there is no whole number of units in the metric system, a/an _____ is written to the left of the decimal point.
3. The basic unit of length in the metric system is the _____.
4. In the metric system, conversions from smaller to larger units require _____ by a power of 10.
5. To change from larger to smaller metric units, _____ by a power of 10.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. To change from smaller units to larger units, multiplication must be used.
7. Units of length in the metric system are named by putting a prefix in front of the basic unit meter, for example, centimeter.
8. In metric units, a square that is 1 centimeter long on each side is said to have an area of 1 centimeter.

Practice

Use this ruler to help measure (or approximate) each length. (Answers will vary.) The ruler shown here is marked with millimeters and centimeters.

1. The width of a pencil
2. The diameter (distance across) of a penny
3. The width of your thumb
4. The width of your little finger
5. The thickness of a watch band
6. The height of a cell phone
7. The width of a paper clip
8. The thickness of a dime
9. The length and width of a dollar bill
10. The length and width of your school ID

What metric unit of length would you use to measure each item? (Choose from millimeters, meters, and kilometers.)

11. A sprint distance on a track

12. The length of a room

13. The height of a building

14. The distance between Houston, TX, and San Antonio, TX

15. The width of the head of a straight pin

16. The thickness of a stick of gum

Convert each measurement. See Examples 1, 3, and 4.

17. 3 m = ___ cm

18. 60 m = ___ dm

19. 0.8 m = ___ cm

20. 1.9 cm = ___ mm

21. 1.5 m = ___mm

22. 13.6 km = ___ m

23. 36 mm = ___ cm

24. 140 cm = ___ dm

25. 82 cm = ___ m

26. 4.8 mm = ___ cm

27. 5.25 cm = ___m

28. 19.77 m = ___km

29. 750 mm = ___ m

30. 185 m = ___ km

31. Change 245 mm to meters.

32. Change 87 mm to meters.

33. Convert 23 cm to meters.

34. Convert 3.2 mm to centimeters.

35. How many kilometers are in 10 000 m?

36. How many meters are in 1100 cm?

37. What number of meters is equivalent to 20 000 cm?

38. What number of kilometers is equivalent to 140 000 m?

39. Express 679 cm in kilometers.

40. Express 3872 mm in kilometers.

Convert each measurement. See Example 5.

41. 150 300 000 000 bytes = ___ GB

42. 6 500 000 hertz = ___ megahertz

43. 30 MB = ___ bytes

44. 24 GB = ___ bytes

Convert each measurement. See Examples 6 through 9.

45. 9.6 cm^2 = _____ mm^2

46. 4.52 cm^2 = _____ mm^2

47. 500 mm^2 = _____ cm^2

48. 39 mm^2 = _____ cm^2

49. 0.5 m^2 = _____ mm^2

50. 3 m^2 = _____ mm^2

51. 13 dm^2 = _____ cm^2 = _____ mm^2

52. 6.4 dm^2 = _____ cm^2 = _____ mm^2

53. 11.5 m^2 = _____ cm^2 = _____ mm^2

54. 3.6 m^2 = _____ cm^2 = _____ mm^2

55. 0.04 m^2 = _____ cm^2 = _____ mm^2

56. 0.6 m^2 = _____ cm^2 = _____ mm^2

57. 6.7 a = _____ m^2

58. 0.45 a = _____ m^2

59. 200 a = _____ m^2

60. 0.8 a = _____ m^2

61. Change 5.75 km^2 to hectares.

62. Change 0.4 km^2 to hectares.

63. 9.56 ha = _____ a = _____ m^2

64. 0.27 ha = _____ a = _____ m^2

65. 6.25 m^2 = _____ a = _____ ha

66. 35 m^2 = _____ a = _____ ha

Applications

Solve.

67. A triangle has a base measuring 4 cm and a height measuring 16 mm. Determine the area of the triangle in cm^2.

68. For the 10-km-long Bridge Run, officials set up water tables every 635 meters, starting at the start line. How many water tables are set up for the Bridge Run?

69. The current world record for the men's long jump is 8.95 m, set by Mike Powell in 1991. Bob Beamon held the previous record of 8.90 m for 23 years. By how many mm did Mike Powell beat Bob Beamon's record?

70. DeQuan has four pieces of metal that must be welded end-to-end in order to form a spill tray. If the pieces measure 35 cm, 112 mm, 4 decimeters, and 1.2 meters, how long will the tray be (in meters)?

71. A rectangular backyard measures 4 hectometers by 11 dekameters. How many square meters is the backyard?

72. A section of railroad track measuring 2.1 km in length needs to be replaced. Each railroad tie is 4 decimeters wide and they are to be spaced 0.8 m apart. How many railroad ties will be needed to complete this section of track?

73. A certain type of computer processor has a speed of 6 000 000 000 hertz. What is the speed of this computer processor in gigahertz?

74. A demolition crew used 5 megatons of dynamite to clear away a rock formation. How many metric tons of dynamite did they use?

75. Two hard drives contain 3.75 terabytes and 4.25 terabytes of information, respectively. What is the difference in the amount of information stored on the two hard drives in gigabytes? (**Hint:** Convert terabytes to bytes, then to gigabytes.)

76. A camera with 14 megapixels has how many more pixels than a camera with 12 megapixels?

77. A farm has an area of 7500 ares. What is the area of the farm in hectares?

78. Arches National Park has an area of 310,300,000 m^2. What is the area of Arches National Park in square kilometers?

79. Two lakes have areas of 1.25 km^2 and 1.18 km^2, respectively. What is the difference in area of the two lakes in square meters?

80. A US nickel has an area of approximately 353.32 mm^2. What is the area of a nickel in square centimeters?

Writing & Thinking

81. Compare and contrast ease of converting units in the US customary system and the metric system.

82. Discuss how to convert units within the metric system.

83. Discuss the meaning of prefixes like milli-, centi-, and kilo- in metric units. Give examples.

84. Discuss the meaning of the prefixes kilo-, mega-, giga-, and tera- in technology.

8. 825 milligrams of table salt are added to a meal. Convert this mass to grams using a unit fraction or a metric conversion line.

Example 8 Application: Converting Metric Units of Weight

A box of detergent weighs 475 grams. Convert this mass to kilograms **a.** using a unit fraction and **b.** using a metric conversion line.

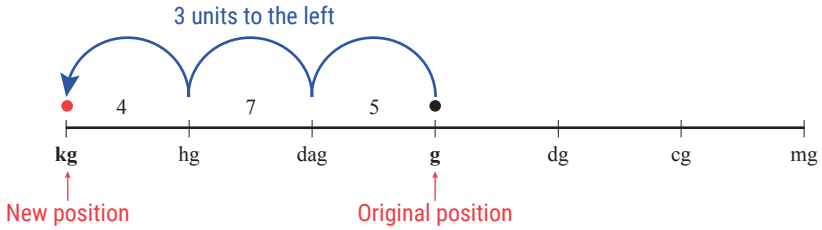
Solution

There are 1000 grams in 1 kilogram. We can convert 475 g to kg as follows.

a. Using a unit fraction:

$$475\ \text{g} = 475\ \cancel{\text{g}} \cdot \frac{1\ \text{kg}}{1000\ \cancel{\text{g}}} = \frac{475}{1000}\ \text{kg} = 0.475\ \text{kg}$$

b. Using a metric conversion line:



Each method shows that 475 g of detergent is equivalent to 0.475 kg of detergent.

Now work margin exercise 8.

Margin Exercise Answers
1. a. milliliters **b.** kiloliters **2.** 2810 L **3.** 0.0133 kL **4.** 49 000 L **5.** 169.208 L
6. a. grams **b.** kilograms **7.** 14 900 g **8.** 0.825 g

6.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Volume measures space occupied and is labeled in _____ units.

2. Liquid volume in the metric system is measured in _____ (L).

3. The force of the Earth's gravitational pull on an object is the object's _____.

4. The amount of material in an object is its _____.

5. The basic unit of mass in the metric system is the _____ (kg).

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. One milliliter is equivalent to one cubic centimeter.

7. Volume is measured in square units.

8. In 1 liter there are 100 milliliters.

9. A metric ton and a US customary ton are equal (a metric ton weighs about 2000 US pounds).

10. A dekagram contains 10 grams.

Practice

What metric unit of capacity (liquid volume) would you use to measure each item? (Choose from milliliters, liters, and kiloliters.) See Example 1.

1. A bottle of perfume
2. A bottle of milk
3. A can of motor oil
4. A large fish tank at an aquarium
5. A drop of water
6. A flu shot

Convert each measurement. See Examples 2 through 4.

7. 2 L = ___ mL
8. 25 L = ___ mL
9. 19 mL = ___ L
10. 75 mL = ___ L
11. 13 L = ___ mL
12. 90 L = ___ mL
13. 500 mL = ___ L
14. 600 mL = ___ L
15. 6.3 kL = ___ L
16. 4.27 kL = ___ L
17. 76.4 L = ___ kL
18. 89.2 L = ___ kL
19. 950 mL = ___ L
20. 125 mL = ___ L
21. 1.25 L = ___ mL
22. 5.45 L = ___ mL
23. Change 5.3 L to milliliters.
24. Change 5.3 mL to liters.

What metric unit of weight (mass) would you use to measure each item? (Choose from milligrams, grams, and kilograms.) See Example 6.

25. A suitcase
26. Your weight
27. A cell phone
28. An apple
29. An aspirin
30. A ladybug

Convert each measurement. See Example 7.

31. 2 g = ___ mg
32. 7 kg = ___ g
33. 7.58 t = ___ kg
34. 5.6 t = ___ kg
35. 0.54 g = ___ mg
36. 3.94 g = ___ mg

37. 2000 g = ___ kg

38. 600 mg = ___ g

39. 34.5 mg = ___ g

40. 92.3 g = ___ kg

41. 91 kg = ___ t

42. 42 kg = ___ t

43. 4.6 kg = ___ mg

44. 19.8 kg = ___ mg

45. 2963 kg = ___ t

46. 3547 kg = ___ t

47. How many kilograms are there in 5 metric tons?

48. How many kilograms are there in 17 metric tons?

49. Express 96 g in milligrams.

50. Express 342 kg in grams.

51. Convert 75 000 g to kilograms.

52. Convert 3000 mg to grams.

53. How many grams are in 1.6 mg?

54. How many milligrams are in 1.6 g?

55. Change 0.34 g to kilograms.

56. Change 8.96 mg to grams.

57. Convert 7 metric tons to grams.

58. Convert 0.4 t to grams.

What metric unit of measurement would you use to measure each item? (Choose from mm, m, km, mL, L, kL, mg, g, and kg.)

59. The depth of a swimming pool

60. The volume of a swimming pool

61. The amount of water in a glass

62. How heavy a microchip is

63. How far you drive to work

64. The diameter of a quarter

65. The volume of toothpaste in a tube

66. The amount of medicine in an injection

67. How heavy an iPod is

68. The weight of a whale

Applications

Solve.

69. How many 5-mL doses of liquid medication can be given from a vial containing 3 deciliters?

70. Salvador's backyard has a swimming pool that holds 960 hectoliters of water. He has a water pump that will pump 16 dekaliters of water per minute into the pool. How long will it take for Salvador to fill his pool?

71. Thomas cleans up after Chemistry lab and part of this job is to empty all of the beakers into a bucket that can be removed for proper disposal. The bucket holds 4 L. There are 5 beakers each containing 50 cL of solution, 3 beakers each with 200 mL of solution, and 2 more beakers with 3 deciliters of solution. Will Thomas be able to dispose of all the leftover solution using just this one bucket? Why or why not?

72. One cup of flour is approximately 120 grams. How many cups of flour can you get out of a bag of flour weighing 2.4 kg?

73. A small dump truck can haul 3 metric tons of sand in one load. If each grain of sand weighs 0.05 mg, how many grains of sand can the dump truck hold in one load?

74. A metric weigh set comes with 10 weights: 2 of them weighing 5 g each, 4 of them weighing 1 g each, and one each weighing 5 mg, 10 mg, 2 cg, and 5 cg. What is the heaviest object that can be weighed using this set (in grams)?

Writing & Thinking

75. In the metric system, the common unit of capacity is the liter. Discuss how you would change from a measure of liters to milliliters.

76. Conlin said that his mass would change if he lived on Jupiter. Is he correct? Explain.

77. If you were to tell someone what your weight is in metric units, what unit of measure would you use, grams or kilograms? Explain briefly.

6.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. In the US customary system, temperature is measured in degrees _____.
2. In the metric system, temperature is measured in degrees _____.
3. One inch equals exactly ____ centimeters.
4. One square foot is about _____ square meters.
5. One quart is approximately _____ liters.
6. One kilogram is approximately _____ pounds.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Water freezes at 32 degrees Celsius.
8. When converting between US customary and metric units, often the results will be approximations.
9. A 5K (km) run is longer than a 5 mile run.
10. One square meter covers more area than one square yard.

Practice

Convert each measurement. Round to the nearest hundredth if necessary. See Examples 2 and 3.

1. 25 °C = ___ °F
2. 80 °C = ___ °F
3. 10 °F = ___ °C
4. 0 °C = ___ °F
5. 113 °F = ___ °C
6. 392 °F = ___ °C
7. 50 °C = ___ °F
8. 35 °C = ___ °F
9. Change 32 °F to degrees Celsius.
10. Change 41 °F to degrees Celsius.
11. Change 15 °C to degrees Fahrenheit.
12. Change 30 °C to degrees Fahrenheit.

Convert each measurement. Round to the nearest hundredth if necessary. See Example 4.

13. 9 ft = ___ m
14. 15 ft = ___ m
15. 11 m = ___ yd
16. 8 m = ___ yd

17. 33 in. = ___ cm

18. 21 in. = ___ cm

19. 8.5 m = ___ ft

20. 40 m = ___ ft

21. 20 mi = ___ km

22. 35 mi = ___ km

23. How many meters are in 3 yd?

24. How many meters are in 5 yd?

25. Change 60 miles to kilometers.

26. Change 100 kilometers to miles.

27. Convert 200 kilometers to miles.

28. Convert 100 miles to kilometers.

29. How many inches are in 50 cm?

30. How many inches are in 100 cm?

31. Convert 14 inches to centimeters.

32. Convert 14 centimeters to inches.

Convert each measurement. Round to the nearest hundredth if necessary. See Example 5.

33. 3 $in.^2$ = _____ cm^2

34. 16 $in.^2$ = _____ cm^2

35. 600 ft^2 = _____ m^2

36. 300 ft^2 = _____ m^2

37. 100 yd^2 = _____ m^2

38. 250 yd^2 = _____ m^2

39. 1000 acres = _____ ha

40. 250 acres = _____ ha

41. How many acres are in 300 ha?

42. How many acres are in 400 ha?

43. Change 5 m^2 to square feet.

44. Change 10 m^2 to square feet.

45. Change 30 cm^2 to square inches.

46. Change 50 cm^2 to square inches.

Convert each measurement. Round to the nearest hundredth if necessary. See Example 6.

47. 4 qt = ___ L

48. 7 qt = ___ L

49. 4 L = ___ gal

50. 9 L = ___ gal

51. 10 qt = ___ L

52. 20 qt = ___ L

53. 78 mL = ___ cc

54. 91 cc = ___ mL

55. 42 L = ___ gal

56. 50 L = ___ gal

57. How many quarts are in 10 L?

58. How many quarts are in 25 L?

59. How many liters are in 50 gal?

60. How many liters are in 36 gal?

Convert each measurement. Round to the nearest hundredth if necessary. See Example 7.

61. 33 kg = ___ lb

62. 95 kg = ___ lb

63. 35 oz = ___ g

64. 55 oz = ___ g

65. 10 lb = ___ kg

66. 70 lb = ___ kg

67. 100 g = ___ oz

68. 53 g = ___ oz

69. Convert 16 oz to grams.

70. Convert 64 oz to grams.

71. How many pounds are in 120 kg?

72. How many pounds are in 500 kg?

Applications

Solve.

73. While visiting her aunt in Germany, Helga wants to surprise her aunt with a cake. She brought her mom's cake recipe with her from Georgia. The recipe says to bake the cake at 350 degrees Fahrenheit but the temperature gauge on her aunt's oven is in degrees Celsius. To what temperature should Helga set her aunt's oven in order to bake the cake at the correct temperature? Round the temperature to the nearest degree.

74. Michael, who lives in Fargo, North Dakota, is packing for his trip to France. He looks up the weather for the duration of his trip to help him determine what type of clothing to pack. The average temperature for the week he will be in France is 24 degrees Celsius. What will the average temperature be in degrees Fahrenheit? Round the temperature to the nearest degree.

75. The Palace at Versailles is 23.5 km from the center of the city of Paris, France. If it takes Pierre 1.5 hours to bike between the two places, what is his speed in miles per hour (to the nearest tenth)?

76. Roger Bannister is famous for being the first man to break the 4-minute mile. He ran the mile in 3 minutes and 59 seconds. Today, most track meets do not have a one mile race. Instead, they have the "metric mile" race, which is 1500 meters. How much further is the actual mile race than the 1500-meter race (rounded to the nearest whole meter)?

77. The Ironman Triathlon championship in Hawaii consists of a swim of 3.86 km, a bike ride of 180.25 km, and finishes with a run equal to the length of a standard marathon. A marathon is typically 26.2 miles. What is the total length of the Ironman Triathlon in kilometers? Round the length to the nearest tenth of a km.

78. Darren needs to retool his widget maker so that it takes measurements in centimeters instead of inches. The widget maker currently makes widgets that are $\frac{3}{4}$ of an inch wide, $\frac{1}{3}$ of an inch deep, and $1\frac{3}{7}$ inches long. Determine the measurements of the widgets in centimeters (to the nearest tenth of a centimeter).

79. Suppose that the home you are buying sits on a rectangular shaped lot that is 270 feet by 121 feet. Convert this area to square meters.

80. A new manufacturing building covers an area of 3 acres. How many hectares of ground does the new building cover?

81. A painting of a landscape is on a rectangular canvas that measures 3 feet by 4 feet.

a. How many square centimeters of wall space will the painting cover when it is hanging?

b. How many square meters?

82. A turkey baster holds 150 mL of liquid. How many times would the turkey baster need to be filled in order to empty a 2-gallon bin of water from a dehumidifier?

83. You are making a large pot of soup and the recipe calls for 2 liters of vegetable stock. (Stock is similar to broth.) If the only cans of vegetable stock available at the super market hold 10 fluid ounces, how many cans will you need?

84. Rachel has taken up French cooking and is making chocolate éclairs. Since the recipe is in French, the measurements are given using metric units. The éclair recipe calls for 150 g butter, 225 mL water, 225 g flour, and 300 mL heavy cream. Convert the measurements given in grams into ounces and the measurements given in milliliters into fluid ounces. Round each measurement to the nearest whole number.

85. Kristy is shopping for gourmet chocolates online. The website of her favorite chocolatier only lists the mass of each box of chocolate in grams. A small box of chocolates has a mass of 153 grams, a medium box has a mass of 309 grams, and a large box has a mass of 595.4 grams. Find the weight in ounces of each box of chocolates. Round each weight to the nearest tenth.

Writing & Thinking

86. Peggy said that water boils at 100 degrees. Joel said it boils at 212 degrees. Who is correct and why?

87. Paola mistakenly thought that a meter stick was the same as a yard stick. Explain her mistake.

88. Most conversions between the US customary system of measure and metric system are not exact. Explain why this is true and give any exceptions.

89. Kai and Kristen were converting between US customary measures and metric measures. Their answers were close but not the same. The teacher said they were both right. How could this be?

$\angle 2$ and $\angle 4$ are vertical angles so $m\angle 4 = 130°$.

Now, because $\angle 1$ and $\angle 5$ are corresponding angles, they have the same measures which means $m\angle 5 = 50°$.

Because $\angle 4$ and $\angle 6$ are alternate interior angles, they have the same measures which means $m\angle 6 = 130°$.

Again, using vertical angles, $m\angle 5 = m\angle 7 = 50°$ and $m\angle 6 = m\angle 8 = 130°$.

Now work margin exercise 8.

Margin Exercise Answers

1. $m\angle 1 = 120°$, $m\angle 2 = 60°$ **2. a.** Right **b.** Obtuse **c.** Straight **3.** Complementary: $\angle MON$ and $\angle POQ$ Supplementary: $\angle QOP$ and $\angle POM$, $\angle QON$ and $\angle NOM$ **4. a.** 110° **b.** No **5.** $\angle ROS \cong \angle TOU$ and $\angle ROU \cong \angle SOT$ **6. a.** 40° **b.** 90° **c.** 50° **d.** 50° **7.** $\angle VQZ$ or $\angle WQX$ **8.** $m\angle 4 = 80°$, $m\angle 5 = 100°$, $m\angle 6 = 80°$

6.5 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A/An _____ has no beginning or end and is labeled with a lowercase letter or by the labels of two points on it.
2. Two rays with a common endpoint, called a vertex, form a/an _____.
3. An angle with a measure less than 90° is a/an _____ angle.
4. An angle that measures 180° is a/an _____ angle.
5. If the sum of the measures of two angles is 180°, they are said to be _____ angles.
6. Two lines are _____ if they intersect and form right angles.

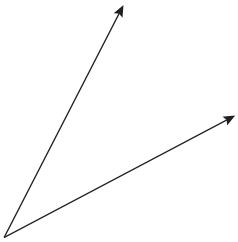
True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The sum of the measures of two complementary angles is equal to the measure of one right angle.
8. The sum of the measures of complementary angles is greater than the sum of the measures of supplementary angles.
9. Adjacent angles are two angles that share a side.
10. If two lines in a plane are not parallel, then they are perpendicular.

Practice

Use a protractor to find the measure of each angle. (**Note:** You may need to extend the rays to be able to read the numbers on your protractor.) See Example 1.

1.



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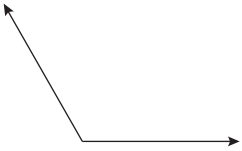

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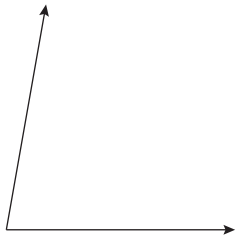

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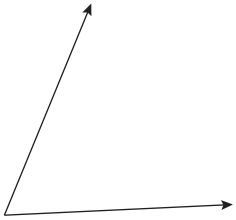
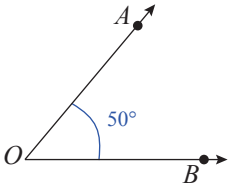


Classify each angle as acute, right, obtuse, or straight. See Example 2.

5.



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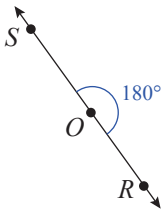

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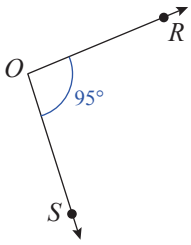

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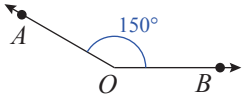

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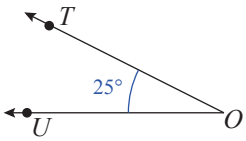
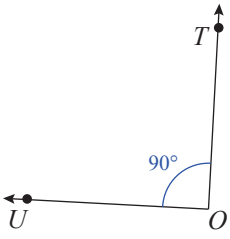
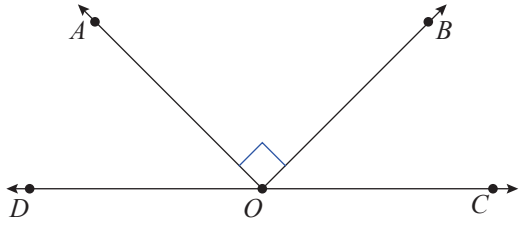


10.



Use the definitions of acute, right, obtuse, and straight angles to answer the questions. See Example 2.

11. In the figure shown, $\overleftrightarrow{DC}$ is a straight line and $m\angle BOA = 90°$.



a. What type of angle is $\angle AOC$?

b. What type of angle is $\angle BOC$?

c. What type of angle is $\angle BOA$?

12. Name the type of angle formed by the hands on a clock.

a. 

b. 

c. 

d. 

d. at six o'clock

e. at three o'clock

f. at one o'clock

g. at five o'clock

13. What is the measure of each angle formed by the hands of the clock in Exercise 12?

Use the definitions of complementary, supplementary, and straight angles to answer each question. See Examples 3 and 4.

14. Assume that $\angle 1$ and $\angle 2$ are complementary.

a. If $m\angle 1 = 15°$, what is $m\angle 2$?

b. If $m\angle 1 = 3°$, what is $m\angle 2$?

c. If $m\angle 1 = 45°$, what is $m\angle 2$?

d. If $m\angle 1 = 75°$, what is $m\angle 2$?

15. Assume $\angle 3$ and $\angle 4$ are supplementary.

a. If $m\angle 3 = 45°$, what is $m\angle 4$?

b. If $m\angle 3 = 90°$, what is $m\angle 4$?

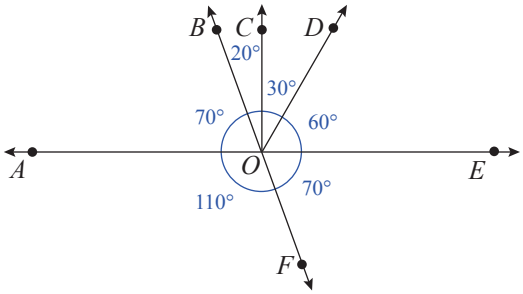
c. If $m\angle 3 = 110°$, what is $m\angle 4$?

d. If $m\angle 3 = 135°$, what is $m\angle 4$?

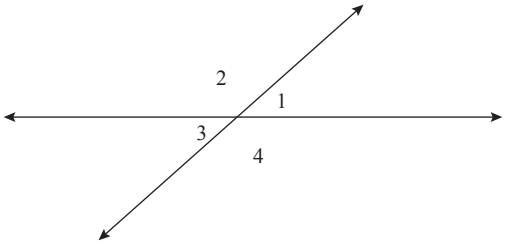
16. In the figure shown,

a. Name all of the pairs of supplementary angles.

b. Name all the pairs of complementary angles.



Use the definitions of adjacent and vertical angles to answer each question. See Examples 6 through 8.



17. The figure shows two intersecting lines.

a. If $m\angle 1 = 30°$, what is $m\angle 2$?

b. Is $m\angle 3 = 30°$? Give a reason for your answer other than the fact that $\angle 1$ and $\angle 3$ are vertical angles.

c. Name two pairs of congruent angles.

d. Name four pairs of adjacent angles.

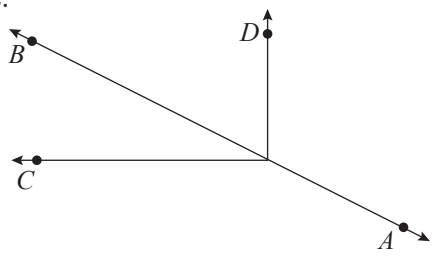
18. The figure shows two intersecting lines where $m\angle 1 = 30°$. Find the measures of the other three angles.

19. Given that $m\angle 1 = 42°$ in the figure, find the measures of the other three angles.

20. In the figure shown, $\overleftrightarrow{AB}$ is a straight line.

a. Name two pairs of adjacent angles.

b. Name two vertical angles, if there are any.

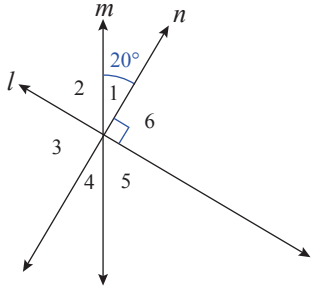


21. In the figure shown, l, m, and n are straight lines with $m\angle 1 = 20°$ and $m\angle 6 = 90°$.

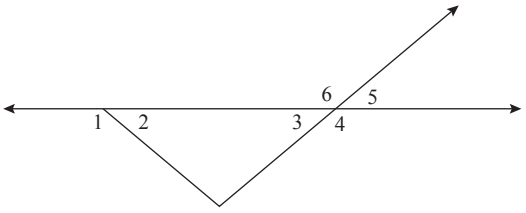
a. Find the measures of the other four angles.

b. Which angle is supplementary to $\angle 6$?

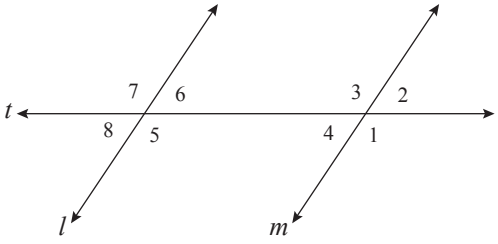
c. Which angles are complementary to $\angle 1$?



22. In the figure, $m\angle 2 = m\angle 3 = 40°$. Find all other pairs of angles that are congruent and their measures.



23. Use the figure to answer each question. Assume l and m are parallel.



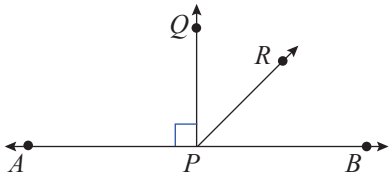
a. If $m\angle 1 = 125°$, then $m\angle 3 =$ _____. Explain your reasoning.

b. If $m\angle 8 = 55°$, then $m\angle 6 =$ _____. Explain your reasoning.

c. What is $m\angle 7$? Explain your reasoning.

d. Does $m\angle 2 = m\angle 6$? Explain your reasoning.

24. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{PQ}$ are perpendicular.



a. Which angle(s) are acute?

b. Which angle(s) are obtuse?

c. Which angle(s) are right angles?

d. Which pair(s) of angles are vertical angles?

e. Which pair(s) of angles are complementary?

f. Which pair(s) of angles are supplementary?

g. Which pair(s) of angles are adjacent?

Writing & Thinking

25. Explain, in your own words, the relationships between vertex, ray, angle, and line.

26. List four types of angles and define each one in terms of measurement.

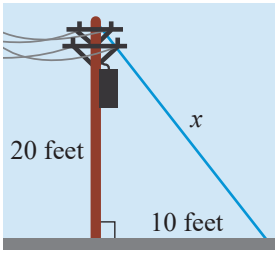
27. a. The supplement of a right angle is what type of angle?

b. The supplement of an obtuse angle is what type of angle?

c. The supplement of an acute angle is what type of angle?

Example 11 Application: Finding the Length of the Hypotenuse

A guy wire is attached to the top of a telephone pole and anchored to the ground 10 feet from the base of the pole. If the pole is 20 feet high, what is the length of the guy wire?



Solution

Let x = the length of the guy wire.

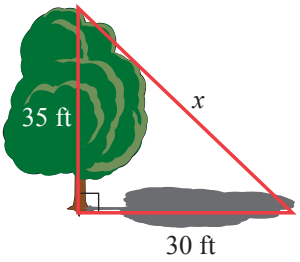
Then, by the Pythagorean Theorem,

$$x^2 = 10^2 + 20^2$$
$$x^2 = 100 + 400$$
$$x^2 = 500$$
$$x = \sqrt{500} \approx 22.36$$

The guy wire is about 22.36 feet long.

Now work margin exercise 11.

11. A 35-foot-tall tree casts a shadow which is 30 feet long. Find the distance from the top of the tree to the end of its shadow to the nearest hundredth.



Margin Exercise Answers

1. Scalene **2.** Yes **3. a.** 90° **b.** $\overline{RO}$ **c.** $\overline{BR}$ and $\overline{BO}$ **d.** Yes, because $m\angle R = 90°$. **4.** b **5.** 2.5 cm **6.** 27 feet **7.** Yes, by ASA **8.** No, $8^2 \neq 7^2 + 4^2$ **9.** 17 cm **10.** $\sqrt{32}$ yd or about 5.66 yd **11.** 46.10 ft

6.6 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A _____ _____ consists of two points on a line and all the points between them.
2. Longer sides are _____ angles with larger measures.
3. In similar triangles, the _____ angles have the same measure.
4. If two triangles are congruent, the lengths of corresponding sides are _____.
5. Two triangles are congruent if they have the same _____ and the same _____.

6. The symbols ≅ is read "is _____ to."

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A scalene triangle has at least two sides of equal lengths.

8. An acute is a triangle where three angles are acute.

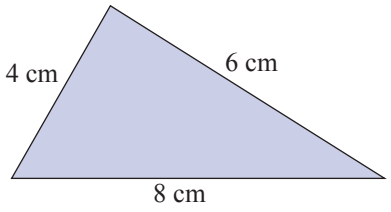
9. In similar triangles, the lengths of corresponding sides are equal.

10. The hypotenuse is the longest side of a triangle.

Practice

Classify each triangle in the most precise way possible, given the indicated lengths of its sides and/or measures of its angles. See Example 1.

1.



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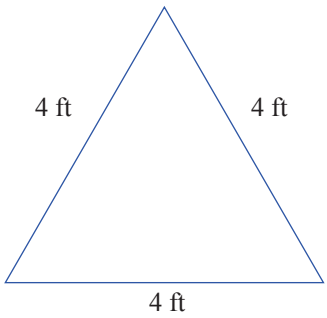

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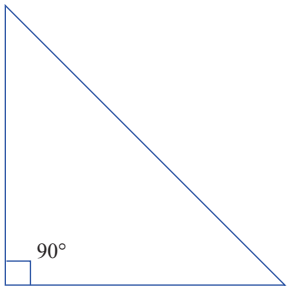

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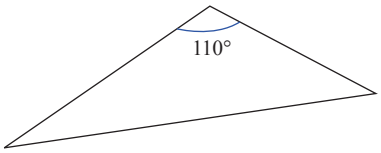

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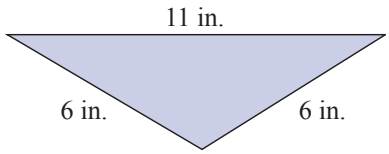

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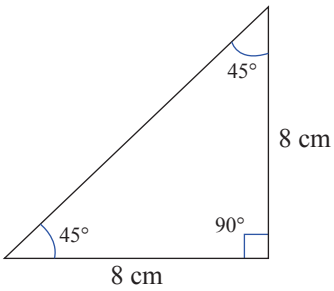

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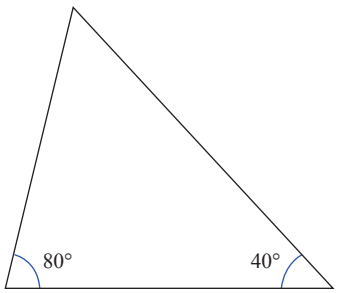

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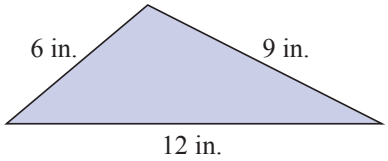
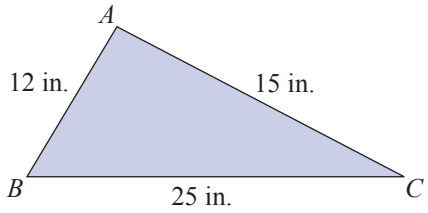
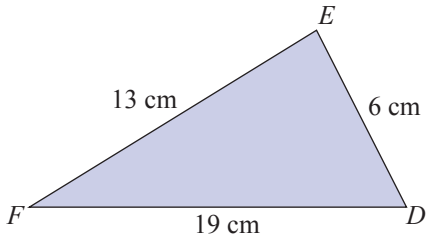


Solve.

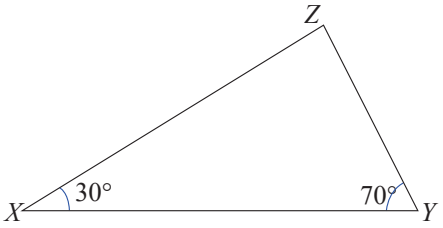
9. Suppose the lengths of the sides of $\triangle ABC$ are as shown in the figure. Is this possible? Explain your reasoning.



10. Suppose the lengths of the sides of $\triangle DEF$ are as shown in the figure. Is this possible? Explain your reasoning.



11. In the triangle shown, $m\angle X = 30°$ and $m\angle Y = 70°$.



a. What is $m\angle Z$?

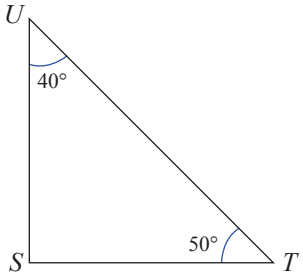
b. What kind of triangle is $\triangle XYZ$?

c. Which side is opposite $\angle X$?

d. Which sides include $\angle X$?

e. Is $\triangle XYZ$ a right triangle?

12. In the triangle shown, $m\angle T = 50°$ and $m\angle U = 40°$.



a. What is $m\angle S$?

b. What kind of triangle is $\triangle STU$?

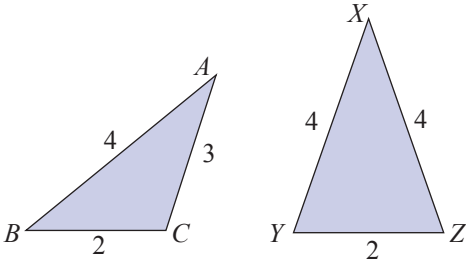
c. Which side is opposite $\angle T$?

d. Which sides include $\angle T$?

e. Is $\triangle STU$ a right triangle?

Determine whether each pair of triangles is similar. If the pair of triangles are similar, explain why and indicate the similarity by using the ~ symbol. See Example 4.

13.



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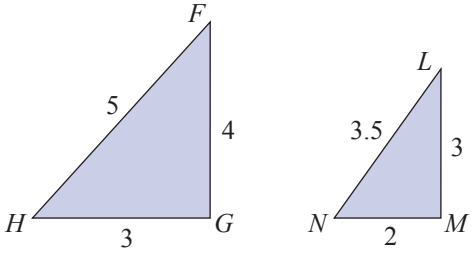

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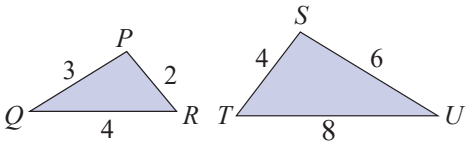

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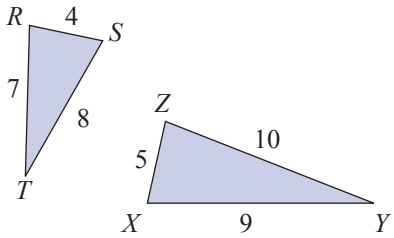

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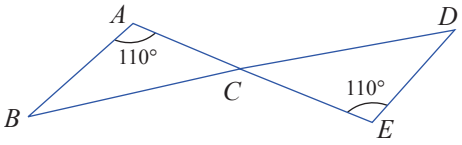

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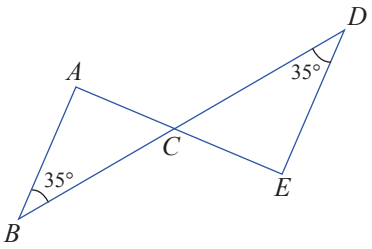
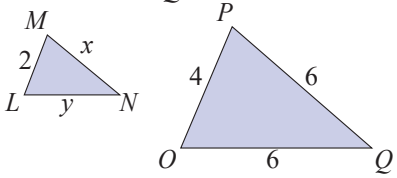
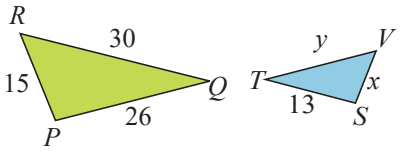


Find the values for x and y. See Example 5.

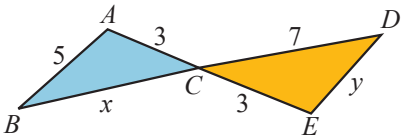
19. $\triangle LMN \sim \triangle OPQ$



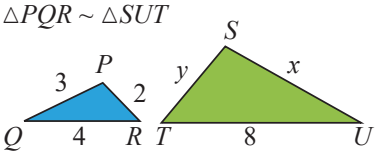
20. $\triangle PQR \sim \triangle STV$



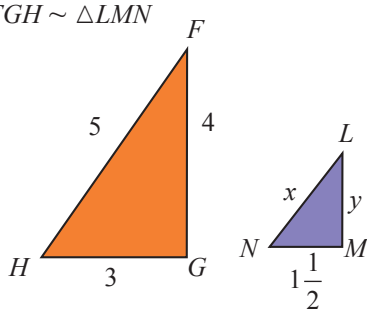
21. $\triangle ABC \sim \triangle EDC$



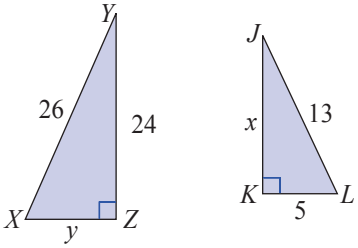
22. $\triangle PQR \sim \triangle SUT$



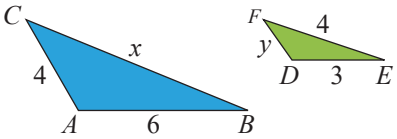
23. $\triangle FGH \sim \triangle LMN$



24. $\triangle XYZ \sim \triangle LJK$

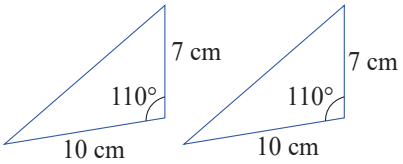


25. $\triangle ABC \sim \triangle DEF$



Determine whether each pair of triangles is congruent. If the pair of triangles is congruent, state the property that confirms that they are congruent. See Example 7.

26.



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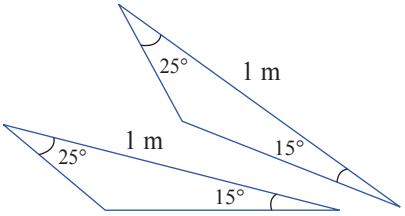

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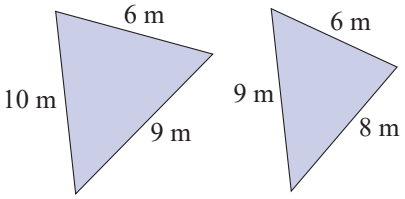

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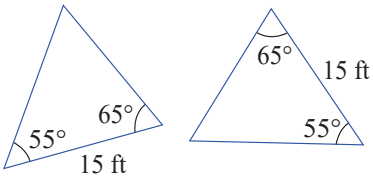

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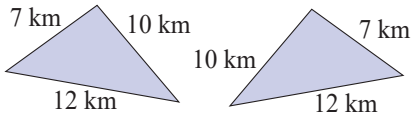

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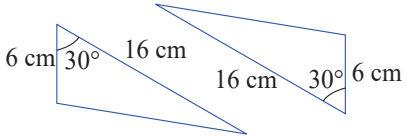

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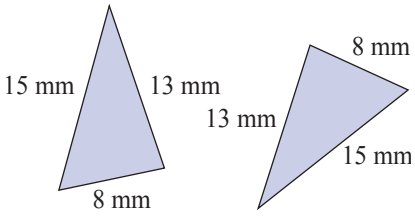

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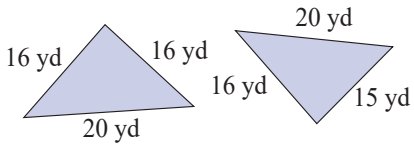
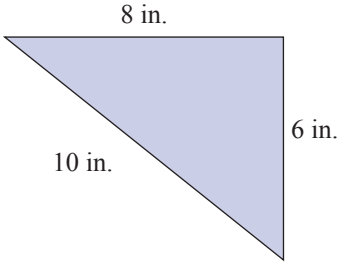


Use the Pythagorean Theorem to determine whether or not each triangle is a right triangle. See Example 8.

34.



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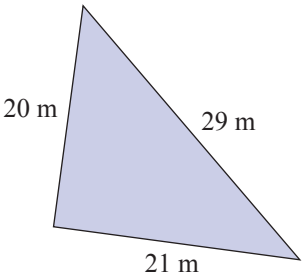

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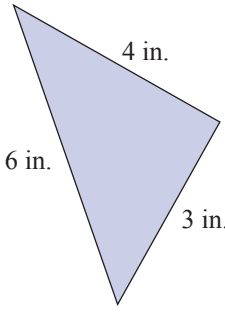

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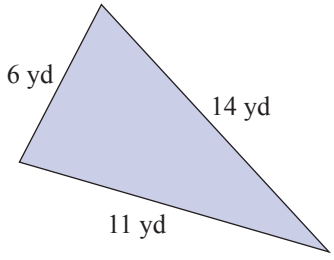

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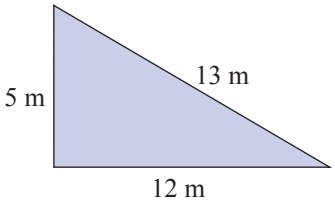

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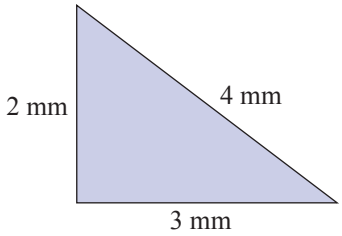
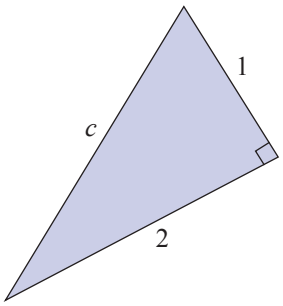


Find the hypotenuse for each right triangle accurate to the nearest hundredth. See Example 9.

40.



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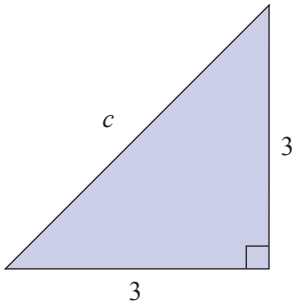

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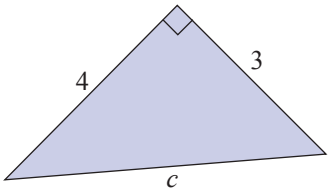

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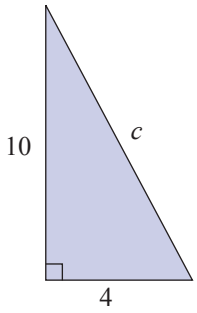

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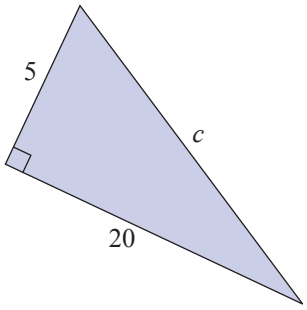

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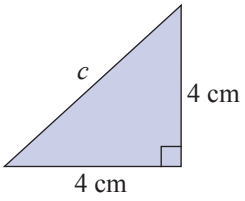

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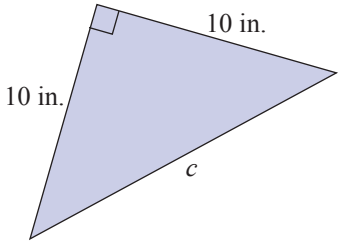

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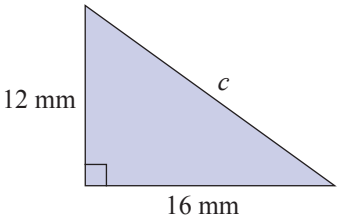
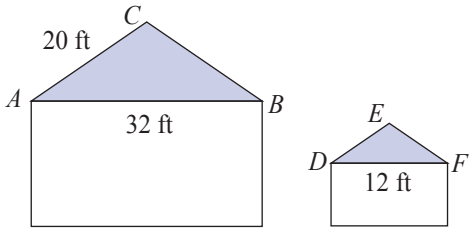


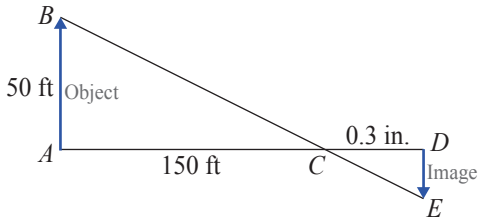
Applications

Solve.

48. A child's playhouse is built to look like a smaller version of the family house, where the ends of the roofs have similar proportions. The width of the main house (AB) is 32 feet and the length from the peak to the gutter of the roof for one of the sides is 20 feet. If the width of the playhouse (DF) is 12 feet, what is the length from the peak to the gutter (DE) of the playhouse roof?

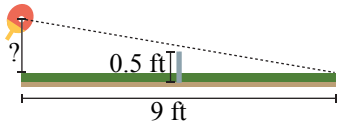


49. A camera uses a lens that will look at a properly focused object (such as a person or a tree) and then display an inverted image of this object on a screen or film which is on the opposite side of the lens as shown in the figure. If a picture of a 50-foot tall building (AB), which is 150 feet from the lens (AC) is photographed, how tall is the image (DE) if the film on the opposite side (CD) is 0.3 inch from the lens.



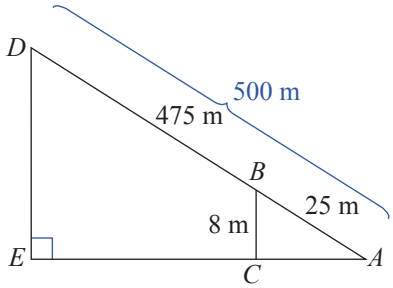
50. You and a friend are walking to class and want to figure out the height of the tree next to your building. Your friend is exactly 6 ft tall and casts 4 ft shadow. The tree casts a 12 ft shadow. How tall is the tree?

51. At what height off the table would you need to hit a ping pong ball for it to skim the net (0.5 feet tall) and hit on the edge of the opposite side of the table 9 feet away from you? Assume that the net is in the exact center of the table (with 4.5 feet on either side) and that your paddle is directly above the edge of the table.

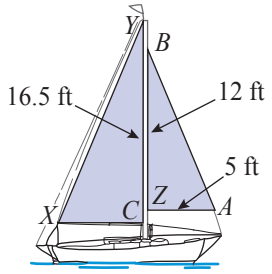


52. Your neighbors are hanging their holiday lights. The ladder they are currently using is 12 feet long and when leaned up against the house just reaches the top of their 8-foot tall porch. How long of a ladder will they need to reach the top of their chimney which is at a height of 32 feet? (Assume that both ladders are placed such that they make the same angle with the ground.)

53. The sloping surface of a hill (from base to peak) is 500 meters long. Paul starts at the base of the hill and walks uphill 25 meters which results in a gain of 8 meters in elevation. What is the height of the hill? (**Hint:** As shown on the accompanying drawing, there are two similar triangles ΔABC and ΔADE. Solve for DE.)



54. A sloop is a sailboat that has two triangular sails on a single mast. If the smaller sail is 12 feet along the mast (CB), and 5 feet along its bottom (AC), and the larger sail is 16.5 feet along the mast (ZY), how wide is the larger sail at the bottom (XZ) if $\triangle ABC$ and $\triangle XYZ$ are similar triangles? Round your answer to the nearest tenth.



55. If an airplane passes directly over your head at an altitude of 1 mile, how far (to the nearest hundredth of a mile) is the airplane from your position after it has flown 2 miles farther at the same altitude?

56. The GE Building in New York is 850 feet tall (70 stories). At a certain time of day, the building casts a shadow 100 feet long. Find the distance from the top of the building to the tip of the shadow (to the nearest tenth of a foot).

57. The base of a fire engine ladder is 30 feet from a building and reaches to a third floor window 50 feet above ground level. Find the length of the ladder to the nearest hundredth of a foot.

58. The Xerox Center building in Chicago is 500 feet tall. At a certain time of day, it casts a shadow that is 150 feet long. At that time of day, what is the distance (to the nearest tenth of a foot) from the tip of the shadow to the top of the Xerox building?

59. Before painting a picture on canvas, an artist must stretch the canvas on a rectangular wooden frame. To be sure that the corners of the canvas are true right angles, the artist can measure the diagonals of the stretched canvas. What should be the diagonal measure, to the nearest tenth of an inch, of a canvas whose sides are 24 inches and 30 inches in length?

60. While installing windows in a new home, a builder measures the diagonals of rectangular window casements to verify that their corners are true right angles. What should be the diagonal measure, to the nearest tenth of an inch, of a window casement with dimensions 36 inches and 54 inches.

61. A hiker hikes 9 kilometers north and then turns left and hikes 11 kilometers west. If she takes the shortest path, how long will she have to walk to get back? Assume the terrain is flat with no obstructions. Round the answer to the closest tenth.

Writing & Thinking

62. Kelly needs to determine whether two triangles are similar. She was given the following information.

For $\triangle ABC$ and $\triangle DEF$, $AB = 3.6$, $AC = 2.4$, $BC = 2$ and $DE = 9$, $DF = 6$, $EF = 5$.

What should be her first step?

63. If three whole numbers satisfy the Pythagorean Theorem, these three numbers are called a Pythagorean triple. For example, 3, 4, and 5 are a Pythagorean triple because $3^2 + 4^2 = 5^2$ (or $9 + 16 = 25$). Another Pythagorean triple is 5, 12, and 13 because $5^2 + 12^2 = 13^2$ (or $25 + 144 = 169$). Complete the following table by finding a, b, and c, and telling which sets of these three numbers (if any) constitute a Pythagorean triple. The first one is done for you.

m	n	$a = 2nm$	$b = m^2 + n^2$	$c = m^2 + n^2$	Pythagorean Triple?
5	1	10	24	26	yes: $10^2 + 24^2 = 26^2$
7	1				
3	2				
7	2				
5	3				
11	3				
13	7				

Extension: Choose some of your own numbers for m and n. Are your results Pythagorean triples? (**Note:** m must be larger than n so $m^2 - n^2$ will be positive.)

$$\text{Area of semicircle} = \frac{1}{2}A = \frac{1}{2}\pi r^2$$
$$\approx \frac{1}{2}\cdot 3.14(5\text{ in.})^2 \qquad \text{In this case, } r = \frac{1}{2}d = \frac{1}{2}\times 10\text{ in.} = 5\text{ in.}$$
$$= 39.25\text{ in.}^2$$

Area of the figure = 100 in.^{2} – 39.25 in.^{2} = 60.75 in.^{2}

The area of the figure is 60.75 in.^{2}

Now work margin exercise 13.

CALCULATOR

Entering π on a Calculator

Many calculators will have a [π] key (may also be accessed using [SHIFT] [EXP] or [SHIFT] [10^x]) which can be used in calculating circumferences involving π. Be aware, though, that the value of π inserted by a calculator will be rounded to a minimum of 10 decimal places. As the problems in this section were calculated using π = 3.14, you may see slight differences in the answers due to rounding. For example, a circle with radius 6 has a circumference of 37.68 ft when calculated using π = 3.14. Alternatively, to find the circumference using a calculator, press the keys

[2] [×] [π] [×] [6]. Then press [=].

The display will read 37.699111843... (your calculator may display more or less digits depending on its settings). Rounding this value to two decimal places gives us 37.70 ft. Even though the two circumferences differ by 0.02, both answers are considered correct.

Margin Exercise Answers

1. 32 ft **2.** 85 ft **3.** 58 cm **4. a.** 29 m **b.** $217.50 **5.** 69.08 m **6.** 66.82 in. **7.** 24.28 ft **8.** 28 mm^2 **9.** 36 cm^2 **10.** 28.26 ft2 **11.** 30 ft^2 **12.** 44 m^2 **13.** 21.87 cm^2

6.7 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A closed plane figure with at least three sides in which each side is a line segment is a/an _____.

2. A four-sided polygon with both pairs of opposite sides parallel is a/an _____.

3. The perimeter of a circle is called the _____.

4. When measuring area use _____ units.

5. The measure of the interior of a plane figure is the _____ of the figure.

6. $A = \frac{1}{2}bh$ is the formula for the area of a/an _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. a. Every square is a rectangle.

b. Every rectangle is a square.

8. A trapezoid has only one pair of parallel lines.

9. The height of a triangle is the distance between the base and the vertex opposite the base.

10. The area formula for a triangle is $A = a + b + c$.

Practice

Calculate the perimeter of each figure described. Use $\pi = 3.14$.

1. A parallelogram with sides of length 15 cm and 7 cm.

2. A parallelogram with sides of length 42 mm and 34 mm.

3. A rectangle with sides of length 24 cm and 34 cm.

4. A rectangle with sides of length $3\frac{1}{4}$ ft and $2\frac{5}{6}$ ft.

5. A square with sides of length $4\frac{1}{2}$ km.

6. A square with sides of length 11 m.

7. A triangle with sides of length 21 in., 67 in., and 55 in.

8. A triangle with sides of length 7.5 in., 17 in., and 13.6 in.

9. A trapezoid with sides of length 14.2 yd, 10.1 yd, 8 yd, and 15.8 yd.

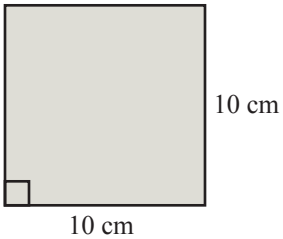
10. A trapezoid with sides of length 31 ft, 39 ft, 45 ft, and 51 ft.

11. A circle with radius 0.5 m.

12. A circle with diameter 14 m.

Calculate the perimeter of each figure. See Examples 1 through 4.

13.



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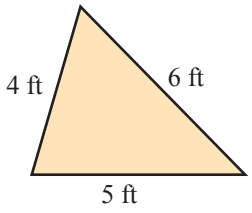

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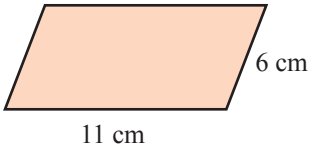

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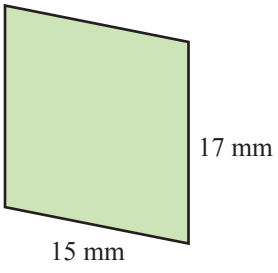

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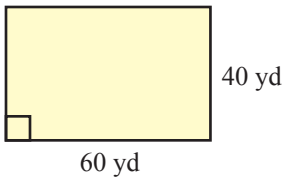

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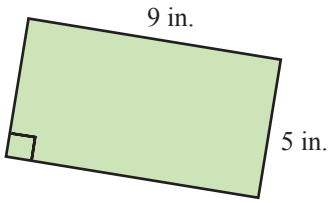

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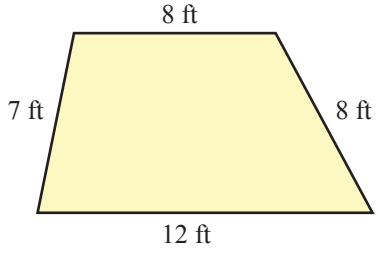

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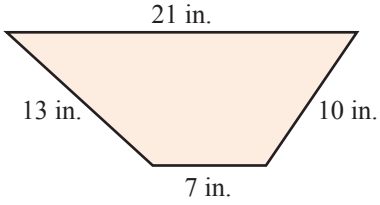

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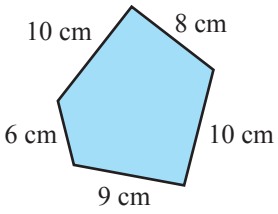

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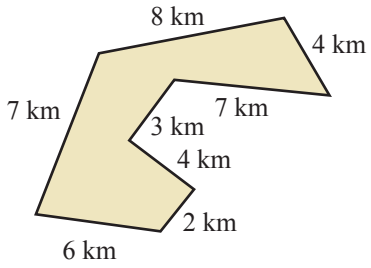

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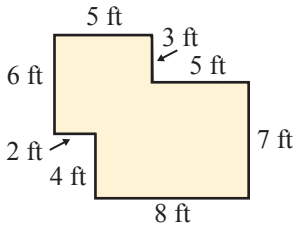

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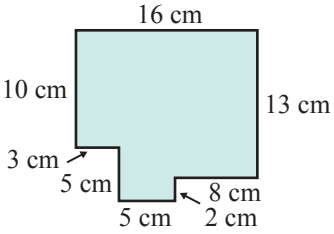

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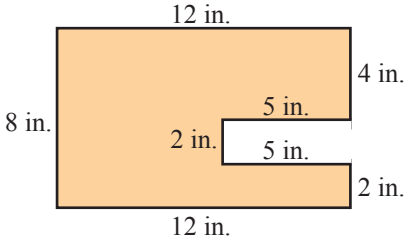

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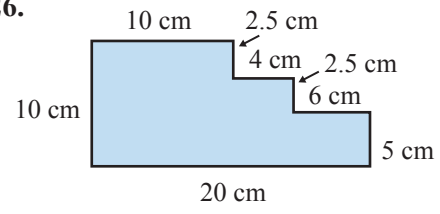

Calculate the perimeter of each figure. Use $\pi = 3.14$. See Examples 5 through 7.

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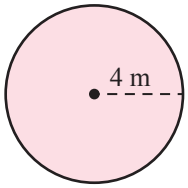

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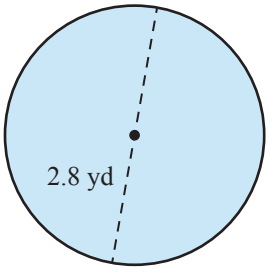

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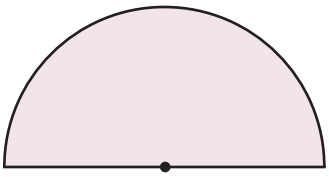

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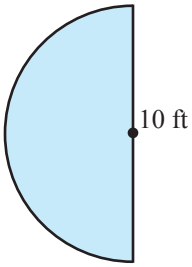

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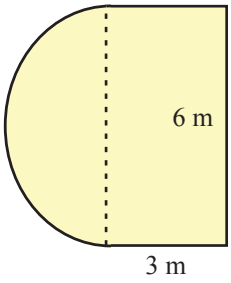

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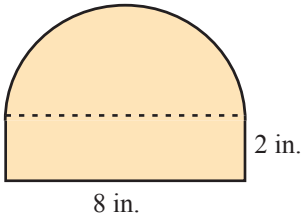

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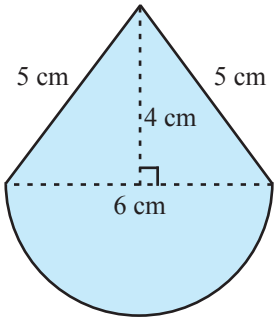

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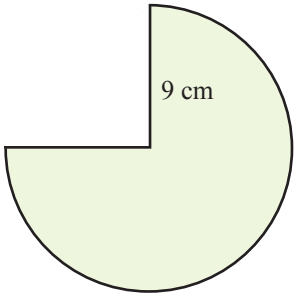


Calculate the area of each figure described. Use $\pi = 3.14$. See Examples 8 through 10.

35. A square with sides of length 9 ft.

36. A square with sides of length 6 in.

37. A rectangle with length 21 km and width 25 km.

38. A rectangle with length $1\frac{1}{4}$ mi and width $2\frac{1}{2}$ mi.

39. A parallelogram with height 2.3 ft and base 11.9 ft.

40. A parallelogram with height 5 m and base 12 m.

41. A triangle with height $\frac{8}{9}$ in. and base $\frac{5}{12}$ in.

42. A triangle with height 16.4 cm and base 8.2 cm.

43. A trapezoid with height 10 cm and parallel sides of length 15 cm and 18 cm.

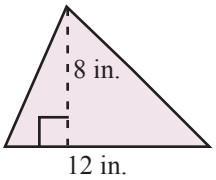
44. A trapezoid with height 30 mm and parallel sides of length 45 mm and 50 mm.

45. A circle with radius $\frac{3}{4}$ ft.

46. A circle with radius $12\frac{1}{5}$ mi.

Calculate the area of each figure. Use $\pi = 3.14$. See Examples 8 through 13.

47.



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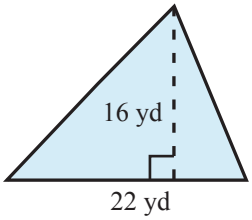

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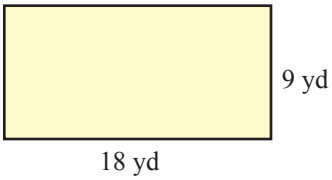

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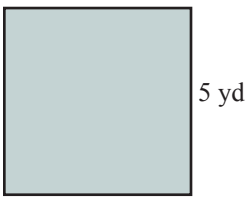

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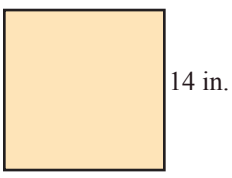

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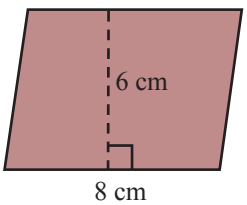

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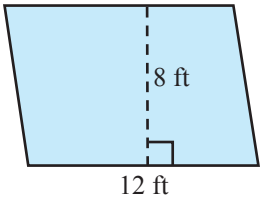

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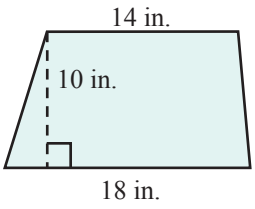

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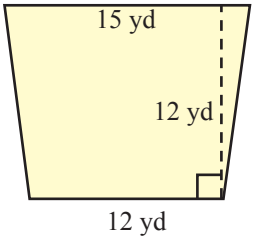

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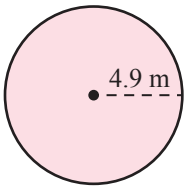

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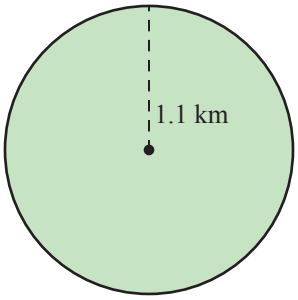

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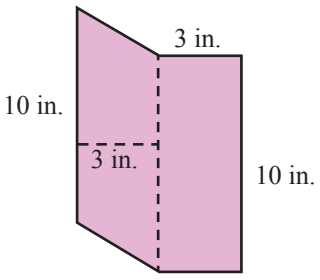

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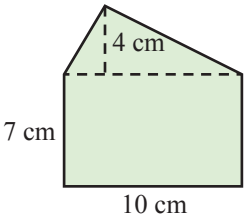

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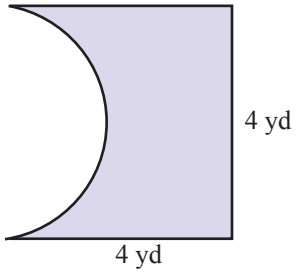

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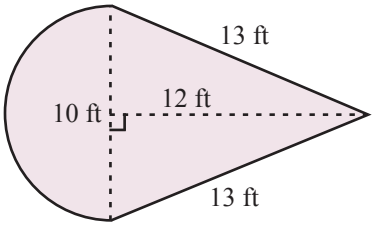

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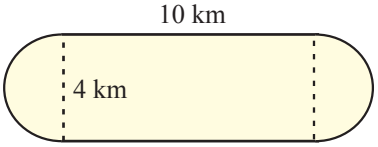

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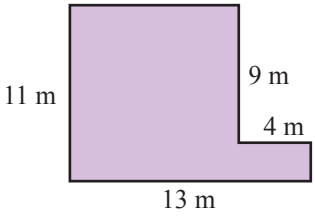

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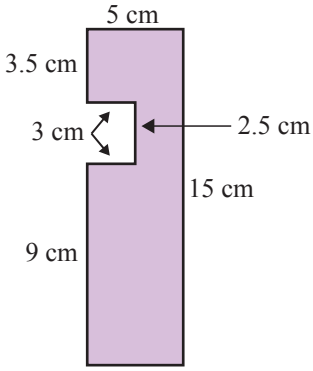

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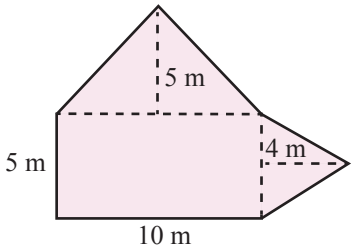

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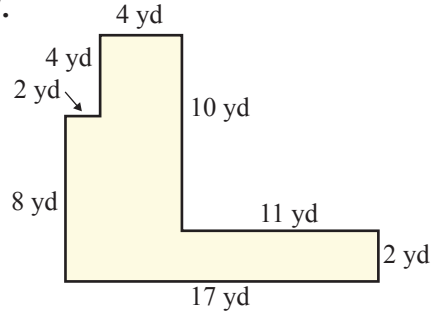
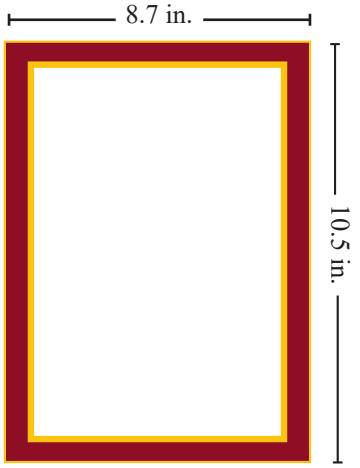


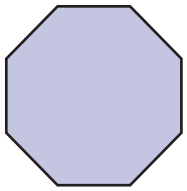
Applications

Solve. Use $\pi = 3.14$.

68. A rectangular picture frame is 10.5 inches high and 8.7 inches wide. How much picture framing material must be used to frame the picture? (**Hint:** This is the same as the perimeter of the outer edge.)



69. A regular octagon is an eight-sided figure with all eight sides equal and all eight angles equal. Find the perimeter of a regular octagon if one side measures 12 inches. (**Note:** Where do you see regular octagons an a regular basis?)

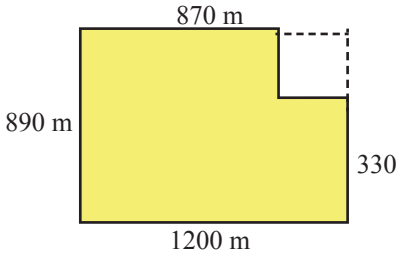


70. The Pentagon near Washington, D.C., is a five-sided building where each outside wall is 921 feet.

a. What is the perimeter of the building?

b. If it takes a person 0.00341 minutes to walk 1 foot, how long will it take the person to walk completely around the building? Round your answer to the nearest tenth of a minute.

71. For exercise, John will walk along the path which is indicated by the solid line in the drawing. Note that he cuts a corner where both sides are the same length.



a. How many meters did John walk?

b. How long would the walk have been if John didn't cut the corner, but rather walked the full rectangle, as indicated by the dotted line?

c. How do these two distances compare?

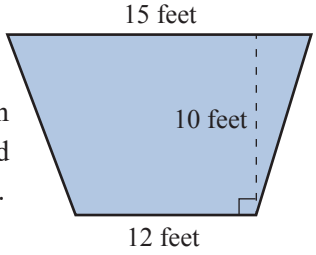
72. An engineer who is designing a new smartphone decides to add a soft neoprene edging to the phone. The phone itself is $4\frac{1}{2}$ inches tall and $2\frac{2}{5}$ inches wide. How much neoprene edging is needed to go along the outside edge of each smartphone?

73. Jessica wants to add a decorative fringe to a throw rug. The rug is a rectangle with length 8 feet and width 5 feet. If Jessica wants to buy 1 foot more than the perimeter of the rug, how many feet of fringe must she buy?

74. The boundaries of a certain small town form a parallelogram with a length of 4.5 miles and a height of 2.6 miles. What is the area within the town limits?

75. Vinyl tile is to be laid on the floor of a rectangular room which is 17 feet long and 12 feet wide. How many square feet of tile must be put down?

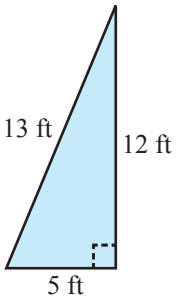
76. The main stage at a theater is in the shape of a trapezoid. The owner of the theater is planning to install a new specially designed flooring system on the stage. The stage is 12 feet wide in the front and 15 feet wide in the back. The stage is 10 feet deep. How much flooring will the manager need?



77. A sailboat has a triangular sail with the dimensions as shown in the drawing. (Note that the 12 foot measurement is the height of the triangle.)

a. What is the area of the sail?

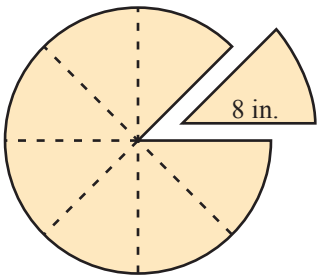
b. What is the perimeter of the sail?



78. A large 16 in. pizza is cut into eight pieces.

a. What is the perimeter of a single piece?

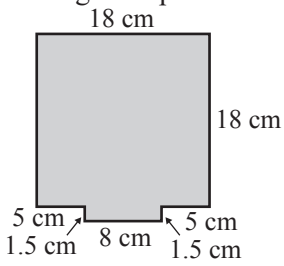
b. What is the area of this piece of pizza?



79. A square electronics circuit board is 18 centimeters on each side. On the center of one of the edges is a 8 by 1.5 centimeter rectangular lip for plugging in.

a. What is the total perimeter of the circuit board, including the lip?

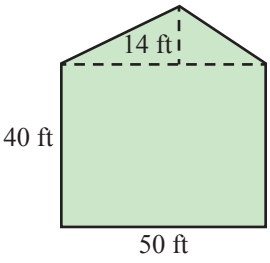
b. What is the area of the circuit board?



80. David is planting a five-sided lawn as shown in the figure below. The lawn consists of a 50 foot by 40 foot rectangle and an attached 14 foot high triangle.

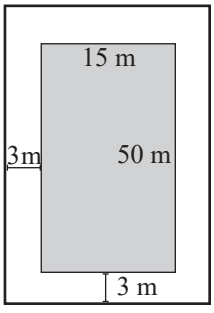
a. What is the area of the lawn to be planted?

b. If one pound of grass seed will cover 200 square feet, how many pounds will be necessary to cover the entire lawn? (**Hint:** Divide the area by the number of square feet that one pound of seed will cover.)



81. A 1-page magazine article must have 1-inch margins of blank space surrounding the content of the page. If the magazine pages are 11 inches by 14 inches, determine the largest amount of space that will contain print on this page.

82. A concrete patio is being poured to surround a rectangular swimming pool. The pool is 15 meters wide by 50 meters long. If the patio is to be a uniform 3 meters width all around the pool, find the area of the concrete patio.



83. A rectangular room is 25 ft by 20 ft with 10-ft high walls. How many gallons of paint will be needed to paint the four walls if each gallon covers 35 m2? Assume that that 1 square foot is approximately equal to 0.093 square meters.

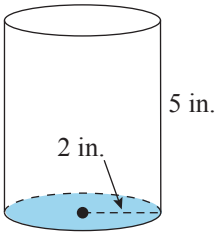
Writing & Thinking

84. The perimeter of a standard sheet of paper ($8\frac{1}{2}$ in. by 11 in.) is $P = 2 \cdot 8\frac{1}{2} + 2 \cdot 11 = 17 + 22 = 39$ inches. Use a pair of scissors to cut a rectangle from one corner of a standard sheet of paper and measure the perimeter of the new figure. Repeat this process several times (in some cases, cut more than one rectangle from a different corner and measure the perimeter each time). Give a brief explanation of the results in each case.

85. Draw a rectangle and choose any point on one side of the rectangle. Draw line segments to the vertices on the opposite side (forming three triangles). Now cut out the two triangles on each end. Place these triangles inside the remaining triangle to show that the total of the two areas is equal to the area of the remaining triangle. Do this three different times choosing a different point each time. What fact does this illustrate about the area of a triangle?

Example 7 Calculating the Surface Area of a Cylinder

Calculate the surface area of a coffee can in the shape of a cylinder with a height of 5 in. and a circular base with a radius of 2 in. (Use $\pi = 3.14$.)



Solution

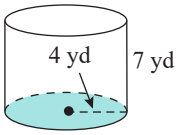
Using the formula for the surface area of a cylinder, we have the following.

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ SA &\approx 2 \cdot 3.14 \cdot (2 \text{ in.})^2 + 2 \cdot 3.14 \cdot 2 \text{ in.} \cdot 5 \text{ in.} \\ &= 25.12 \text{ in.}^2 + 62.8 \text{ in.}^2 \\ &= 87.92 \text{ in.}^2 \end{aligned}$$

The surface area of the cylinder is 87.92 in.^{2}

Now work margin exercise 7.

7. Calculate the surface area of a cylinder with a height of 7 yd and a circular base with a radius of 4 yd.



Margin Exercise Answers

1. 810 in.^{3} **2.** 113.04 ft^3 **3.** 28.26 m^3 **4.** 678.24 cm^3 **5.** 1728 in.^{3} or 1 ft^3 **6.** 576 ft^2 **7.** 276.32 yd^2

6.8 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The measure of the space enclosed by a three-dimensional figure is its _____.
2. Volume is measured in _____ units.
3. The measure of the area of the outside surface of a geometric solid is the solid's _____ _____.
4. Surface area is measured in _____ units.
5. $SA = 2\pi r^2 + 2\pi rh$ is the formula for the surface area of a/an _____ _____ _____.
6. $SA = 4\pi r^2$ is the formula for the surface area of a/an _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. To find the volume of a can of corn, the formula $V = \pi r^2 h$ would be used.

8. $V = lwh$ is the formula for the surface area of a rectangular solid.

9. The area of the paper label on a can of peaches is an example of surface area.

10. To find the volume of a rectangular solid, the areas of each surface are added together.

Match each formula for volume to its corresponding geometric figure.

11.

a.	Rectangular solid	**A.**	$V = \frac{4}{3}\pi r^3$
b.	Rectangular pyramid	**B.**	$V = \frac{1}{3}\pi r^2 h$
c.	Right circular cylinder	**C.**	$V = lwh$
d.	Right circular cone	**D.**	$V = \pi r^2 h$
e.	Sphere	**E.**	$V = \frac{1}{3}lwh$

Practice

Calculate the volume of each solid. See Examples 1 through 5. Use $\pi \approx 3.14$.

1. A rectangular solid with length 5 in., width 2 in., and height 7 in.

2. A right circular cylinder 15 in. high and 1 ft in diameter.

3. A sphere with radius 4.5 cm.

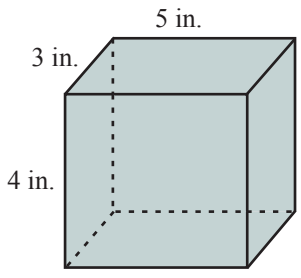
4. A sphere with diameter 12 ft.

5. A right circular cone 3 mm high with a 2 mm radius.

6. A rectangular pyramid with length 8 cm, width 1 cm, and height 30 cm.

Calculate the volume of each solid. See Examples 1 through 5. Use $\pi \approx 3.14$.

7.



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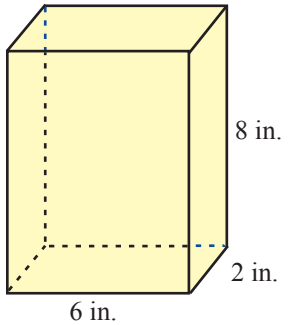

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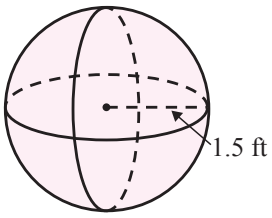

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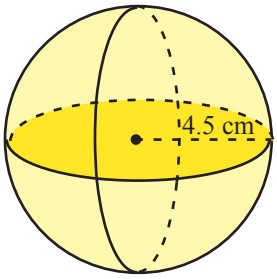

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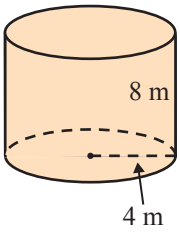

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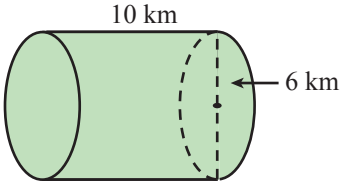

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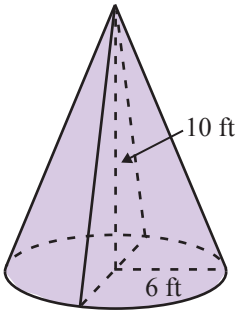

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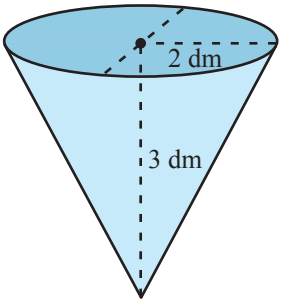

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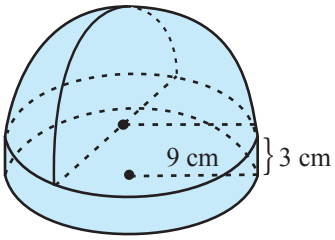

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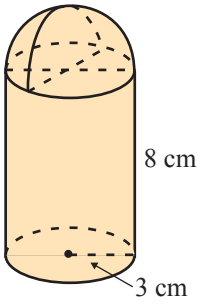

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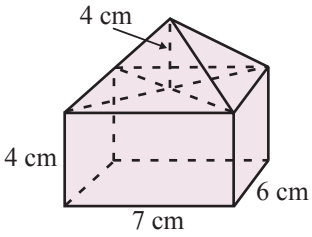

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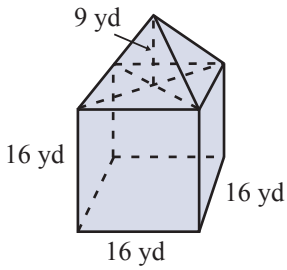

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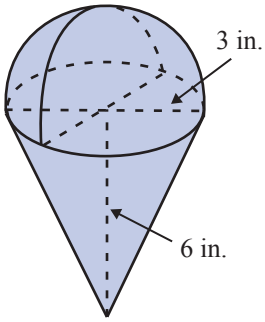

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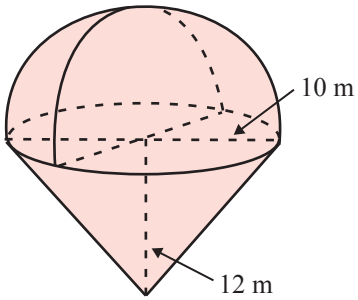

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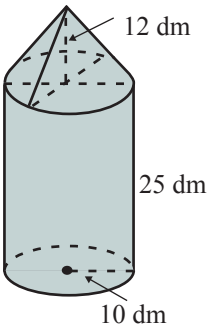

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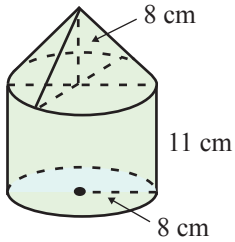

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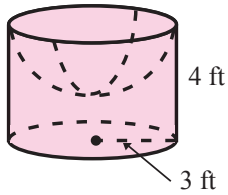

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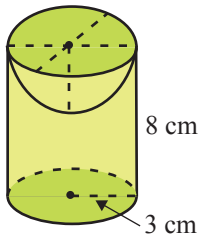

Calculate the surface area of each solid. See Examples 6 and 7. Use $\pi \approx 3.14$.

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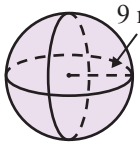

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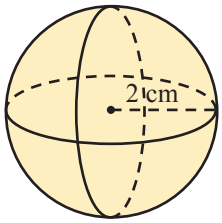

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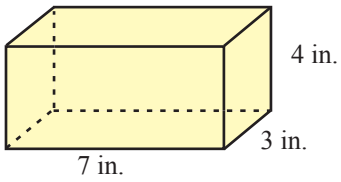

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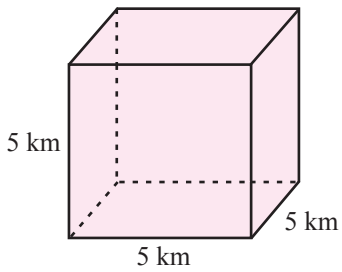

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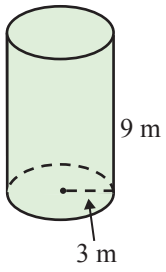

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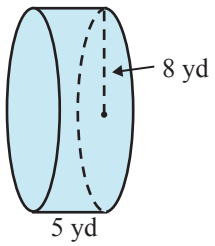


Solve. Use $\pi = 3.14$.

31. Find the volume of a rectangular solid with length 5 in., width 2 in., and height 7 in., in both cubic inches and cubic centimeters.

32. Find the volume of right circular cylinder 1.5 ft in height and 1 ft in diameter, in both cubic feet and cubic meters.

33. Find the volume of a right circular cone 3 dm high with a 2 dm radius, in both cubic decimeters and cubic meters.

34. Find the volume of a rectangular pyramid with length 18 cm, width 10 cm, and altitude 3 cm, in both cubic centimeters and cubic millimeters.

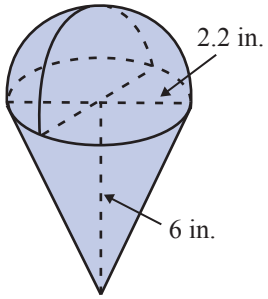
Applications

Solve. Use $\pi \approx 3.14$.

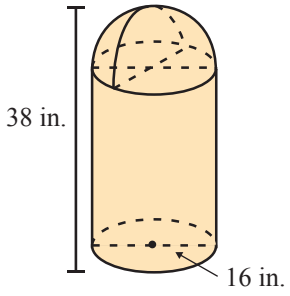
35. The Great Pyramid of Giza, which is located in Egypt, has a square base of 231 m on each side, and its height is 146 m. What is its volume?

36. A standard 55 gallon round steel drum is about 23 in. in diameter and 34.5 in. high. Assuming that the drum is totally enclosed, what is its surface area?

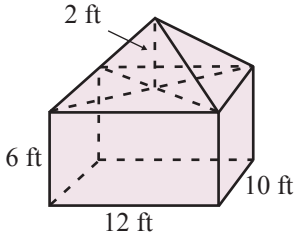
37. A 6 in. tall ice cream cone is filled solid with ice cream where the final scoop of ice cream forms a perfect hemisphere above the top of the cone. What is the total volume of ice cream in the cone if the top of the cone has a 2.2 in. opening? Round your answer to the nearest hundredth.



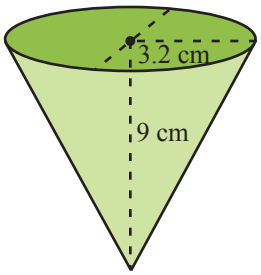
38. A cylindrical trash can has a hemispherical top (with a trap door for the trash). If the diameter of the can is 16 in. and its total height is 38 in., find its volume. (**Hint:** Begin by finding the height of the straight part of the can.)



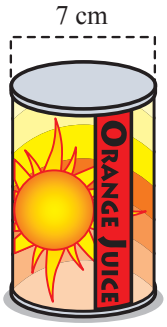
39. A rectangular tent with straight sides has a pyramidal shaped roof. The dimensions of the rectangular portion are 12 ft long, 10 ft wide, and 6 ft high. The peak of the pyramid is 2 ft above the top edge of the walls. What is the volume of the inside of the tent?



40. Disposable paper drinking cups like those used at water coolers are often cone-shaped. Find the volume of such a cup that is 9 cm high with a 3.2 cm radius. Express the answer to the nearest milliliter.



41. A manufacturer is to design a can in the shape of a right circular cylinder to hold 0.5 L of juice concentrate. If the can must have a diameter of 7 cm, how tall will the can be (to the nearest centimeter)?



42. A specialty lined storage chest has the inside dimensions of 5 ft long, 3 ft wide, and 2 ft high.

a. What is its volume?

b. What is its surface area?

43. A cubic footstool is 1.5 ft long in each direction.

a. What is the volume of the footstool?

b. How many square feet of material are necessary to cover the footstool? Assume the bottom is also being covered by the material.

44. A group of college students went to the beach and inflated their 2-ft spherical beach ball, whose radius is 1 ft.

a. What is the volume of the ball? Round your answer to the nearest hundredth.

b. What is the surface area of the ball?

45. Alan plans to weld pieces of metal together to create a metal cube. The length of each side of the cube will be 16 inches. How much metal will he need to create the cube?

46. A soup can has a diameter of 3 inches and a height of 5 inches.

a. Approximately how much material is needed to create the soup can? (**Hint:** It is the same as the surface area.)

b. What volume of soup can fit inside of the soup can? Round to the nearest tenth.

Writing & Thinking

47. Discuss the type of units used for volume and explain why.

48. List the steps and formulas you would use to find the volume of an ice cream cone (assuming the ice cream itself forms a perfect half sphere).

49. Discuss what you think would be more important to a UPS driver when loading his truck, surface area or volume.

50. No formula was given in the text for the surface area of a rectangular pyramid. Create a plan to find the surface area of this type of figure. Include formulas and operations you would use.

6.9 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. An acute angle has a measure between ____ and _____.
2. A right triangle has one _____ angle.
3. ____ trigonometric functions of the angle θ can be defined as ratios using the three sides of a right triangle.
4. The function ____ is the reciprocal of sine.
5. In one common application of right triangles, the lengths of two sides are given and you are to find one of the _____ angles.
6. The notation $\tan^{-1}$ stands for the _____ _____ function.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

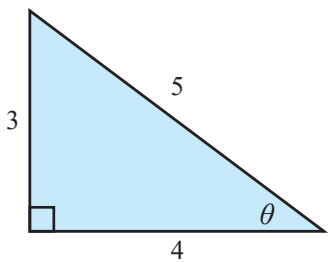
7. The hypotenuse of a right triangle is the side opposite the right angle.
8. If θ is an acute angle of a right triangle, then $\sin\theta = \frac{opp}{hyp}$.
9. If θ is an acute angle of a right triangle, then $\tan\theta = \frac{opp}{adj}$.
10. If the hypotenuse of a right triangle is 12 and the length of the side adjacent to angle θ is 5, then $\cos\theta = \frac{5}{12}$.

Practice

For the given right triangle, answer the following questions. See Example 1.

a. Which side is opposite angle θ?
b. Which side is adjacent to angle θ?
c. What are the values of sin θ, cos θ, and tan θ?

1.



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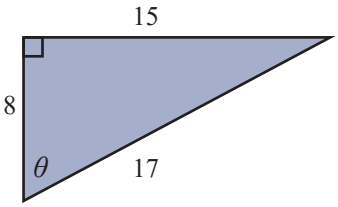

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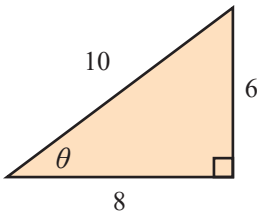

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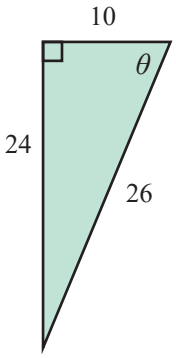


Use a calculator to determine the value of each of the following trigonometric functions (accurate to the nearest thousandth). See Example 2.

5. sin50.2°

6. sin64.5°

7. cos24.5°

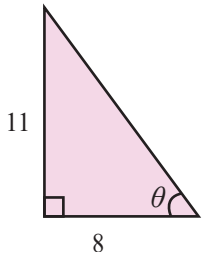
8. tan80°

9. tan30.42°

10. cos2.596°

The lengths of two sides of each right triangle are given. Determine the measure of the angle θ in each triangle (accurate to the nearest tenth of a degree). See Example 4.

11.



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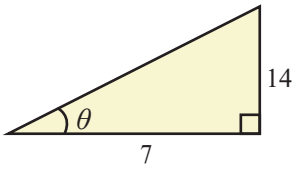

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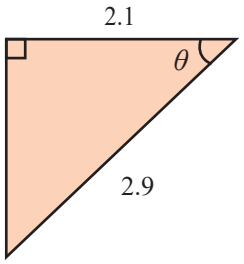

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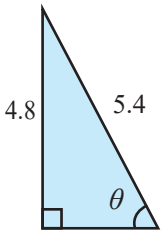
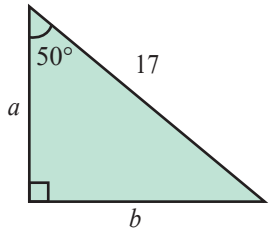


Use trigonometric functions to determine the lengths of the sides labeled *a* and *b* in each right triangle (to the nearest tenth).

15.



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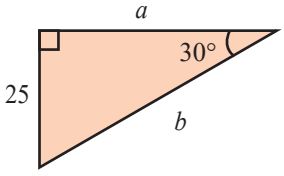

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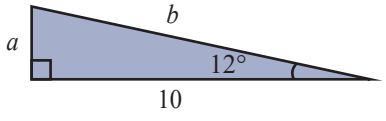

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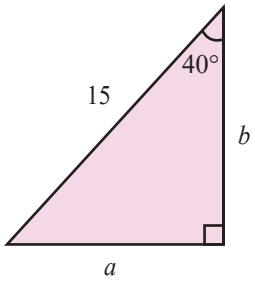


Applications

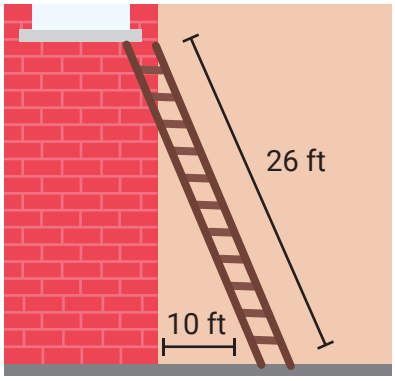
Solve. For the following problems, make sure your calculator is set to display angles in degrees. Round your each length to the nearest tenth, if necessary. Round each angle to the nearest hundredth, if necessary.

19. A ladder 26 feet long is leaning against a wall of a building. The bottom of the ladder is 10 feet from the wall.

a. What is the measure of the angle (to the nearest tenth of a degree) the ladder makes with the wall?

b. What is the measure of the angle (to the nearest tenth of a degree) the ladder makes with the ground?

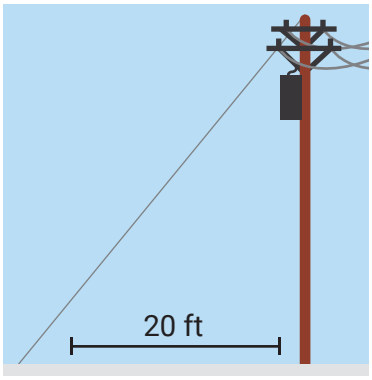
c. How many feet above the ground is the point where the ladder touches the wall?



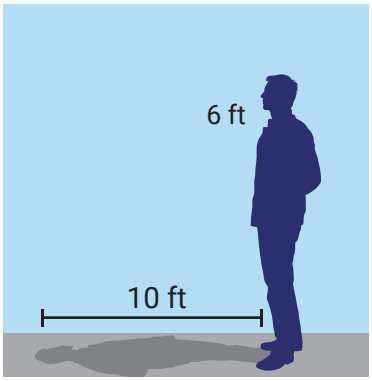
20. A telephone pole is 30 feet tall. A guy wire reaches from the top of the pole to a point 20 feet from the base of the pole.

a. Approximately how long is the guy wire (to the nearest tenth of a foot)?

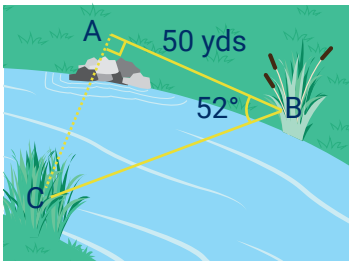
b. What is the measure of the angle that the guy wire makes with the ground (to the nearest tenth of a degree)?



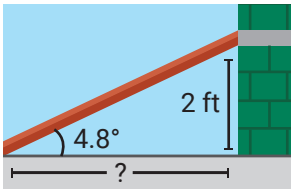
21. The tip of the shadow of a person 6 feet tall is 10 feet from the person's feet. What is the angle of elevation (to the nearest tenth of a degree) of the sun at this time? (See figure.)



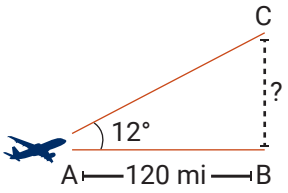
22. To find the width of a river an ecologist measures 50 yards from a point A to point B on one side of a river. Then, from point B, she sights a point C on the shore directly opposite point A. With a sextant she measures the angle to be 52°. What is the approximate width of the river (accurate to the nearest yard)?



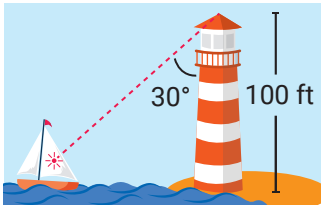
23. A carpenter must construct a wheelchair access ramp according to certain specifications. The ramp must be 2 feet high and have a 4.8° incline. Determine how far away from the building the ramp must start.



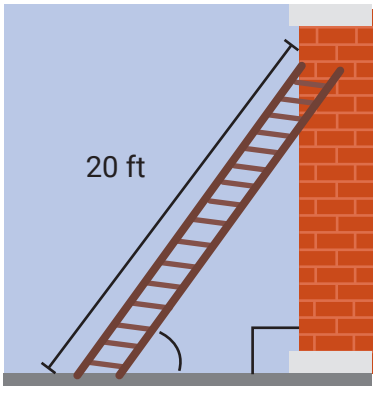
24. A pilot was flying directly from point A to point B. A storm forced her to deviate from the original course by 12°, making her land at point C. How far away is the pilot from her original destination if the distance between points A and C is 120 miles.



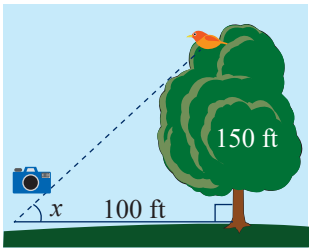
25. A person standing atop of 100 ft tall a lighthouse points a laser beam towards a ship. The angle between the laser beam and the lighthouse is 30°. How far away is the ship from the base of the lighthouse?



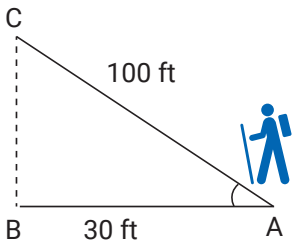
26. The safety instructions for a 20 ft ladder state that it is only safe to use when the ladder's angle with a level floor is between 60° and 70°. Determine the lowest point and the highest point up a wall that the top of the ladder can safely rest against the wall.



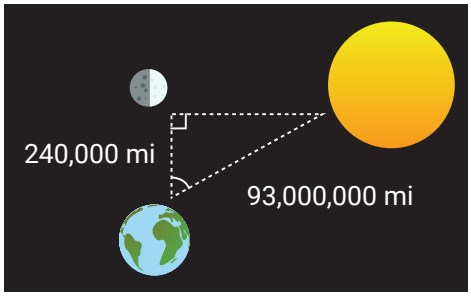
27. A biologist is 100 ft away from the bottom of a tree. She is trying to take pictures of birds that are perched on top of the tree. She knows that the height of the tree is 150 ft. Determine the angle she must elevated her camera in order to photograph the birds, assuming the camera is at ground level.



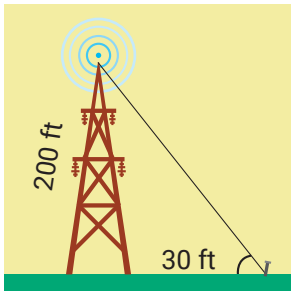
28. A hiker planned to walk 8 miles west, in a straight line, from point A to point B. Unfortunately, he got slightly disoriented and end up at point C, which is 8.5 miles from point A and directly north of point B. Determine how many degrees off-course the hiker was.



29. During a quarter moon (the phase when half of the moon is illuminated) the Earth, Moon, and Sun create a right triangle. The distance between the Moon and the Earth is approximately 240,000 miles while the distance between the Sun and the Earth is approximately 93,000,000 miles. Determine the angle between these two distances.



30. A cellphone tower is 200 ft tall. An engineer specifies that a support cable must but attached to the top of the tower and to a point 30 ft away from the base of the tower. Determine the angle between the ground and the tethering cable.



Writing & Thinking

31. Explain why it is not possible for the cosine of any angle to be equal to 2. Think about what would have to be true about the relative size of the adjacent side and the hypotenuse of a right triangle with such an angle.

32. Now, generalize your thoughts for question 31 and explain why neither the cosine nor sine of any angle can never be a number greater than 1.

33. Explain why any positive number can be the tangent of an angle.

Example 11 Application: Calculating the Percent of Profit

Calculate the percent of profit for both **a.** and **b.** and tell which is the better investment.

a. \$300 profit on an investment of \$2400 or

b. \$500 profit on an investment of \$5000

Solution

Set up ratios and find the corresponding percents.

a. $$\frac{\$300 \text{ profit}}{\$2400 \text{ invested}} = \frac{\cancel{300} \cdot 1}{\cancel{300} \cdot 8} = \frac{1}{8} = 0.125 = 12.5\%$$ percent of profit

b. $$\frac{\$500 \text{ profit}}{\$5000 \text{ invested}} = \frac{\cancel{500} \cdot 1}{\cancel{500} \cdot 10} = \frac{1}{10} = 0.1 = 10\%$$ percent of profit

Clearly, \$500 is more than \$300, but 12.5% is greater than 10%, and investment **a.** is the better investment.

Now work margin exercise 11.

11. Calculate the percent of profit for both **a.** and **b.** and tell which is the better investment.

a. \$900 profit on an investment of \$2700

b. \$400 profit on an investment of \$1100

Margin Exercise Answers

1. a. 0.35% **b.** 0.40 **2. a.** $\frac{1}{16}$ **b.** 18.75% **3.** 198 **4.** 336.8 **5.** 20% **6.** \$624 **7.** \$648.96 **8.** \$10,300 **9.** 4% **10.** 3% **11. a.** has a percent of profit of 33.3% and **b.** has a 36.4% percent of profit. Therefore, **b.** is a better investment.

7.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. To change a decimal number to a percent, move the decimal point _____ places to the _____.
2. To change a percent to a decimal number, move the decimal point _____ places to the _____.
3. The amount of reduction in the original selling price is called a _____. The reduced price is the _____ price.
4. Sales tax is a percentage of the _____ price. This tax is added to the buyer's cost.
5. The fee paid to an agent or salesperson for a service is called a _____.
6. If the value of an item increases, the increase is value can be called _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. To change a decimal number to a percent, move the decimal point two places to the left and add the % sign.

8. When using the basic formula $R \cdot B = A$, the word "of" means to divide.

9. If an item is selling for a 35% discount, the customer will pay 65% of the original price.

10. A car was purchased in 1965 for $3800. It sold for $1200 in 2011. This is an example of depreciation.

Practice

Change each decimal number to a percent. See Example 1.

1. 0.02

2. 0.09

3. 0.1

4. 0.7

5. 0.36

6. 0.52

7. 0.128

8. 0.368

9. 1.12

10. 1.75

11. 2

12. 25

Change each percent to a decimal number. See Example 1.

13. 2%

14. 7%

15. 18%

16. 42%

17. 60%

18. 30%

19. 125%

20. 120%

21. 17.3%

22. 10.1%

23. 0.26%

24. 0.52%

Change each fraction or mixed number to a percent. If necessary, round to the nearest tenth of a percent. See Example 2.

25. $\frac{7}{100}$

26. $\frac{16}{100}$

27. $\frac{1}{2}$

28. $\frac{1}{4}$

29. $\frac{11}{20}$

30. $\frac{35}{56}$

31. $\frac{1}{8}$

32. $\frac{7}{12}$

33. $1\frac{1}{4}$

34. $5\frac{3}{10}$

35. $2\frac{1}{15}$

36. $4\frac{1}{18}$

Change each percent to a fraction or mixed number and reduce, if possible. See Example 2.

37. 4%

38. 15%

39. 25%

40. 66%

41. 300%

42. 125%

43. 0.75%

44. 0.2%

45. $\frac{1}{2}\%$

46. $\frac{1}{4}\%$

47. $12\frac{1}{2}\%$

48. $16\frac{2}{3}\%$

Use the equation $R \cdot B = A$ to find each unknown quantity. Round percents to the nearest tenth of a percent. All other answers should be rounded to the nearest hundredth, if necessary. See Examples 3 through 5.

49. 10% of 70 is what number?

50. 6% of 75 is what number?

51. Find 75% of 12.

52. Find 60% of 35.

53. 150% of ____ is 63.

54. 110% of ____ is 330.

55. ____% of 60 is 90.

56. ____% of 60 is 150.

57. What percent of 75 is 15?

58. What percent of 120 is 90?

59. 3 is 2% of what number?

60. 17 is 20% of what number?

61. ____% of 34 is 17.

62. ____% of 30 is 6.

63. 100% of 36 is ____.

64. 1% of 148 is ____.

65. What percent of 48 is 16?

66. What percent of 100 is 35?

67. 82% of 50 is ____.

68. 22 is 20% of ____.

69. 18 is what percent of 10?

70. 15 is what percent of 10?

71. 92.1 is 15% of what number?

72. 119.6 is 23% of what number?

73. 9.5 is 25% of ____.

74. 29.2 is 40% of ____.

75. 100 is 125% of ____.

76. 30 is 120% of ____.

77. 36 is ____% of 18.

78. 60 is ____% of 40.

Applications

Solve.

79. A new briefcase was priced at $275. If it were to be marked down 30%:

a. What would be the amount of the discount?

b. What would be the new price?

80. A store owner received a 3% discount from the manufacturer when she bought $15,500 worth of dresses.

a. What was the amount of the discount?

b. What did she pay for the dresses?

81. Sheets are marked $22.50 and pillowcases $7.50. What is the sale price of each item if each item is discounted 25% off the marked price?

82. Towels were on sale at a discount of 30%. If the sale price was $3.01, what was the original price?

83. Headphones were on sale for $49.00. What was the original price if the sale price represents a discount of 20%?

84. If the sales tax rate is 6.5%, what is the tax on an $800 purchase?

85. If the sales tax in a certain state is figured at 6%:

a. How much tax is there on a purchase of $30.20?

b. What is the total amount paid for the purchase?

86. If sales tax is figured at 6%:

a. How much tax would be paid on the purchase of three textbooks priced at $55.00, $25.50, and $43.95?

b. What would be the total cost of all three books?

87. If sales tax is figured at 7.25%, how much tax will be added to the total purchase price of three textbooks priced at $25.00, $35.00, and $52.00?

88. A real estate agent works on 6% commission. What is his commission on a house he sold for $195,000?

89. A car saleswoman earns a commission of 7% on each car she sells. How much did she earn on the sale of a car for $12,500?

90. A real estate agent works on 6% commission. What is his commission on a house he sold for $125,000?

91. If a salesman works on a 10% commission (no monthly salary), how much merchandise will he have to sell to earn $2800 in one month?

92. A real estate agent works on a 5% commission. How much would she need to sell a house for in order to earn $24,250 in commission?

93. A sales clerk receives a monthly salary of $500 plus a commission of 6% on all sales over $3500. What did the clerk earn the month she sold $8000 in merchandise?

94. Central Valley Community College had 48 teams compete at their 3rd annual corn hole tournament. The following year they had 54 teams compete. What was the percent increase in competing teams?

95. Due to the increasing cost of breakfast cereals, more and more people are buying private-label brands rather than national brands. In a recent year, the sale of private-label cereals rose from 170 million boxes to 180 million boxes. What was the percent increase in sales (to the nearest tenth of a percent)?

96. The average attendance to a Yankees game in 2009 was 45,364 fans. In 2010 the average attendance grew to 46,491 fans. Find the percent increase in attendance. Round your answer to the nearest thousandth.

97. In 1966 the student enrollment at California Polytechnic State University in San Luis Obispo, CA, was 7740. In 1977 the university had 15,502 students. Since that time the enrollment growth has slowed. What was the percent increase of student enrollment during that eleven year period? Round your answer to the nearest tenth of a percent.[1]

98. Conroe City, Texas, had the largest population growth among large cities according to the Census Bureau. In 2015, the population of Conroe City was 76,332. The percent increase between 2015 and 2016 was 7.8%. What was the population of Conroe City in 2016 (to the nearest whole number)?

99. The 2010 population of Wheeling, WV, was 43,002 while the 2015 population dropped to 42,573. What was the percent decrease of population in that five year period? Round your answer to the nearest tenth of a percent.[2]

100. The circulation of the Washington Post newspaper was approximately 633,100 in 2009, and it dropped to 395,234 in 2015. What was the percent decrease in circulation? Round your answer to the nearest percent.[3]

101. The Dow Jones Industrial Index had a peak of 13,930 in October of 2007, but dropped to a minimum of 7,063 in February 2009. Fortunately this dip was short lived, and the market started increasing again. What was the percent decrease in the stock market drop according to the Dow Jones Industrial Index during this sixteen month interval? Round your answer to the nearest tenth of a percent.[4]

1 Source: http://lib.calpoly.edu/universityarchives/history/timeline/
2 Source:http://www.city-data.com/zips/26003.html
3 Source: http://www.capitolcommunicator.com/washington-post-circulation-drops-37-percent-since-2009-states-dcrtv/
4 Source: http://stockcharts.com/charts/historical/djia2000.html

102. A company manufactures and sells plastic boxes that cost $21 each to produce, and that sell for $28 each.

a. How much profit does the company make on each box?

b. What is the percent of profit based on cost?

c. What is the percent of profit based on selling price?

103. Men's suits were on sale for $300. Each one cost the store owner $250.

a. What was the profit for the store?

b. What as the store's percent profit based on cost?

c. What was the store's percent profit based on selling price?

104. The cost of a 20-inch television set to a store owner was $450, and she sold the set for $630.

a. What was her profit?

b. What was her percent of profit based on cost?

c. What as her percent of profit based on selling price?

105. An art gallery sells paintings by a well-known artist for $2500 each. The gallery owner has agreed to pay the artist $2000 for each painting of a certain size.

a. What is the profit on each painting?

b. What is the percent of profit based on cost?

c. What is the percent of profit based on selling price?

106. The Golf Pro Shop had a set of 10 golf clubs that were marked on sale for $860. This was a discount of 20% off the original selling price.

a. What was the original selling price?

b. If the clubs cost the Golf Pro Shop $602, what was its profit?

c. What was the shop's percent of profit based on the original selling price?

d. What was the percent of profit based on the sale price?

Writing & Thinking

107. Determine how to calculate sales tax when eating out and relate this process to either a proportion and/or using the amount/base/rate equation. Give an example.

108. A man weighed 200 pounds. He lost 20 pounds in 3 months. Then he gained back 20 pounds 2 months later.

a. What percent of his weight did he lose in the first 3 months?

b. What percent of his weight did he gain back?

c. The loss and gain are the same, but the two percentages are different. Explain why.

109. Explain the process to determine how to find percent of profit based on

a. cost.

b. selling price.

Collaborative Learning

With the class separated into teams of two to four students, each team is to analyze the following problem and decide how to answer the related questions. Then each team leader is to present the team's answers and related ideas to the class for general discussion.

110. Jerry works in a bookstore and gets a salary of $500 per month plus a commission of 3% on whatever he sells. Wilma works in the same store, but she decided to work on a straight 8% commission.

a. At what amount of sales will Jerry and Wilma make the same amount of money?

b. Up to that point, who will be making more?

c. After that point, who would be making more? Explain briefly. (If you were offered a job at this bookstore, which method of payment would you choose?)

12. Suppose that a certain boat depreciates at a rate of 18% each year. Find the current market value of the boat if it is 7 years old and its original value was $13,000.

Example 12 Application: Calculating the Current Value Due to Depreciation

Suppose that a certain make of automobile depreciates 15% each year. Find the current market value of one of these automobiles if it is 5 years old and its original value was $40,000.

Solution

$P = \$40{,}000, \quad r = 15\% = 0.15, \quad t = 5$ years

Using the formula for the current value due to depreciation, we have the following.

$$V = P(1-r)^t = 40{,}000(1-0.15)^5$$

Putting this in the calculator gives us 17,748.2125.

The current market value of the automobile is $17,748.21.

Now work margin exercise 12.

Completion Example Answers

2. 0.05; $\frac{1}{4}$ year; $I = 2400 \cdot 0.05 \cdot \frac{1}{4} = 30$; $30

10. a. $\left(1+\frac{0.12}{4}\right)^{(4\cdot 4)}$; $(1+0.03)^{16}$; $(1.03)^{16}$; (1.604706439); 9628.24; $9628.24 **b.** $9628.24, $3628.24

Margin Exercise Answers

1. $127.50 **2.** $35 **3.** $2150 **4.** $25,000 **5.** $\frac{5}{6}$ year (or 10 months) **6.** $95.51 **7.** $3215.57 **8.** $3590.44 **9.** $590.44 **10. a.** $15,529.69 **b.** $5529.69 **11.** $38,900.38 **12.** $3240.71

7.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Money paid for the use of money is _____.
2. The amount of money being invested or borrowed is known as the _____.
3. Money borrowed and paid back in one payment is calculated with _____ interest.
4. In the formula $I = P \cdot r \cdot t$, the units for time must be in _____ .
5. If interest is compounded four times per year, it is said to be compounded _____.
6. Interest paid on interest earned is known as _____ interest.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. In the simple interest formula, the rate can be written as a decimal number or a fraction.

8. Simple interest can be compounded monthly or quarterly.

9. Interest cannot be earned on interest, only the principal.

10. Inflation can be treated in the same manner as simple interest.

Applications

Solve each problem using the formula for simple interest. Round your answer to the nearest cent, if necessary. Assume 1 year equals 360 days. See Examples 1 through 5.

1. What is the simple interest paid on $500 at 6% for one year?

2. What is the simple interest paid on $2000 at 8% for one year?

3. What will be the interest earned in one year on a savings account of $800 if the bank pays 4% interest?

4. If interest is paid at 6% for one year, what will a principal of $1800 earn?

5. How much interest would be paid on a loan of $5000 at 8% for 6 months?

6. How much interest would be paid on a loan of $3000 at 5% for 9 months?

7. Stacey loaned her brother $1500 for 8 months at 10% interest. How much interest did she earn?

8. Find the simple interest paid on a savings account of $2800 for 120 days at 3.5%.

9. If you were to borrow $1000 at 5% for nine months, how much interest would you pay?

10. What principal will earn $50 in interest if it is invested at 6% for one year?

11. What principal will earn $50 in interest if it is invested at 8% for 90 days?

12. What principal will earn $75 in interest if it is invested for 60 days at 9%?

13. What principal will earn $75 in interest if it is invested at 5% for 6 months?

14. What principal would have to be invested at 8% for 60 days to earn interest of $500?

15. How long will it take for $1000 invested at 5% to earn $50 in simple interest?

16. What length of time will it take to earn $70 in simple interest if $2000 is invested at 7%?

17. How many days must you leave $1000 in a savings account at 5.5% to have a balance of $1011?

18. If interest is paid at 6% for one year, what will a principal of $1800 earn?

19. If a principal of $900 is invested at a rate of 4% for 90 days, what will be the interest earned?

20. If you borrow $750 for 30 days at 9%, how much interest will you pay?

21. How much interest would be paid on a 60-day loan of $500 at 4%?

22. What interest rate would you be paying if you borrowed $1000 for 6 months and paid $60 in interest?

23. What rate of interest is charged if a loan of $2500 for 90 days is paid off with $2562.50?

24. A friend wants to borrow $500 from you for 8 months and is willing to pay you interest at 6%. How much would he owe you at the end of the 8 months?

25. If you charge $1000 worth of merchandise at a local department store at 18% interest, how much will you owe at the end of 60 days?

26. A bank decides to loan $5 million to a contractor to build new homes. How much interest will the bank earn in one year if the interest rate is 9.2%?

27. Every 6 months a stock pays 10% in dividends (interest on investment). What will be the earnings of $14,600 invested for 6 months? (Remember, the rates of interest are given as annual rates.)

28. A credit card company has $120 million loaned to its customers at 18.9%. How much interest will it earn in one month?

29. A department store keeps $15 million in merchandise in stock. If the store pays interest at 9% on a bank loan for this stock, how much interest will the store pay in 3 months' time?

30. You buy an oven on sale from $500 to $450, but you don't make a payment for 60 days and are charged interest at a rate of 18%.

- **a.** How much do you pay for the oven by waiting 60 days to pay?
- **b.** How much do you save by buying the oven on sale? (Sales tax is not included here.)

31. Carlos has a savings account of $25,000 drawing interest at 8%.

- **a.** How much interest will he earn in 6 months?
- **b.** How long must he leave the money in the account to earn $1500?

32. A savings account of \$5300 is left for 90 days drawing interest at a rate of 5%.

a. How much interest is earned?

b. What is the amount in the account at the end of 90 days?

33. If you charge \$1000 worth of merchandise at a local department store at 18% interest, how much will you owe at the end of 60 days?

34. Ms. Lee accumulated \$240,000 and she wants to live on the interest each year. If she needs \$2000 a month to live on, what interest rate must she earn on her money?

35. Mr. Smith has a savings account of \$2500 that draws 4.5% interest. How many days will it take for him to earn \$75?

36. A small airline company borrowed \$7.5 million to buy some new airplanes. The loan rate was 7.5%, and the airline paid \$562,500 in interest. What was the length of time of the loan?

37. Determine the missing item in each row.

Principal	Rate	Time	Interest
\$400	16%	90 days	**a.**
b.	15%	120 days	\$5.00
\$560	12%	**c.**	\$5.60
\$2700	**d.**	40 days	\$25.50

38. Determine the missing item in each row.

Principal	Rate	Time	Interest
\$600	15%	30 days	**a.**
\$500	18%	**b.**	\$15.00
\$450	**c.**	90 days	\$22.50
d.	10%	30 days	\$1.50

Solve each problem by repeatedly using the formula for calculating simple interest. Round your answer to the nearest cent, if necessary. See Examples 6 and 7.

39. You loan your cousin \$2000 at 5% compounded annually for 3 years. How much interest will your cousin owe you?

a. First year: $I = 2000 \cdot 0.05 \cdot 1 =$ _____

b. Second year: $I =$ _____ $\cdot 0.05 \cdot 1 =$ _____

c. Third year: $I =$ _____ $\cdot 0.05 \cdot 1 =$ _____

d. The total interest is _____.

40. John borrowed \$5000 from his uncle at 6% compounded annually for 4 years. How much interest will he owe his uncle at the end of 4 years?

a. First year: $I = 5000 \cdot 0.06 \cdot 1 =$ _____

b. Second year: $I =$ _____ $\cdot 0.06 \cdot 1 =$ _____

c. Third year: $I =$ _____ $\cdot 0.06 \cdot 1 =$ _____

d. Fourth year: $I =$ _____ $\cdot 0.06 \cdot 1 =$ _____

e. The total interest is _____.

41. If \$9000 is deposited in a savings account at 4% compounded monthly, what will be the balance in the account in 4 months?

a. First month: $I = 9000 \cdot 0.04 \cdot \frac{1}{12} =$ _____

b. Second month: $I =$ _____ $\cdot 0.04 \cdot \frac{1}{12} =$ _____

c. Third month: $I =$ _____ $\cdot 0.04 \cdot \frac{1}{12} =$ _____

d. Fourth month: $I =$ _____ $\cdot 0.04 \cdot \frac{1}{12} =$ _____

e. The total interest earned is _____.

f. The balance in the account is _____.

42. Jeremy put \$3500 in a savings account at 5.5% compounded quarterly for 6 months. What will be the balance on the account at the end of 6 months?

a. First quarter: $I = 3500 \cdot 0.055 \cdot \frac{1}{4} =$ _____

b. Second quarter: $I =$ _____ $\cdot 0.055 \cdot \frac{1}{4} =$ _____

c. The total interest earned is _____.

d. The balance in the account is _____.

43. Your cousin loans you \$3000 compounded annually at 4% for 4 years. How much interest will your cousin owe you?

44. Keri borrowed \$4000 from her aunt compounded annually at 5% for 6 years. How much interest will she owe her aunt at the end of 6 years?

45. If \$9000 is deposited in a savings account compounded monthly at 4%, what will be the balance in the account in 3 months?

46. If interest is calculated at 10% compounded quarterly, what will be the value of \$15,000 in 9 months?

Solve each problem by using the compound interest formula. Round your answer to the nearest cent, if necessary. See Examples 8 and 9.

47. You deposit \$1500 at 4% to be compounded semiannually. How much interest will you earn in 3 years?

48. A principal of $2500 is deposited at 6% to be compounded monthly. How much will the account be worth in 6 months?

49. A savings account with $10,000 is compounded daily at 10%.

a. How much interested will be earned after one year?

b. What is the difference between the compound interest earned and simple interest at 10% for one year?

50. A savings account with $5000 is compounded quarterly at 8%.

a. Find the value of the account after 4 years.

b. What do you think the difference in interest would be if the money were compounded daily: about $5, $20, or over $100?

c. Find the exact difference in interest.

51. A savings account with $20,000 is compounded annually at 7%.

a. Find the value of the account after 5 years.

b. How much more would be earned if the interest were compounded daily?

52. Suppose that $3000 is invested at 5% and compounded monthly for one year.

a. Find the accumulated value.

b. Is the accumulated amount the same if the original principal of $3000 is compounded annually for 12 years? If not, what is the difference?

53. An account with $5000 is compounded monthly at 12%.

a. How much interested will be earned after one year?

b. Suppose the interest is compounded semiannually. Is the accumulated value the same?

c. If not, explain why not in your own words.

54. A savings account with $20,000 compounds annually at 10%.

a. Find the value of the account after 3 years.

b. Suppose the interest is compounded semiannually. What is the value? Is the value the same?

c. If not, explain why not in your own words.

55. An account with $10,000 is compounded monthly at 14%.

a. How much interested will be earned after one year?

b. What is the difference between this and simple interest at 14% for 1 year?

56. Suppose that $50,000 is invested in a certificate of deposit for 5 years and the interest rate is 8%.

a. What will be the interest earned if it is compounded monthly?

b. How much more interest would be earned if it were compounded daily?

57. An account with $25,000 is compounded daily at 5%.

a. Find the value of the account after 20 years.

b. Do you think that the amount will be doubled or more than doubled if the rate is doubled to 10%?

c. Find the amount if the rate is 10%.

Find the amount (A) and the interest (I) for the given information.

	Compounding Period	Principal	Annual Rate	Time	A	$I = A - P$
58.	Quarterly	$1000	10%	5 yr	**a.**	**b.**
59.	Monthly	$1000	10%	5 yr	**a.**	**b.**
60.	Daily	$1000	10%	5 yr	**a.**	**b.**
61.	Monthly	$5000	7.5%	10 yr	**a.**	**b.**
62.	Daily	$25,000	8%	20 yr	**a.**	**b.**
63.	Daily	$25,000	12%	20 yr	**a.**	**b.**

Solve. Round your answer to the nearest cent, if necessary. See Examples 11 and 12.

64. Kevin is currently spending $1300 per month on rent and utilities. Assuming an annual inflation rate of 3%, how much should he plan to spend on rent and utilities per month 2 years from now?

65. In 2013, the average price for a gallon of milk was $3.00, and the average price for a loaf of bread was $2.50. If the inflation rate was 6% per year, how much did these items cost in 2016?

66. Sam receives a cost-of-living raise equal to inflation each year. If inflation was steady at 5% annually, and his current yearly income is $56,800, what was Sam's yearly income 4 years ago?

67. The current market value of Brenda's car is $24,000. She plans to trade in her current car and buy a new one in 5 years. If the car depreciates at 12% per year, what will be the market value of her car when she is ready to trade it in?

68. Stephen has a fishing boat that he bought 3 years ago. He has decided to sell it, and the boat is valued at $8500. If the yearly rate of depreciation for his boat is 13.2%, how much did he originally pay for the boat?

69. Stan bought a new truck last year for $29,900. This year, he decided that he wants to trade it in for a smaller car. He can resell the truck for 26,500. What was the rate of depreciation for the year?

70. A house is appraised at $125,000. Assuming 3% constant inflation, what will be its value, to the nearest thousand dollars, in 30 years?

71. If a new pickup truck is valued at $18,000, what will be its value in 3 years if it depreciates 22% each year?

72. Suppose that an apartment complex is purchased for $1,500,000. For property tax purposes, the land is considered to be 30% of the value of the property. For income tax purposes, the owners are allowed to depreciate the value of the buildings by 5% per year. What will be the value of the apartment complex (buildings and land) in 10 years? (**Note:** This will not be the market value, but it will form the basis for capital gains taxes when the property is sold.)

Writing & Thinking

73. List the four parts involved in the simple interest formula. In your own words, define each one.

74. Compare and contrast simple interest with compound interest.

75. **a.** What will be the value of $10,000 compounded weekly at 10% for 3 years?

b. Use your calculator to choose values for t to use in the formula until you find approximately how many years of daily compounding are needed for the value to accumulate to $20,000.

t	$A = P\left(1 + \frac{0.10}{365}\right)^{365t}$	A

Collaborative Learning

With the class separated into teams of two to four students, each team is to analyze the following problem related to compound interest. Each team leader is to discuss the results found by the team and how the team arrived at these results. A general classroom discussion should follow with the class coming to an understanding of the concepts of present value and future value.

76. Suppose that you would like to set aside some money today for your child's college education. Your child is 3 years old and would be starting college at the age of 18. What amount should you invest today (called the present value) at 8% compounded daily to accumulate $40,000 (called the future value) for your child's education?

In groups of three to four students, work through the following problem. Discuss your answers in class.

77. Three monthly incomes are listed in the table and each receives a yearly cost-of-living raise.

Monthly Income	4%	6%	8%
$2000			
$2500			
$4000			

a. Find each monthly income after 5 years for annual inflation rates of 4%, 6%, and 8%. Round to the nearest cent.

b. Discuss the difference between starting with a $2000 monthly income with an 8% yearly pay raise and starting with a $4000 monthly income with a 4% yearly pay raise. (**Hint:** Compare each starting monthly income with the monthly income after 5 years.)

c. Suppose that the cost-of-living increases at a rate of 6% each year and you only received a 4% raise each year. Discuss how this might affect your way of living and what actions you might take.

Total car costs per year = $3909.12 + $752.84 + $1560 + $800 = $7021.96

Next, divide by 12 to determine the amount Liam should budget per month.

Monthly car expenses = $7021.96 ÷ 12 ≈ $585.16

Thus, Liam should plan to budget at least $585.16 per month for car expenses. He should re-evaluate this amount yearly to determine if his expenses have changed.

Now work margin exercise 4.

Margin Exercise Answers
1. The down payment is $3560 and the amount financed is $14,240 **2. a.** $212.84 **b.** $1070.40
3. 9% **4.** $735.99

7.3 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. _____ _____is the amount agreed to be paid to the seller in exchange for the car.
2. Finance charge is the amount of _____ paid over the life of the loan.
3. The loans taken out to purchase a car are typically _____ _____ loans.
4. The cost of gasoline, oil, tires, and tunes are considered _____ costs for a car.
5. _____ is the amount financed from the lender.
6. In addition to the _____ _____, you must pay for auto insurance, any necessary repairs, and general maintenance costs.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The down payment is the amount the seller pays upfront to sell a car.
8. Auto insurance covers a wide variety of situations with your car, including theft and accidents.
9. You should include operating costs and repairs in your monthly budget.
10. APR reflects the true finance charge of your loan.

Practice

Determine the down payment amount and the amount financed. See Example 1.

1. A car with a purchase price of $18,450 and a 10% down payment

2. A car with a purchase price of $15,000 and a 20% down payment

3. An SUV with a purchase price of $24,600 and a 15% down payment

4. A truck with a purchase price of $21,850 and a 10% down payment

5. A van with a purchase price of $27,250 and a 20% down payment

6. An SUV with a purchase price of $36,500 and a 15% down payment

Determine the monthly payment for an auto loan with the given information. Round to the nearest cent, if necessary. See Example 2.

7. A $15,000 loan at 6% APR over 3 years

8. A $23,400 loan at 6.5% APR over 5 years

9. A $19,240 loan at 8% APR over 6 years

10. A $16,800 loan at 4% APR over 4 years

11. A $18,750 loan at 4.5% APR over 5 years

12. A $21,300 loan at 5.75% APR over 4 years

Determine the total finance charge over the life of the loan with the given information. See Example 2.

13. A 4-year loan with monthly payments of $378.54 and a starting principal of $16,600

14. A 5-year loan with monthly payments of $361.01 and a starting principal of $18,900

15. A 3-year loan with monthly payments of $431.99 and a starting principal of $14,200

16. A 5-year loan with monthly payments of $418.94 and a starting principal of $22,200

17. A 4-year loan with monthly payments of $497.79 and a starting principal of $19,750

18. A 6-year loan with monthly payments of $279.72 and a starting principal of $17,750

Determine the APR for each loan. See Example 3.

19. A 5-year loan with monthly payments of $293.49 and a starting principal of $15,000

20. A 4-year loan with monthly payments of $471.49 and a starting principal of $19,500

21. A 3-year loan with monthly payments of $580.49 and a starting principal of $18,800

22. A 4-year loan with monthly payments of $546.85 and a starting principal of $22,400

23. A 5-year loan with monthly payments of $330.25 and a starting principal of $17,500

24. A 3-year loan with monthly payments of $619.11 and a starting principal of $20,200

Applications.

Solve.

25. After purchasing a car, Eleanor adjusts her budget to cover most car-related expenses that may occur. The monthly payment for her loan is $297.64, the cost of car insurance for 6 months is $276.73, the estimated fuel cost per month is $145, and she wants to set aside $950 per year for repairs and other maintenance. How much should Elanor budget per month for car expenses?

26. After purchasing a car, Tom adjusts his budget to cover most car-related expenses that may occur. The monthly payment for his loan is $322.76, the cost of car insurance for 6 months is $376.32, the estimated fuel cost per month is $195, and he wants to set aside $1200 per year for repairs and other maintenance. How much should Tom budget per month for car expenses?

27. Stephen is considering purchasing a new car and wants to make sure all of the car expenses fit into his budget of $550 per month. The estimated monthly payment for a loan is $324.32, the cost of car insurance for the year is $589.44, the estimated fuel cost per month is $100, and he wants to set aside $750 per year for maintenance. What is the estimated monthly cost for buying a new car and will this fit into Stephen's budget?

28. Maya is considering purchasing a new car and wants to make sure all of the car expenses fit into her budget of $675 per month. The estimated monthly payment for a loan is $302.97, the cost of car insurance for the year is $404.88, the estimated fuel cost per month is $200, and she wants to set aside $1800 per year for maintenance. What is the estimated monthly cost for buying a new car and will this fit into Maya's budget?

29. Alexander purchases a car and takes out a 4-year loan for $17,400. The loan officer tells him that with an interest rate of 7%, his monthly payment will be $424.78. What is the APR of the loan?

30. Kelli purchases a car and takes out a 5-year loan for $20,500. The loan officer tells her that with an interest rate of 5.5%, her monthly payment will be $410.78. What is the APR of the loan?

31. Kylie purchases a car that has a purchase price of $18,700 and makes a 15% down payment. She finances the remaining cost with a 4-year loan at 4.5% APR.

a. Calculate the monthly payment for the car loan.

b. Calculate the finance charge on the car loan after 4 years.

32. Doug purchases a car that has a purchase price of $23,600 and makes a 20% down payment. He finances the remaining cost with a 5-year loan at 6% APR.

a. Calculate the monthly payment for the car loan.

b. Calculate the finance charge on the car loan after 5 years.

33. You are buying a car that has a purchase price of $24,900. You plan to make a 25% down payment and finance the rest. How much is the down payment and what is the amount you will finance?

34. You are buying a car that has a purchase price of $17,700. You plan to make a 15% down payment and finance the rest. How much is the down payment and what is the amount you will finance?

Margin Exercise Answers

1. a. \$153,000 **b.** \$22,054 **2. a.** \$697.87 **b.** \$992.87 **3. a.** 1299.97 **b.** This payment fits into the monthly budget of \$1500

4.

Payment Number	Interest Payment	Principal Payment	Mortgage Balance
4	\$672.32	\$239.71	\$179,046.53
5	\$671.42	\$240.61	\$178,805.93
6	\$670.52	\$241.51	\$178,564.42

7.4 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. _____ _____ is the selling price that you agree to pay for the house.
2. Cash paid to the seller that is not part of the loan is called the _____ _____.
3. An amortization schedule breaks down your monthly payment to show how much is applied to the _____ and how much goes towards _____.
4. The general guideline is to spend at most 30% of your income (before taxes) on _____ _____.
5. _____ _____ are the amount you need to pay to finalize the purchase of a house.
6. Points are a fee paid to the lender at closing in exchange for a lower _____ _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Private mortgage insurance is required on all mortgages.
8. One point is equal to 1% of the mortgage.
9. When budgeting for housing costs, you should factor in homeowners insurance and property tax.
10. During initial payments of a mortgage, more money goes towards the principal than towards interest.

Practice

Determine the down payment amount and the amount financed. See Example 1.

1. A house sells for \$160,000 and the buyer makes at 15% down payment.
2. A house sells for \$250,000 and the buyer makes an 18% down payment.
3. A house sells for \$195,000 and the buyer makes a 20% down payment.

4. A house sells for $176,000 and the buyer makes a 10% down payment.

5. A house sells for $385,900 and the buyer makes a 20% down payment.

6. A house sells for $495,000 and the buyer makes a 15% down payment.

Calculate the monthly mortgage payment. Round to the nearest cent. See Example 2.

7. $155,000 is financed for 30 years at 4.5% APR

8. $190,000 is financed for 15 years at 4% APR

9. $175,000 is financed for 15 years at 4.25% APR

10. $214,000 is financed for 30 years at 5% APR

11. $388,000 is financed for 30 years at 4.25% APR

12. $412,600 is financed for 30 years at 4.75% APR

Fill in an amortization table for the first three months for each mortgage. See Example 4.

Payment Number	Interest Payment	Principal Payment	Mortgage Balance
1			
2			
3			

13. A 30-year $200,000 mortgage at an APR of 4% has a monthly payment of $954.83.

14. A 30-year $160,000 mortgage at an APR of 5% has a monthly payment of $858.91.

15. A 30-year $250,000 mortgage at an APR of 4.5% has a monthly payment of $1266.71.

16. A 15-year $185,000 mortgage at an APR of 4% has a monthly payment of $1368.42.

Applications

Solve.

17. Andrea is buying a house for $130,000. She plans to make a 20% down payment. Closing costs include $400 for 6 months of homeowners insurance, $900 for 6 months of property tax, $150 for the title fee, and $450 in transaction fees. Andrea also agreed to pay two points in exchange for a 0.5% reduction in interest rate.

    a. Determine the mortgage amount.

    b. Determine the amount of money Andrea needs to cover closing costs.

18. Kylie and Joel are buying a house for $220,000. They plan to make a 10% down payment. Since the down payment is less than 20%, they will also have to pay for PMI that costs 0.5% of the mortgage value and will be included in the closing costs. Other closing costs include $675 for 6 months of homeowners insurance, $1300 for 6 months of property tax, $150 for the title fee, and $600 in transaction fees.

a. Determine the mortgage amount.

b. Determine the amount of money Kylie and Joel need to cover closing costs.

19. Manuel purchases a house and gets a 30-year mortgage for $170,500 at 4.25% APR. In addition to the monthly payment, the lender requires him to pay into an escrow account for the homeowners insurance and property tax. His homeowners insurance is $1050 per year and the property tax is $2200 per year.

a. Determine the monthly mortgage payment for this loan. Round to the nearest cent, if necessary.

b. Determine the monthly payment to the lender that includes the insurance and property tax.

20. Sarah purchases a house and gets a 30-year mortgage for $142,000 at 4.5% APR. In addition to the monthly payment, the lender requires her to pay into an escrow account for the homeowners insurance and property tax. Her homeowners insurance is $850 per year and the property tax is $1700 per year.

a. Determine the monthly mortgage payment for this loan. Round to the nearest cent, if necessary.

b. Determine the monthly payment to the lender that includes the insurance and property tax.

21. You and your spouse make $4800 per month and are looking to buy a house. You want to spend no more than 30% of your monthly budget on mortgage payments. You've saved $45,000 for a down payment and closing costs. You are interested in a house that is listed at $270,000. The estimated yearly property tax for the house is $2700 and the estimated homeowners insurance for a year is $1250.

a. Suppose you purchase the house for the price listed and make a 15% down payment. Determine the estimated monthly payment, including homeowners insurance and property tax, on a 30-year mortgage at 4.5% APR.

b. Does this monthly payment fit into your budget?

22. You and your spouse make $3600 per month and are looking to buy a house. You want to spend no more than 30% of your monthly budget on mortgage payments. You've saved $25,000 for a down payment and closing costs. You are interested in a house that is listed at $150,000. The estimated yearly property tax for the house is $1900 and the estimated homeowners insurance for a year is $970.

a. Suppose you purchase the house for the price listed and make a 10% down payment. Determine the estimated monthly payment, including homeowners insurance and property tax, on a 30-year mortgage at 4.75% APR.

b. Does this monthly payment fit into your budget?

23. You are taking out a 30-year mortgage on a house for $276,400. The loan officer tells you that you can decrease your interest rate from 4.75% to 4.25% if you pay 2 points on your mortgage up front.

a. Determine the total finance cost of the mortgage at 4.75% APR.

b. Determine the total finance cost of the mortgage at 4.25% APR, including the cost of the points paid up front.

c. Is it worth paying points up front to decrease the interest rate? Explain why or why not.

24. You are taking out a mortgage on a house for $312,760. The loan officer tells you that you can get a 15-year mortgage at 4% APR or a 30-year mortgage at 4.5% APR.

a. Determine the total finance cost of the 15-year mortgage.

b. Determine the total finance cost of the 30-year mortgage.

c. Describe the pros and cons of each mortgage option. Which would you go with, a 15-year mortgage or a 30-year mortgage?

c. $\{1, 2, 3, 4\} \subseteq A$. $\{1, 2, 3, 4\}$ contains all the elements of A. This means it is a subset of A, but not a proper subset of A.

Now work margin exercise 9.

Margin exercise answers

1. $W = \{0, 1, 2, 3\}$ **2.** $T = \{a, b, c, d, e, f, g, h\}$ **3.** $F = \{x \mid x \text{ is an NFL team}\}$ **4. a.** T is the set of Philadelphia professional sports teams **b.** $T = \{\text{Phillies, Eagles, 76ers, Flyers, Union}\}$ **c.** $T = \{x \mid x \text{ is a Philadelphia professional sports team}\}$ **5. a.** Florida $\in S$; **b.** Los Angeles $\notin S$ **6. a.** Paris $\in C$; **b.** London $\notin E$ **7. a.** and **c.** are empty sets. **8.** There are 7 proper subsets of B. $\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$. **9. a.** $\varnothing \subseteq$ and $\subset A$. **b.** No symbol can be inserted here. While bird $\in A$, bird is a not a subset of A. In order to be a *subset*, it is necessary to be a *set*. **c.** No symbol can be inserted here. The element in this set, camel, is not an element of A.

8.1 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. A _____ is a collection of items.

2. To write a set using _____ notation, list each member of the set, separating members by commas, and enclosing them in braces, { }

3. _____is method of writing a set that it uses the features of both the word description method and the roster method.

4. A set with no elements is called the _____ or the null set.

5. A _____ is a set that consists of elements of another set.

6. If there are n elements in a set, use the formula _____ to determine the number of proper subsets of that set.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The only ways to write sets are with word descriptions and set-builder notation.

8. Given $A = \{1, 2, 3, 4, 5\}$, it is correct to write that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $\{5\} \subseteq A$.

9. A set cannot be considered a subset of itself.

10. The set of all negative natural numbers is the empty set.

Practice

Write a word description of these sets. There may be more than one correct answer.

1. {January, February, March, April, May, June, July, August, September, October, November, December}

2. $\{m, o, z, a, r, t\}$

3. $\{2, 4, 6, 8, 10\}$

4. $\{-9, -7, -5, -3, -1\}$

5. $\{x \mid x \text{ is a book written by J.R.R. Tolkien}\}$

6. $\{x \mid x \text{ is an appetizer on sale during happy hour}\}$

Write these sets using roster notation. See Examples 1 and 2.

7. The set of months of the year with less than 30 days.

8. The set of days in the week that begin with letter S.

9. The planets in our solar system that begin with the letter M.

10. $\{x \mid x \text{ is a letter in the word "Bookkeeper"}\}$

11. $\{x \mid x \text{ is a natural number and } 5 \le x < 9\}$

12. $\{x \mid x \text{ is a prime number between 4 and 20}\}$

Write these sets using set-builder notation. There may be more than one correct answer. See Example 3.

13. $\{\text{Winter, Spring, Summer, Autumn}\}$

14. $\{\text{red, yellow, blue}\}$

15. $\{j, u, l, y\}$

16. $\{p, i, c, k, l, e\}$

17. $\{1, 2, 3, 4, 5\}$

18. $\{-10, -8, -6, -4\}$

Fill in the blank with the correct symbol, $\in$ or $\notin$. See Examples 5 and 6.

$A = \{1, 3, 5, 7, 9\}$

19. 1 _____ A

20. 2 _____ A

21. $\{1\}$ _____ A

22. $\varnothing$ _____ A

Determine if the set is empty. See Example 7.

23. $\{x \mid x \text{ is an even prime number greater than } 4\}$

24. $\{x \mid x \text{ is a positive number and } x < 0\}$

25. The set of all crew members who journeyed to America with Christopher Columbus in 1492 who are alive today.

26. The set of all states of the United States.

Determine the number of proper subsets of the given set. See Example 8.

27. $A = \{1, 2, 3\}$

28. A is the set of the 7 Wonders of the Ancient World

29. $\{x \mid x \text{ is a day of the week that begins with the letter } T\}$

30. $\{x \mid x \text{ is a month of the year that begins with the letter } A\}$

List all the subsets of the given set. See Example 8.

31. $A = \{\text{Jack, Jill}\}$

32. $B = \{x \mid x \text{ is a letter in the word "cat"}\}$

33. $C = \varnothing$

34. $D = \{\text{North, South, East, West}\}$

Determine which symbol or symbols $(\subset, \subseteq)$, if any, can be placed in the blank to form a true statement. See Example 9.

35. $\{\text{warm}\}$ _____ $\{\text{cool, warm, hot}\}$

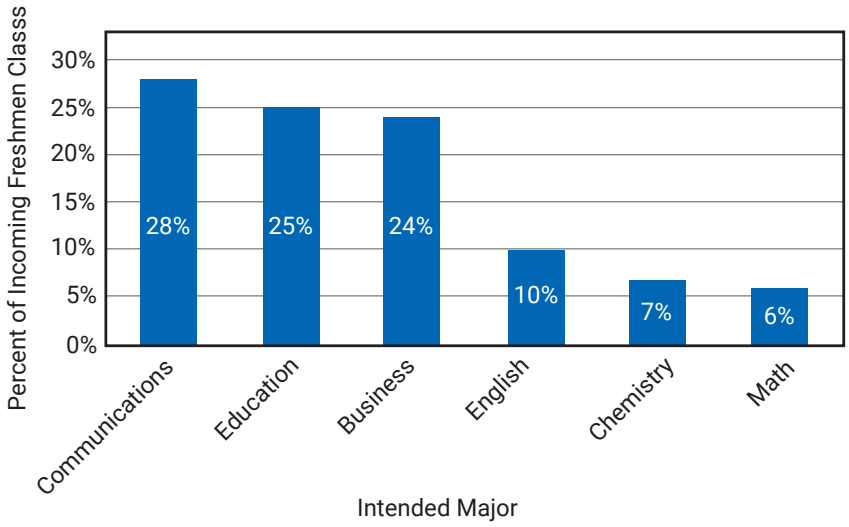
36. Mets _____ $\{\text{Mets, Yankees, Jets, Knicks, Rangers}\}$

37. $\{1, 2, 3\}$ _____ $\{3, 2, 1\}$

38. $\varnothing$ _____ $\{\text{elephant, camel}\}$

Applications

The bar graph below shows the intended major of the incoming freshmen class at a local university by percentage. Represent each set by roster notation or use appropriate notation to indicate the empty set.



39. Set of intended majors in which the percent of incoming freshmen exceeds 12%.

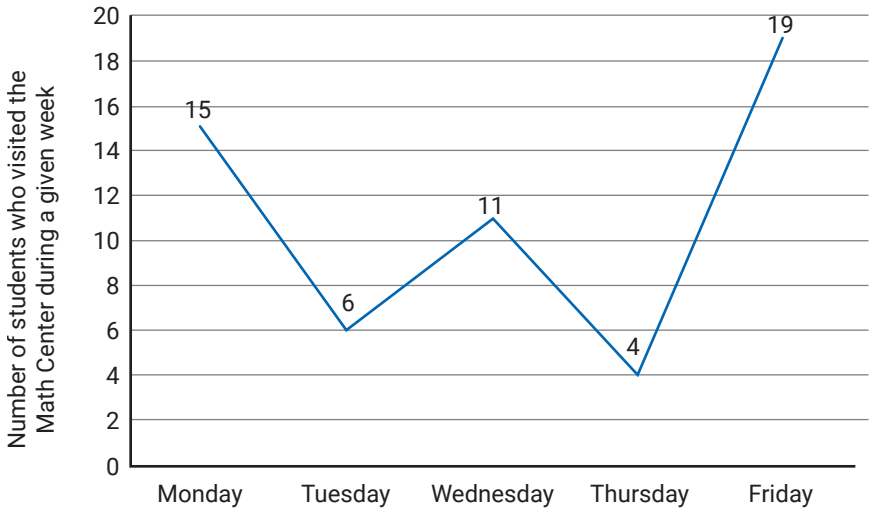
40. Set of intended majors in which the percent of incoming freshmen exceeds 30%.

41. Set of intended majors in which the percent of incoming freshmen is at least 9% and at most 26%.

42. $\{x \mid x \text{ is the intended major of at least 40\% of the incoming freshmen}\}$

43. $\{x \mid x \text{ is the intended major of between 23\% and 26\% of the incoming freshmen}\}$

The line graph below shows the number of students who visited the Math Center each weekday during a given week. Represent each set by roster notation or use appropriate notation to indicate the empty set.



44. The set of days in which between 5 and 12 students visited.

45. The set of days in which more than 10 students visited.

46. The set of days in which less than 3 students visited.

47. $\{x \mid x \text{ is a day in which an odd number of students visited}\}$

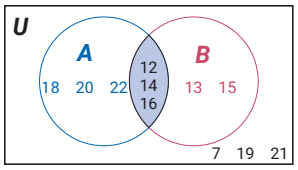
48. $\{x \mid x \text{ is day in which the number of students that visited is a prime number}\}$

Writing & Thinking

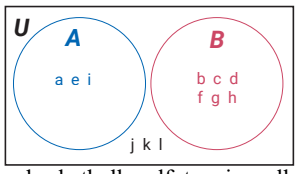
49. Do $\{\varnothing\}$ and $\varnothing$ represent the same thing? Explain your answer.

50. If a set has 15 proper subsets, how many elements are in the set?

6.



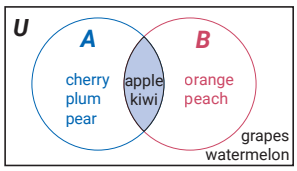
7.



8. $A \cup B = \{\text{football, baseball, hockey, basketball, golf, tennis, volleyball}\}$

9. $X \cup Y = \{12, 15, 16, 18\}$

10.



11. $A' = \{\text{penny, nickel, half-dollar, dollar}\}$

12. $A' = \{12, 13, 14, 16, 18, 19, 20\}$

8.2 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. In a Venn diagram, the large rectangle represents the _____ _____.
2. In a Venn diagram, the circles represent _____.
3. Sets that have no elements in common are said to be _____.
4. The set operation _____ is used to find the elements that two sets have in common.
5. The set operation _____ corresponds to the word "or" because it is used to find all the elements in one set or the other set.
6. The _____ of a set contains all the elements of the universal set that are not contained in the set itself.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A Venn diagram illustrates the relationship between sets.
8. The union of a set and its complement is the universal set.
9. A set and its complement always have at least one element in common.
10. The sets in a Venn diagram are subsets of the universal set.

Practice

Given:

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8, 10\}$

$C = \{1, 3, 5\}$

$D = \{7\}$

Write the elements of the sets below using roster notation.

1. $A \cap B$

2. $A \cup B$

3. A'

4. $A \cap B'$

5. $A' \cap B$

6. $C \cup D$

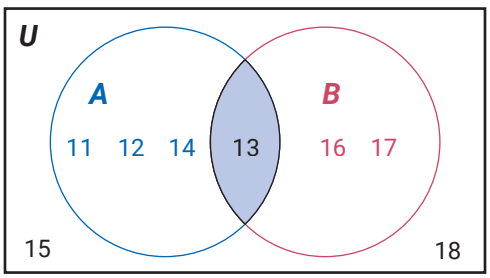
7. $C \cap D$

8. $C \cap D'$

9. $C \cup \varnothing$

10. $D \cap \varnothing$

Given:



Write the elements of the sets below using roster notation.

11. $A \cap B$

12. $A \cup B$

13. A'

14. $A \cap B'$

15. $A' \cap B$

16. U

17. U'

18. $A \cup B'$

19. $A' \cap B'$

20. $A' \cap \varnothing$

Given:

$U = \{ x \mid x \text{ is a natural number} \leq 20\}$

$A = \{ x \mid x \text{ is a prime number} \leq 20\}$

$B = \{ x \mid x \text{ is an odd number} \leq 20\}$

Write the elements of the sets below using roster notation. It may be helpful to first rewrite the sets in roster notation.

21. $A \cap B$

22. $A \cup B$

23. A'

24. $A \cap B'$

25. $A' \cap B$

26. U

27. U'

28. $A \cup B'$

29. $A' \cap B'$

30. $A' \cap \varnothing$

Given:

U is the set of states in New England.

A is the set of New England states that beginning with the letter "M".

B is the set of New England states north of Massachusetts.

Write the elements of the sets below using roster notation.

31. U

32. A

33. B

34. $A \cup B$

35. $A \cap B$

36. A'

37. B'

Applications

Use the following situation for Exercises 38 through 50.

Professor Summers teaches a Math class with ten students: Ann, Ben, Carol, David, Emily, Frank, George, Harry, Ian, and Joe. To help her students prepare for a test, she posts an online practice test and conducts a weekend review session. Ann, Carol, David, Harry, Ian, and Joe complete the online practice test, while Ann, Ben, David, and Emily attend the weekend review session. Let T be the set of students who complete the online practice test and R be the set of students who attend the weekend review.

38. Draw a Venn diagram to represent the above scenario. Using the Venn diagram, represent each set by roster notation or use appropriate notation to indicate the empty set.

39. Set of students who completed the online practice test and attended the weekend review.

40. Set of students who completed the online practice test but did not attend the weekend review.

41. Set of students who did not complete the online practice test.

42. Set of students who did not attend the weekend review.

43. $\{x \mid x$ is a student who did not complete the online practice test and did not attend the weekend review$\}$

44. $\{x \mid x$ is a student who attended the weekend review but did not complete the online practice test$\}$

45. $R' \cup T$

46. $R \cap T'$

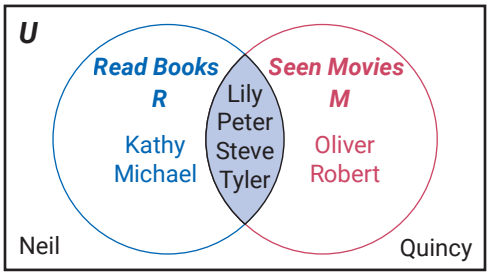
47. $R' \cup T'$

48. $U \cup \varnothing$

49. $R' \cap \varnothing$

50. U'

The Venn Diagram below shows the students of Professor Horn's English class who have read Harry Potter books, seen Harry Potter movies, or have both read Harry Potter books and seen Harry Potter movies. Use the Venn Diagram to represent each set by roster notation or use appropriate notation to indicate the empty set.



51. The set of students who have both read the books and seen the movies.

52. The set of students who have read the books, but not seen the movies.

53. The set of students who have neither read the books nor seen the movies.

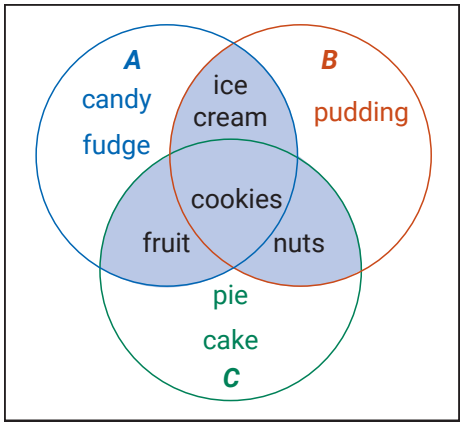
54. $\{x \mid x$ is a student who has seen the movies, but not read the books$\}$

55. $\{x \mid x$ is student who has not read the books$\}$

56. U

Writing and Thinking

A Venn Diagram can also be used to show the relationships among three sets.



Refer to the Venn Diagram above to answer these questions.

57. Use roster notation to list the elements of the set $A \cap B \cap C$. (**Hint:** Where do the three circles intersect?)

58. Use roster notation to list the elements of the set $(A \cap B) \cap C$. (**Hint:** List the elements of $A \cap B$. Now, list the elements of C. What elements do the two have in common?)

8.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. _____ _____ is a process of reaching a generalization or conclusion from specific incidents or observations.
2. A _____ is an educated guess based on inductive reasoning.
3. A _____ is an instance or time when a conjecture does not hold true.
4. _____ _____ is the process of reaching a specific conclusion from generalized statements.
5. _____ _____ uses specific incidents and observations to reach a general conclusion.
6. _____ _____ uses general statements to a specific conclusion.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. It is possible to reach the same conclusion using either inductive reasoning or deductive reasoning.
8. Deductive reasoning is used to identify patterns.
9. Concluding that there will be an unannounced quiz in Biology at the end of each section because there have been unannounced quizzes in Biology at the end of the last three sections is an example of deductive reasoning.
10. Concluding that spaghetti cooks in water because all pasta cooks in water and spaghetti is a type of pasta is an example of deductive reasoning.

Practice

Find a counterexample to disprove each conjecture. There may be more than one correct answer. See Example 1.

1. All odd numbers less than 10 are prime numbers.
2. All even numbers between 11 and 19 are divisible by 3.
3. All days of the week contain exactly two vowels.
4. No month of the year contains the vowel "i."
5. No month of the year contains less than 5 letters.
6. All cards in a standard deck of cards are black cards.

7. No whole numbers between 7 and 14 are divisible by 5.

8. No perfect square less than 100 is also a perfect cube less than 100.

9. All of the United States touch another state.

10. No ocean touches the United States.

11. All integers are whole numbers.

12. All real numbers are rational numbers.

13. No planet starts with a vowel.

14. All continents start with the letter A.

Identify a pattern in the list and use the pattern and inductive reasoning to find the next entry in the list. See Example 2.

15. 3, 8, 13, 18, _____

16. 4, 8, 16, 32, _____

17. 12, 10, 8, 6, _____

18. 10000, 5000, 2500, 1250, _____

19. 7, 8, 10, 13, _____

20. 2, 4, 12, 48, _____

21. 25, 5, 1, _____

22. a, c, e, g, _____

23. m, p, s, v, _____

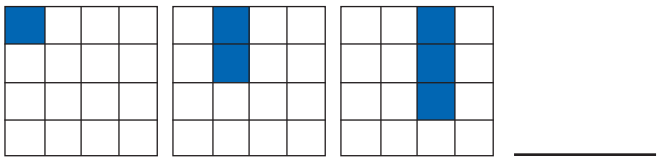
24. a, bb, ccc, dddd, _____

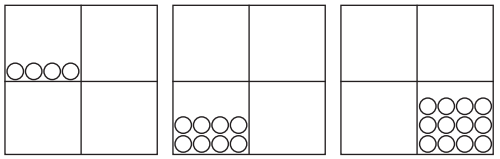
25. abcde, bcdea, cdeab, deabc, _____

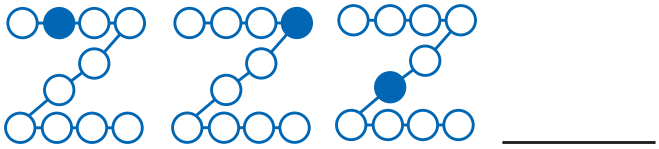
26. dog, 1, dog, 2, dog, _____

27. Sunday, Monday, Tuesday, Wednesday, _____

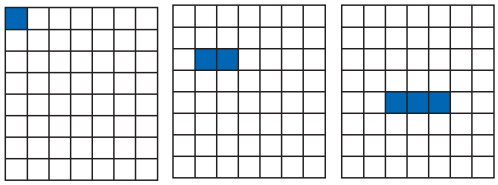
Describe the pattern and use the pattern to draw the next figure in the sequence. See Example 3.

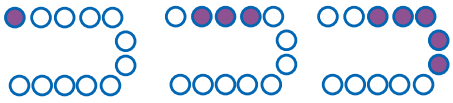
28.  _____

29. 

30. 

31. 

32. 

33. 

Use deductive reasoning to determine a conclusion that can be drawn from each set of facts. See Example 4.

34. All apples have cores and stems. A MacIntosh is a type of apple.

35. All states in the United States have capital cities. New York is a state in the United States.

36. All fish have fins. A trout is a fish.

37. All basketball players on the college team must attend mandatory study halls every weekday night. Harry is a basketball player on the college team.

38. All professors in the Math Department teach four classes per semester. Dr. Adams is a professor in the Math Department.

39. Today is Tuesday. All Tuesdays are 2-for-1 burger nights at Bobby's Burger Joint.

40. Franklin is my cat. All my pets have black fur.

41. Every year Christmas is celebrated on December 25. Today Christmas is being celebrated.

42. All milkshakes sold at Bobby's Burger Joint contain at least 500 calories. Phil buys a milkshake from Bobby's Burger Joint.

43. Marie wraps all Christmas presents with red bows. Marie wrapped a Christmas present for Neil.

Determine if inductive reasoning or deductive reasoning was used to reach the conclusion. See Example 5.

44. The last four times Joe went to the movies, Joe ate a large popcorn with butter and drank a large soda. Tonight when Joe goes to the movies, he will eat a large popcorn with butter and drink a large soda.

45. All moviegoers at the Party Cinema order two large boxes of candy before the movie. Joe went to the Party Cinema to see a movie. Joe ordered two large boxes of candy before the movie.

46. All cats purr when they feel loved. My cat Troubles is purring. My cat Troubles feels loved.

47. The last five times I opened the drapes, my cat Troubles jumped up on the window seat and meowed. I just opened the drapes near the window seat. My cat Troubles jumped up on the window seat and meowed.

48. In this list of numbers: 5, 9, 13, 17, the next number will be 21.

49. Carol observed that $5 + 7 = 12$, $9 + 5 = 14$, $11 + 13 = 26$, and concluded that the sum of two odd numbers is an even number.

50. In this list of letters: ffffff, eeeee, dddd, ccc, the next entry will be bb.

51. Ben noticed that $6 \cdot 4 = 24$, $4 \cdot 2 = 8$, and $10 \cdot 12 = 120$. He concluded that the product of two even numbers is an even number.

52. Andy looked at a map of the world and concluded that the names of all seven continents end in a vowel.

53. All police officers are trained in first aid. Jason is a police officer. Jason is trained in first aid.

Writing & Thinking

54. Write a list of 4 numbers that uses a pattern of adding 3 to the preceding number so that the 5^{th} number on the list is 19.

55. Write a list of 3 numbers that uses a pattern of multiplying the preceding number by 3 so that the 4^{th} number on the list is 999.

56. Two identical twin brothers live together. One of the brothers always tells the truth, while the other always lies. The police arrive at their place and one of them opens the door. What is one question that the police officer can ask the brother who opened the door to know if the other one is home?

Margin Exercise Answers

1. a. Statement **b.** Not a statement but an opinion **c.** Statement **2. a.** The Winter Olympics does not occur every 4 years **b.** SUV is not a model of vehicle **c.** The flight is late **3.** p: We do not have exams after each chapter; $\sim q$: Traveling is not educational; r: The game has started **4.** p: It is going to be hot today; q: There is no class today **5.** a: Today was rainy; b: Regan is graduating; c: It is New Year's Ever **6. a.** Quantified **b.** Quantified **c.** Not Quantified **d.** Quantified **7.** a: Some students make As in college algebra; b: No sea animals are mammals; c: Some birds are carnivorous, d: All classes are interesting.

8.4 Exercises

Concept Check:

Fill-in-the-Blank: Complete the sentences using information found in this section.

1. A _____ is a declaration that can be determined to be true or false, but not both.
2. A _____ is the logical opposite of a statement.
3. A _____ indicates the extent or scope of the term it refers to.
4. A statement containing one or more of the words all, none, some, or no is called a _____ _____.
5. The symbol ~ represents the _____ of a statement.
6. The negation of the quantifier some are is _____ _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. An opinion is not a valid statement.
8. All statements can be negated by adding the word "not".
9. "Not all trees are green" is the negation of the statement "All trees are green."
10. A statement can be both true and false at the same time.

Practice

Determine whether each of the following are statement or not. If it is not a statement, explain why it is not a statement. See Example 1.

1. I like horses.
2. $3 + 5 = 8$
3. Christmas is always on a Tuesday.
4. Game of Thrones is a very good show.

5. $5 + 3 = 1$

6. The Final Exam had 51 multiple choice questions.

Find the negations of the following statements. See Example 2.

7. A penny is a coin.

8. You got an A.

9. The course was easy.

10. $x + 2 = 5$.

11. There are 100 pennies in a dollar.

12. A week has seven days.

13. I do not like math.

14. A quarter is not a coin.

15. I will not receive any change.

16. You will not take a nap.

Find the negation for the following statements. See Examples 3 and 4.

17. p: You are not finished eating dinner.

18. p: Your stomach feels full.

19. p: You will not cook dinner.

20. p: You passed the course.

21. p: You thought the course was easy.

22. p: You studied a lot.

23. $\sim p$: The soup is not hot.

24. $\sim p$: The math test is tomorrow.

25. $\sim p$: Avocados are not green

26. $\sim p$: The sunflowers are not tall.

27. $\sim p$: The number is prime.

28. $\sim p$: Tomorrow is not payday.

Determine which of the following are quantified statements. If it is a quantified statement, identify the quantifier. See Example 6.

29. Some math courses are at capacity.

30. The teacher is sick today.

31. All prime numbers are odd.

32. None of the pants are khaki.

33. At least one butterfly has blue on it.

34. Twenty-five is a composite number.

Determine the negation of the given quantified statement. There may be more than one correct answer. See Example 7.

35. All even numbers are divisible by two.

36. Some of the tickets have been sold.

37. Some of the animals are not restless.

38. None of the audience members were paying attention.

39. Some of the art is for sale.

40. None of the desserts are chocolate.

41. All of the sodas are cold.

42. Some of the dentists do not recommend the toothpaste.

Writing & Thinking

43. Why is an opinion not considered a statement in logic?

44. Explain why the quantified statements "some p are not q" and "all p are q" are negations of each other.

$p \wedge \sim q$

b. p: Today is Tuesday.

q: Tomorrow is Friday.

$p \vee q$

c. Notice that this compound statement can be rewritten as "If the weather is cool, then I will exercise."

p: The weather is cool.

q: I will exercise.

$p \Rightarrow q$

d. p: I have a pint of milk.

q: I have 16 fl oz of milk.

$p \Leftrightarrow q$

Now work margin exercise 9.

Margin exercise answers

1. a. Compound statement **b.** Simple statement **c.** Compound statement **2. a.** Today is Monday and I work today. **b.** Today is Monday and I have a meeting. **c.** Today is Monday, I work today, and I have a meeting. **3. a.** I walk my dog in the morning or after work. **b.** I take my dog to the park or to the agility course. **c.** I take my dog to the park or I do not take my dog to the agility course. **4. a.** If I eat a lot of candy, then I get cavities. **b.** If a number is divisible by 1 and itself, then the number is prime. **5. a.** p: I will go to school; q: It is Wednesday **b.** p: The weather is sunny; q: I want to sleep in; $\sim q$: I don't want to sleep in. **6. a.** The measure of angle A is 90 degrees if and only if angle A is a right angle. **b.** A triangle has 3 equal angles if and only if the triangle has 3 equal sides. **7.** p: The milk will be chocolate; q: The cow is brown; $\sim q$: The cow is not brown. **8. a.** I will study and I will walk. **b.** I will study, I will walk, or I will eat lunch. **c.** If the toast does not have an avocado topping, then I will eat toast. **d.** I will study if and only if I eat toast. **9. a.** p: Today the weather is sunny; q: Today the weather is rainy; $p \vee q$ **b.** p: I will drive a car; q: I have a driver's license; $p \Leftrightarrow q$ **c.** p: I like hamsters; q: I like gerbils; $\sim p \wedge q$ **d.** p: I drink coffee; q: I have creamer; $p \Rightarrow q$

8.5 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. A _____ _____ is a word or phrase that connects two or more simple statements.

2. A _____ _____ can be determined to be either true or false, but not both.

3. A _____ is a compound statement made up of two or more simple statements with the logical connector _____.

4. A _____ is a compound statement made up of two or more simple statements with the logical connector _____.

5. A _____ _____ is a compound statement created when a simple statement p implies the simple statement q.

6. A _____ _____ is a compound statement created with a simple statement p implies the simple statement q and q also implies p.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. "If you take this class, then you will pass" is an example of a biconditional statement.

8. "Tomorrow will be sunny or it will rain" is an example of a conjunction statement.

9. Joining two or more simple statements with a logical connective creates a compound statement.

10. The symbol $\wedge$ represents the logical connective *and*.

Practice

Determine if the following statements are simple or compound. See Example 1.

1. A quarter is a coin.

2. You got an A or B.

3. If I study, then I will get an A on the test.

4. There are 100 pennies or 4 quarters in a dollar.

5. A week has seven days.

6. I do like math and science.

Determine the conjunction of the following statements. See Example 2.

p: You are finished eating dinner

q: You cooked dinner

r: Your stomach is full.

7. p and $\sim q$

8. $\sim p \wedge r$

9. $\sim r \wedge p \wedge \sim q$

10. q and r

11. r and q and $\sim p$

Determine the simple statements used in each conjunction and the compound statement symbolically.

12. Space is vast and the ocean is deep

13. Jets are flying east and ships are not going south

14. Pigs are pink and horses are not pink.

15. No shows are funny and movies aren't free.

16. My classes start at 8:00 am and end at 2:00 pm.

17. I like to swim and don't like jumping rope.

Determine the disjunction of the following statements in words. See Example 3.

p: Bears are big.

q: Lakes are deep.

r: Hills are steep.

18. p or not r

19. $\sim r \vee \sim q$

20. r or q

21. $\sim p \vee \sim r$

22. Not q or not p

23. $p \vee q$

Determine the simple statements used in each conjunction and the compound statement symbolically.

24. Tomorrow is Tuesday or Wednesday.

25. I will not go to the museum or I will not go to the zoo.

26. The monkeys were eating or they were asleep.

27. The winter is cold or not rainy.

28. Hot dogs are not sold at baseball games or sodas are sold football games.

Use the given simple statements to write the indicated conditional statement. See Example 4.

p: The earth is white.

q: The sky is dark.

r: The night has come.

s: The wind is blowing.

t: The snow is falling.

29. p implies q

30. q implies r

31. $r \Rightarrow s$

32. s implies t

33. $t \Rightarrow p$

34. $r \Rightarrow q$

Identify the simple statements that were used to form the given conditional statement and write the compound statement symbolically. See Example 5.

35. We will go to the beach if Liam finishes his work on time.

36. If the sunset today is red, then tomorrow it will rain.

37. Whenever a storm begins, Jane is afraid to leave the house.

38. I will pay for a taxi if Mike pays for pizza.

39. Every time Mary skips her classes, I don't want to help her prepare for the tests.

40. We will not go ice skating if the pond does not thoroughly freeze.

Use the given simple statements to write the indicated biconditional statements. See Example 6.

p: Zoe will buy a house.

q: The house is in the countryside.

r: The house has a backyard.

s: James will buy a car.

t: Margaret will get a driver's license.

u: John will change his job.

41. $p \Leftrightarrow r$

42. r if and only if q

43. s if and only if q

44. $t \Leftrightarrow s$

45. p if and only if s

46. $u \Leftrightarrow t$

Write the symbolic representation for the given biconditional statement. See Example 6.

47. I can play tennis if and only if I don't play polo.

48. Robots are useful if and only if they work.

49. I can't work on the project if and only if I have a lot of work to do.

50. Swimming in the ocean is fun if and only if there aren't any sharks.

Identify the simple statements that were used to form the given biconditional statement and write the compound statement symbolically. See Example 7.

51. The workers will receive a bonus if and only if they finish their work ahead of time.

52. I will be able to master the technique if and only if I learn it step by step.

53. Dominic will get this job if and only if Neil does not come to compete with him.

54. Jessica won't stop worrying about the person if and only if this person is extremely important to her.

55. Matthew will get to work on time if and only if he is not late for the train.

56. The shop owner will not give you a discount is and only if you do not greet him.

Writing & Thinking

57. Compare and contrast conditional and biconditional statements.

5.

p	q	r	$q \wedge r$	$p \Rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

8.6 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A truth table describes the possible truth values of a _____ _____.

2. A truth table organizes all possible _____ of truth values of the individual simple statements.

3. The number of _____ in a truth a table with vary depending on how many simple statements are involved.

4. The number of _____ in a truth table depends on the logical connectors used and whether any of the statements are negations.

5. A conjunction $p \wedge q$ is true only when both p and q are _____.

6. The conditional statement $p \Rightarrow q$ is false only when p is _____ and q is _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement is true. (**Note:** There may be more than one acceptable change.)

7. A compound statement can have different truth values depending on the truth values of the simple statements it contains.

8. A disjunction $p \vee q$ is always true regardless of the truth values if p and q.

9. The number of columns in a truth table is increased when negations are involved.

10. A truth table must contain every possible combination of true and false for the simple statements involved.

Practice

Write each logical statement symbolically. Then, create a truth table for the statement.

1. He went to the store.
2. One mile is 5280 feet.
3. The teacher said this test isn't hard.
4. I did not watch television last night.
5. She did not want her favorite team to lose.
6. It is sunny and she has sunglasses.
7. My dog is asleep and I hear him snoring.
8. It is not New Year's Day and he is having a party.
9. It is raining and he doesn't want to go outside.
10. She did not go to the store and she did not buy anything.
11. The sky is blue or there are clouds.
12. The cell phone is charged or the battery is dead.
13. He has children or he does not have children.
14. She did not stay up late or she read a book.
15. He played baseball or she did not play softball.
16. The clock is not accurate or I am not on time.
17. If her son studies hard, she will make him cookies.
18. If he watches the first Harry Potter movie, then he has to watch all of them.
19. If I read Stephen King, then I cannot sleep at night.
20. If he doesn't go to the store today, then he has to go tomorrow.
21. If it does not rain, then I did not wash my car.
22. If I read Stephen King, then I will be awake all night and I will be tired tomorrow.
23. My dog is not awake and I hear him snoring, or my dog is awake.
24. If he doesn't go to the store today, then he has to go tomorrow or he will be hungry.
25. If it is raining and I have an umbrella, then I won't get wet.

Writing & Thinking

26. Why can't a truth table for "Stop the car!" be created?

27. A truth table with the last column consisting of all T values is called a tautology. Create a statement, like Exercise 13, that is a tautology.

9.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. In an experiment, a _____ is a repetition of the same procedure.
2. In a random experiment, there is one distinct _____ for each trial of the experiment.
3. Empirical probability of an event is calculated by finding the ratio of the number of _____ in which a specific event occurs to the total number of _____.
4. The law of _____ numbers says that the _____ the number of trials an experiment has, the closer the empirical probability will be to the actual probability.
5. Classical probability of an event is calculated by finding the ratio of the number of _____ in the event to the total number of outcomes in the _____.
6. A _____ diagram can be used to organize outcomes in a systematic manner.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. In an experiment where a coin is tossed several times, the trials are a head or a tail.
8. In an experiment where a standard die is rolled, the sample space would be $S = \{1,2,3,4,5,6\}$.
9. An empirical probability is a probability that is determined by observing a random experiment.
10. Classical probability can be measured as a simple proportion: the number of trials in an experiment divided by the number of outcomes in the sample space.

Practice

For each situation, explain whether each condition of a random experiment is met. See Examples 1 through 4.

1. You draw a card from a well-shuffled deck of 52 cards and observe if the card is a jack.
2. A bag contains three red balls, four green balls, and three blue balls. You pull two balls from the bag and observe if their colors are the same.

3. You roll a 6-sided die and observe whether the number rolled is less than seven.

4. You roll two dice and observe the sum of the numbers on the uppermost surfaces.

5. A spinner with the numbers 1 through 8 is spun and the outcome is observed.

6. A quality inspector takes 20 details from the last batch of parts made on a machine and observes the number of defects.

7. You select a piece of candy from bowl of candy that contains only butterscotch discs.

8. A researcher selects seven mice for the experiment and observes the number of mice that have lost weight.

Find the probability for each situation. Round your answers to the nearest thousandth, if necessary. See Examples 5 and 6.

9. A football team won 24 of 44 games.
   a. Determine the empirical probability that the team will win the next game.
   b. Determine the empirical probability that the team will not win the next game.

10. A bakery sells all of the fruitcakes they make on 16 days out of 30.
    a. What is the empirical probability that the bakery will sell all of the fruitcakes tomorrow?
    b. What is the empirical probability that the bakery will not sell all of the fruitcakes tomorrow?

11. An archer hits the target 18 times out of 20.
    a. What is the empirical probability that he will miss the next shot?
    b. What is the empirical probability that he will hit the next shot?

12. Of the 56 donuts tasted by a group of friends, 21 had caramel filling.
    a. What is the empirical probability that the next donut they try will not have caramel filling?
    b. What is the empirical probability that the next donut they try will have a caramel filling?

13. You asked 54 people whether they like Hawaiian pizza and found that 18 of them like Hawaiian pizza. What is the empirical probability that the next person you ask does not like Hawaiian pizza?

14. Fourteen employees in a company of 105 people got a bonus last month. Determine the empirical probability of getting a bonus at this company.

15. Of this year's graduates at a local university, 190 found a full-time job and 110 are still searching for one. Determine the empirical probability of finding a full-time job for a graduate of this university.

16. A new video watched by one thousand people got 190 likes and 70 dislikes. What is the empirical probability that the next viewer will neither like nor dislike the video?

17. A mobile phone company conducted a survey of 280 new customers to determine whether they were interested in using mobile internet hotspot. Using the results from the following table, determine the empirical probability that a new customer will be interested in mobile internet hotspot.

Interested	Not Interested
156	124

18. A research center tests a new medicine for headache on 300 patients. The results of the study are shown in the following table.

Decreased Pain	Increased Pain	No Effect
230	6	64

What is the empirical probability that the medicine will decrease a patient's pain?

19. A candidate's team conducts a telephone survey of 1000 people to find out if they intend to vote for the team's candidate. The results were as follows.

Votes for the Team's Candidate	Votes for a Different Candidate	Undecided
365	521	114

a. What is the empirical probability that a person will vote for the team's candidate?

b. What is the empirical probability that a person has already decided how he or she will vote?

20. A survey was conducted to identify the number of children in 74 local families. The following table shows the results.

Number of Children	0	1	2	3	More than 3
Number of Families	12	32	19	8	3

a. What is the empirical probability that a family has one child?

b. What is the empirical probability that a family has more than one child?

For each experiment, determine the empirical probability of each outcome. Round your answer to the nearest thousandth, if necessary. See Example 7.

21. A football referee is testing a new coin that he intends to use in the next season. The results of 182 tosses are shown in the following table.

Outcome	Tails	Heads
Number of Observations	89	93

22. A convenience store manager wants to compare the popularity of the different berry jams they sell. She records the last 150 purchases, getting the following results.

Outcome	Strawberry	Raspberry	Cherry
Number of Observations	65	30	55

23. A manufacturer tests a new 4-sided die for fairness. The results of 400 throws are as follows.

Outcome	1	2	3	4
Number of Observations	98	105	107	90

24. The developers of a random number generator want to test their application. They set the generator to produce 1000 random integers from 0 to 9 and obtain the following frequencies.

Outcome	0	1	2	3	4	5	6	7	8	9
Number of Observations	110	90	100	110	120	90	80	100	90	110

Find the indicated probability. Round your answer to the nearest ten thousandth, if necessary. See Examples 8 through 10.

25. A 4-sided die is rolled once. Let E be the event of observing a 3. What is $P(E)$?

26. A card is drawn from a standard deck of 52 cards.

a. Let E be the event of drawing a card of a red suit (red suits are diamonds and hearts). What is $P(E)$?

b. Let E be the event of drawing a queen of a red suit. What is $P(E)$?

27. An application generates a random number from 1 to 9.

a. Let E be the event of getting an odd number. What is $P(E)$?

b. Let E be the event of getting an even number. What is $P(E)$?

28. A 6-sided die is rolled twice. Let E be the event of rolling the same number on two throws of the die. What is $P(E)$?

29. A coin was tossed three times. The sample space consists of 8 outcomes, {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. Let E be the event of getting three tails. What is $P(E)$?

30. An application generates a random number from 1 through 100. Let E be the event of observing a number greater than 30. What is $P(E)$?

31. Twelve students expressed the desire to take the position of a class president. Let E be the event that the youngest student wins. What is $P(E)$?

32. Thirty men and forty women participate in a lottery. Let E be the event that a woman wins. What is $P(E)$?

33. Eight-hundred people participate in a marathon. Six of them are your friends. Let E be the event that the winner is one of your friends. What is $P(E)$?

34. A bag contains 15 chocolate, 7 mint, and 11 caramel candies. You randomly choose a piece candy from the bag. Let E be the event that candy drawn from the bag is chocolate. What is $P(E)$?

35. Four coins are tossed simultaneously. The sample space is {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THTH, TTHH, THHT, HTTT, THTT, TTHT, TTTH, TTTT}

a. Let E be the event of getting exactly two heads. What is $P(E)$?

b. Let E be the event of getting one or four tails. What is $P(E)$?

36. A 20-sided die is rolled.

a. Let E be the event of the number being no less than 4. What is $P(E)$?

b. Let E be the event of the number being even. What is $P(E)$?

Solve and round your answer to the nearest thousandth, if necessary. See Example 11.

37. Two 6-sided dice are rolled.

a. Draw a tree diagram for the possible outcomes of this experiment.

b. Determine the probability of getting two different numbers.

38. Two 4-sided dice are rolled, and we observe the sum of the numbers.

a. Draw a tree diagram for the possible outcomes of this experiment.

b. Determine the probability of the sum being greater than 4.

39. In a family of tennis players, a brother has equal chances to win or lose when playing against his sister. Consider they play four matches.

a. Draw a tree diagram for the possible outcomes of this experiment.

b. Determine the probability of the sister winning at least three matches.

40. Two people play rock-paper-scissors. In this game, rock beats scissors, scissors beats paper, and papers beats rock.

a. Draw a tree diagram for the possible outcomes of this experiment.

b. Determine the probability of the first player winning.

41. Consider we choose a random number from 3 to 7 and then a random even number from 2 to 8.

a. Draw a tree diagram for the possible outcomes of this experiment.

b. Determine the probability of the first and second numbers being the same.

42. Five managers should gather for the company meeting. Each of them has equal chances to arrive on time (T) or to be late (L).

a. Draw a tree diagram for the possible outcomes of this experiment.

b. Determine the probability that exactly three managers will arrive on time.

Writing & Thinking

43. Give at least two examples of random experiments.

44. In your own words, explain the difference between empirical probability and classical probability.

45. After tossing a fair coin 1000 times, we got 510 tails and 490 heads. Is the actual probability of observing a tail is greater than the probability of observing a head?

46. When can you use a tree diagram to calculate probability?

Finding Probability Given the Odds

If we are given the odds in favor of an event A as n to m, then the probability of event A can be calculated by

$$P(A) = \frac{n}{n+m}.$$

FORMULA

Using this formula, we can find the probability of Audible winning the Kentucky Derby.

$$P(\text{Audible wins Kentucky derby}) = \frac{1}{1+8} = \frac{1}{9} \approx 0.11111$$

The probability of Audible winning the Kentucky Derby is slightly more than 11%.

Example 12 Calculating Probability Given the Odds

Dominic sees that his college's soccer team has 3 : 2 odds in favor of winning their game this weekend. What is the probability implied by these odds?

Solution

Using the formula for finding probability given the odds, we have the following.

$$P(\text{wins game}) = \frac{3}{3+2} = \frac{3}{5} = 0.6$$

Thus, the soccer team has a 60% chance of winning their game this weekend.

Now work margin exercise 12.

12. Patricia sees that her college's baseball team has 5 : 3 odds in favor of winning their game this weekend. What is the probability implied by these odds?

Margin Exercise Answers

1. No **2.** Yes **3.** Yes **4.** Yes **5.** Yes **6.** $\frac{2}{3}$ **7.** $\frac{1}{2}$ **8.** $\frac{10}{13}$ **9.** $\frac{3}{5}$ **10.** 0.96 **11. a.** $\frac{1}{19}$ **b.** $\frac{18}{19}$ **c.** $\frac{1}{18}$ or 1 : 18 **12.** $\frac{5}{8} = 0.625$

9.2 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. Two events are mutually exclusive if they cannot _____ at the same time.

2. If two events are mutually exclusive, the probability of the outcome of the two events can be found by _____ their probabilities.

3. When calculating the probability of non-mutually exclusive events, the _____ of the two events needs to be subtracted.

4. The _____ of a set is all outcomes in the universal set that are not part of the set.

5. The sum of the probabilities of a set and its _____ equals 1.

6. The odds _____ an event occurring are calculated by the ratio of the probability of the event to the probability of the complement of the event.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If it is possible for two events to occur at the same time, then those two events are mutually exclusive.

8. If a certain team is given 1 to 3 odds of winning a game, this means that for every dollar bet on the team, a person would win $3 if the team wins.

9. An event A and its complement A' make up the entire sample space.

10. If you have 1 to 5 odds in favor of winning a game, then the probability of winning is $\frac{1}{5}$.

Practice

Determine whether the given events are mutually exclusive. See Examples 1 through 5.

1. Are the events $A = \{1, 3, 5, 7, 9, 11\}$ and $B = \{4, 8, 12\}$ mutually exclusive?

2. Are the events $A = \{1, 2, 3, 5, 8, 13\}$ and $B = \{3, 6, 9, 12\}$ mutually exclusive?

3. Consider the set of whole numbers 1 through 10. Are the events $A = \{\text{the whole numbers in the set divisible by } 5\}$ and $B = \{\text{the whole numbers in the set no less than } 5\}$ mutually exclusive?

4. Consider the set of whole numbers 10 through 20. Are the events $A = \{\text{the whole numbers in the set greater than } 18\}$ and $B = \{\text{the whole numbers in the set divisible by } 6\}$ mutually exclusive?

5. In one trial of drawing a card from a deck of 52 cards, are the events "drawing a 2 of hearts" and "drawing a 10 of spades" mutually exclusive?

6. In one trial of drawing two cards from a deck of 52 cards, are the events "drawing two queens" and "drawing two kings" mutually exclusive?

7. In a single trial of rolling a 6-sided die, are the events "getting a number less than 3" and "getting a number greater than 4" mutually exclusive?

8. In a single trial of rolling a 6-sided die, are the events "getting a number less than 5" and "getting an even number" mutually exclusive?

9. Are the events "the archer hits the target" and "the archer misses the target" mutually exclusive?

10. Are the events "the weather is cold" and "the weather is sunny" mutually exclusive?

11. Are the events "a swimmer takes first place in the competition" and "a swimmer takes second place in the competition" mutually exclusive?

12. Are the events "a cup is filled with black tea" and "a cup is made of porcelain" mutually exclusive?

Find the indicated probability. Round your answer to the nearest ten thousandth, if necessary. See Examples 6 and 7.

13. In one trial of rolling a 6-sided die, what is the probability of observing a number less than 3 or greater than 4?

14. When selecting a card from a deck of 52 cards, what is the probability of drawing a 7 of spades or a 9 of hearts?

15. A bag contains 12 balls; 4 of the balls are white, 2 are green, and 6 are black. What is the probability that a randomly chosen ball is white or black?

16. Three coins are tossed. Find the probability of observing either three heads or three tails.

17. If you randomly choose a whole number from 1 to 50 (inclusive), what is the probability that the number will be even or will end with 5?

18. Several bottles are placed on a table. The labels indicate that 6 bottles contain black tea, 5 bottles contain milk, 7 bottles contain coffee, and the remaining 8 bottles contain apple juice. If you randomly choose a bottle from the table, what is the probability that the bottle you chose contains coffee or milk?

19. An analyst estimates that the price of wheat will increase with a probability of $\frac{7}{12}$ or will remain at the same price with a probability of $\frac{1}{6}$. Find the probability that the price will not decrease.

20. The probabilities of getting each of the three main prizes in a lottery are $\frac{1}{10,000}$, $\frac{1}{500}$, and $\frac{1}{100}$, respectively. If each participant can win only one prize, what is the probability of getting one of the three main prizes?

Find the indicated probability. Round your answer to the nearest ten thousandth, if necessary. See Examples 8 and 9.

21. When rolling a 6-sided die, what is the probability of observing a number less than 5 or an odd number?

22. When selecting a card from a deck of 52 cards, what is the probability of getting a card of a black suit or an ace?

23. When tossing two coins, what is the probability of the coins landing on different faces or the first coin landing on tails?

24. When choosing a random whole number from 1 to 30 (inclusive), what is the probability that the number will be divisible by six or divisible by four?

25. At an animal shelter, 20% of cats have white fur, 25% have black spots, and 5% have white fur and black spots. If you randomly choose one cat from the shelter, what is the probability that it will be white or will have black spots?

26. In a bag of candies, 35% of candies are chocolate and 25% contain nuts. In addition, 10% of candies are chocolate with nuts. What is the probability that a randomly chosen candy will either be chocolate or contain nuts?

27. While choosing a melon, Kate notices that 17 out of 20 melons on a counter have no visible flaws, 14 are ripe, and 12 both have no flaws and are ripe. Had she randomly picked one, what would be the probability of getting a flawless and ripe melon?

28. A box contains 34 pieces of paper covered with strange colored symbols. There are 20 pieces with some of the symbols colored red, 18 pieces with some of the symbols colored green, and 10 pieces that have both red and green symbols. Find the probability that a randomly chosen piece have symbols that are colored red or green or both red and green.

Solve and round your answer to the nearest ten thousandth, if necessary. See Examples 10 and 11.

29. Thirty percent of applicants pass the interview for a job position. Determine the probability that an applicant will not pass the interview.

30. A study shows that 5% of people who register for an event end up not going to it. Determine the probability that a person will go to the event for which they registered.

31. An archer has an 89% chance of hitting the target. Determine the probability that the archer will miss the target.

32. In a certain country, 59.7% of households own at least one car. Determine the probability that a randomly chosen household does not own a car.

33. The statistical data show that police officers make up $\frac{5}{1000}$ of the total population of a country. Find the probability that a randomly chosen citizen is not a police officer.

34. When tossing a coin 4 times, the probability that the number of heads is equal to the number of tails is $\frac{3}{8}$. What is the probability that the number of heads is different from the number of tails?

35. A 20-sided die is rolled.

a. Calculate the probability of getting a number greater than 18.

b. Calculate the probability of getting a number that is not greater than 18.

c. Calculate the odds in favor of getting a number greater than 18.

36. In a game of chance, a player is shown a table with 6 rows and 6 columns. In the table, each cell hides the name of a prize and one of the cells hides the jackpot. The player chooses one cell and wins the prize hidden there.

a. Calculate the probability of getting the jackpot.

b. Calculate the probability of not getting the jackpot.

c. Calculate the odds in favor of getting the jackpot.

37. A recent quality test found that a machine produces 6 defective parts out of every 500 parts manufactured.

a. Calculate the probability of a component having defects.

b. Calculate the probability of a component having no defects.

c. Calculate the odds in favor of a component having no defects.

38. The results of a study show that a new drug stops the development of a certain disease for 10 out of 25 study participants.

a. Calculate the probability of the drug stopping the development of the disease.

b. Calculate the probability of the drug not stopping the development of the disease.

c. Calculate the odds in favor of the drug stopping the development of the disease.

39. A company assesses the reliability of a new car and states that 498 out of 500 cars will not need repairs within the first 10,000 miles.

a. Calculate the probability of a car driving the first 10,000 miles without needing repairs.

b. Calculate the probability of a car needing repairs within the first 10,000 miles.

c. Calculate the odds in favor of a car driving the first 10,000 miles without repairs.

40. An athlete completed the 100-meters sprint in less than 11 seconds in 32 of the last 76 attempts.

a. Calculate the probability of the athlete completing the next attempt in less than 11 seconds.

b. Calculate the probability of the athlete completing the next attempt in 11 seconds or more.

c. Calculate the odds in favor of the athlete completing the next attempt in less than 11 seconds.

Determine the probability implied by each of the following odds. Round your answer to the nearest ten thousandth, if necessary. See Example 12.

41. If we disregard the individual strength of the teams, any team that wins the quarterfinals has 1 : 3 odds in favor of winning the tournament.

42. For the car you have chosen to win the race, the odds in favor of placing in the top three are 3 : 17.

43. The odds of a certain disease worsening without proper treatment are 5 : 2.

44. The odds of a new power supply unit failing in the first two years are 3 : 22.

45. The odds of making a correct decision on the first try are 2 : 7.

46. With the cards you were dealt, the odds of winning the round of the game are 2 : 11.

47. The leader of the chess club has 12 : 13 odds in favor of getting the first place in the local tournament.

48. The odds of house prices falling in the next year are 2 : 9.

Writing & Thinking

49. In your own words, explain the difference between mutually exclusive and non-mutually exclusive events.

50. Give an example of the event with 1 : 1 odds.

Multiplication Rule for Dependent Events

If two events, A and B, are dependent, then

$$P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B).$$

FORMULA

9. Assume that there are 15 juniors and 18 seniors in a computer science club. Two members are chosen at random to participate in a new project. What is the probability that the first member chosen is a junior and the second member chosen is a senior?

Example 9 Using the Multiplication Rule for Dependent Events

Assume that there are 17 men and 24 women in the Lions Club. Two members are chosen at random each year to serve on the hospitality committee. What is the probability of choosing two members at random where the first member chosen is a man and the second is a woman?

Solution

Note that since we are choosing two members, the first choice will influence the probability for the second choice, assuming we do not want to choose the same member twice. This means we are dealing with dependent events. We want to find $P(\text{man} \cap \text{woman})$ which, according to the multiplication rule for dependent events, equals $P(\text{man}) \cdot P(\text{woman}|\text{man})$. When the first member is picked, there are 17 men out of 41 members. When the second member is picked, we assume that we have already picked a man, so that leaves all 24 women, but only 40 remaining members. The calculation is as follows.

$$P(\text{man} \cap \text{woman}) = P(\text{man}) \cdot P(\text{woman}|\text{man}) = \frac{17}{41} \cdot \frac{24}{40} \approx 0.2488$$

Now work margin exercise 9.

Margin Exercise Answers

1. $\frac{83}{253} \approx 0.3281$ **2. a.** $\frac{79}{156} \approx 0.5064$ **b.** $\frac{79}{96} \approx 0.8229$ **3.** Yes **4. a.** $\frac{1}{8}$ **b.** $\frac{1}{8}$ **5.** $\frac{1}{78} \approx 0.0128$ **6.** $\frac{1}{208} \approx 0.0048$ **7. a.** 0.1176 **b.** 0.9784 **8.** $\frac{1}{204} \approx 0.0049$ **9.** $\frac{45}{176} \approx 0.2557$

9.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. The probability that an event will occur given that some other event has already occurred is called _____ probability.

2. The notation $P(A \mid B)$ is read as "the probability of _____ given the occurrence of _____."

3. The vertical bar within a probability statement will always mean _____.

4. Finding the conditional probability of B given that A has occurred implies that $P(A)$ _____.

5. The multiplication rule for independent elements states that the probability that mutually independent events occur at the same time is the _____ of their probabilities.

6. When two events are not independent, the occurrence of one _____ the occurrence of the other.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The probability of an event B, given that event A has occurred, is always equal to the probability of an event A, given that event B has occurred.

8. Two events are said to be independent if knowing that one event occurred does not provide information regarding whether the other event has occurred.

9. The probability that mutually dependent events occur at the same time is the product of their probabilities.

10. Flipping a coin and rolling a die are independent events.

Practice

Use the data tables to find the indicated probabilities. Round your answer to the nearest ten thousandth, if necessary. See Example 1.

A restaurant manager asks customers what they think about the restaurant's new menu. Use the table below to answer the following questions.

	Like	Dislike	Neutral/ Undecided	Total
Soups	39	8	7	54
Salads	28	3	8	39
Main courses	33	5	9	47
Total	100	16	24	140

1. If a customer orders a soup, what is the probability that he or she will like it?

2. If a customer likes a dish, what is the probability that this dish is a soup?

3. If a customer orders a salad, what is the probability that he or she will not dislike it?

4. If a customer does not dislike a dish, what is the probability that this dish is a salad?

An airline collects information about a drink preference during a flight depending on the age of a passenger. Use the table below to answer the following questions.

Age	Water	Juice	Soft Drinks	Total
Below 18	50	43	53	146
18-30	43	31	46	120
31-50	121	203	115	439
Over 50	150	105	40	295
Total	364	382	254	1000

5. If a passenger is over 50 years old, what is the probability that he or she will choose a juice?

6. If a passenger chooses a juice, what is the probability that he or she is over 50 years old?

7. If a passenger is between 18 and 30 years old, what is the probability that he or she will not choose water?

8. If a passenger does not choose water, what is the probability that he or she is between 18 and 30 years old?

The results of a final exam for each year of study are given in the table below. Use the table to answer the following questions.

Year of study	A	B	C	D	Total
Freshman	95	65	12	5	177
Sophomore	85	70	6	3	164
Junior	77	67	15	2	161
Senior	72	74	5	1	152
Total	329	276	38	11	654

9. If a student is a sophomore, what is the probability that he or she will get C?

10. If a student gets C, what is the probability that he or she is a sophomore?

11. If a student is a senior, what is the probability that he or she will get either A or B?

12. If a student gets either A or B, what is the probability that he or she is a senior?

Determine the indicated conditional probability. Round your answer to the nearest ten thousandth, if necessary. See Example 2.

13. The best 8 of 32 teams go on to the playoffs. Of the teams going on to the playoffs, six participated in last year's tournament.

a. What is the probability that a randomly chosen team participated in last year's tournament, given that the team goes on to the playoffs?

b. If 22 of the teams participated in last year's tournament, what is the probability that a randomly chosen team goes to the playoffs, given that the team participated in last year's tournament?

14. Twelve out of 50 employees were offered a promotion during the last review session. Five of them had worked at the company for more than 5 years.

a. What is the probability that a randomly chosen employee has worked at the company for more than 5 years, given that he or she is offered a promotion?

b. If 16 employees have worked at the company for more than 5 years, what is the probability that a randomly chosen employee is offered a promotion, given that he or she has worked at the company for more than 5 years?

15. Out of 100 students, 85 say that they are satisfied with their specialization choice. Of those satisfied, 15 specialize in mathematics.

a. What is the probability that a randomly chosen student specializes in mathematics, given that he or she is satisfied with the choice?

b. If 21 students specialize in mathematics, what is the probability that a randomly chosen student is satisfied with the specialization choice, given that he or she specializes in mathematics?

16. Out of 150 university staff members, 96 are graduates of the university. Of the graduates, 12 are graduates of this year.

a. What is the probability that a random staff member is a graduate of this year, given that he or she graduated from the university?

b. If 18 university staff members are graduates of this year, what is the probability that a random university staff member graduated from the university, given that he or she is a graduate of this year?

17. Out of 500 new companies, 240 made a profit last year. Of the profitable ones, 105 companies were related to the IT industry.

a. Determine the probability that a new company is related to the IT industry, given that it made a profit last year?

b. If 202 new companies are related to the IT industry, what is the probability that a new company made a profit last year, given that the company is related to the IT industry?

18. A study involving 250 people was conducted to determine the effects of a new medicine. Of the participants, 150 received the medicine and the rest were given a placebo. Out of the people who took the medicine, 102 felt their health improving. Of all the participants, 114 people felt the improvement.

a. What is the probability that a randomly chosen participant felt the improvement, given that he or she took the medicine?

b. What is the probability that a randomly chosen participant took the medicine, given that he or she felt the improvement?

Determine whether the given events are independent. See Example 3.

19. A coin is tossed and a card is drawn from a standard deck. Are the events $A = \{\text{observing a tail}\}$ and $B = \{\text{drawing a spade}\}$ independent?

20. Two games of American roulette are played. Consider the two events

$A = \{\text{a ball landing on red in the first game}\}$ and

$B = \{\text{a ball landing on red in the second game}\}$.

Are these two events independent?

21. A 6-sided die is rolled. Are the events $\{\text{rolling a number smaller than } 4\}$ and $\{\text{rolling an even number}\}$ independent?

22. A coin is tossed two times. Consider the two events

$A = \{\text{observing a head on the first toss}\}$ and

$B = \{\text{observing the same outcome on both tosses}\}$.

Are these two events independent?

23. When being randomly assigned into groups, are the events "getting into the first group" and "getting into the second group" independent?

24. Are the events of correctly guessing the first, second, and third digits in an arbitrary 3-digit code independent?

Find the probability of the given event using **a.** a tree diagram and **b.** the multiplication rule for independent events. See Example 4.

25. When playing against a computer, a chess player has equal chances to win and to lose. Find the probability that the player will win two games in a row.

26. Two people play rock-paper-scissors. Find the probability of the first player showing rock and the second player showing paper.

27. Olivia has 6 pencils and 4 erasers. Ann likes 2 of the pencils and 3 of the erasers. Before the game, Olivia randomly gives a pencil and an eraser to each of her four playmates, including Ann. What is the probability that Ann will receive a pencil and an eraser that she likes?

28. A small test consists of three questions. Each question has three answer choices, only one of which is correct. If a person randomly chooses an answer for each question, what is the probability that he or she will give correct answers for the first two questions and an incorrect answer for the last question?

Use the multiplication rule for independent events to find the probability of the given event. Round your answer to the nearest ten thousandth, if necessary. See Examples 5 and 6.

29. A coin is tossed and a card is drawn from a standard deck of 52 cards. Find the probability of getting a head on the coin and a ten from the deck of cards.

30. Two 6-sided dice are rolled. Find the probability of getting a 5 or a 6 on both dice.

31. Find the probability that the first digit of a random whole number from 10 to 99 is even and the second digit is odd.

32. Four differently sized coins are tossed. Find the probability of getting a head on the largest two coins.

33. Suppose a playlist contains 15 songs, and each song played is selected completely at random. If we choose one song from the playlist, what is the probability that this song will be played twice in a row at the start?

34. A bag contains 9 balls numbered from 1 to 9. We draw a ball from the bag, look at its number, and then return it into the bag. If we repeat this process three times, what is the probability that the ball number each time will be divisible by 4?

Use the multiplication rule for independent events to find the probability of the given event. Round your answer to the nearest ten thousandth, if necessary. See Example 7.

35. Three cards are drawn with replacement from a standard deck of 52 cards.

a. What is the probability that all three cards are not diamonds?

b. What is the probability that at least one card is not a diamond?

36. When installing new software, the provider guarantees three months of free consultations. The probabilities that a client will ask for consultations in each of the first three months are estimated as $P(I) = 0.6, P(II) = 0.4,$ and $P(III) = 0.2.$

a. Find the probability that a client will not use free consultations.

b. Find the probability that a client will not use free consultations in at least one of the three months.

37. According to market analysts, the probabilities that the company's profit for each of the next four years will be greater than planned are $P(I) = 0.7, P(II) = 0.8, P(III) = 0.9,$ and $P(IV) = 0.95.$

a. What is the probability that the company will fail to exceed the plan in each of the four years?

b. What is the probability that the company will fail to exceed the plan at least once?

38. A car is sold as a base model with four possible additional options (A, B, C, D). The probabilities that a customer will install each of these options are estimated as $P(A) = 0.2$, $P(B) = 0.35$, $P(C) = 0.16$, and $P(D) = 0.3$.

a. What is the probability that a customer will not choose any options while buying the car?

b. What is the probability that a customer will choose some, but not all, of the possible options?

Use the multiplication rule for dependent events to find the probability of the given event. Round your answer to the nearest ten thousandth, if necessary. See Examples 8 and 9.

39. What is the probability of drawing two face cards from a standard deck of 52 cards, if the cards are drawn without replacement?

40. The host of a game holds 10 sticks, one of which is shorter than the rest. Players randomly draw a stick until the short one is drawn out. What is the probability that the second player will draw the short stick?

41. What is the probability of drawing a jack, then a queen, and finally a king from a standard deck of 52 cards, if the cards are drawn without replacement?

42. A bowl contains 12 candies, three of which are chocolate. What is the probability that the first three candies randomly picked from the bowl are chocolate?

43. Basketball qualifying matches were held during two days, with eight teams selected on each day. Then, the selected teams are being randomly assigned into pairs. What is the probability that in the first pair, the first team qualified during the first day, and the second team qualified during the second day?

44. Zoe and her brother, along with 48 other people, participate in a marathon. Assuming all participants have an equal chance of winning, what is the probability that Zoe will take the second place and her brother will place first?

45. In a batch of 100 decals, four decals have defects. Two decals are randomly selected from the batch for testing. Determine the probability that the first decal has no defects and the second decal is defective.

46. King Arthur randomly chooses two knights from 12 Knights of the Round Table to set off on the quest for the Holy Grail. Determine the probability that King Arthur will first choose Lancelot and then choose either Gawain or Percival.

47. Assume that a hotel has 10 studio rooms, 20 single rooms, 15 twin rooms, and 10 double rooms. The hotel management plans to conduct a sanitary inspection in two randomly selected rooms. Determine the probability that the first room selected is a single room, and the second is a double room.

48. A restaurant menu has 9 soups, 5 salads, 17 main dishes, and 4 desserts. A restaurant critic usually chooses two dishes for his magazine review. What is the probability that he will first order a salad and then a main dish?

Writing & Thinking

49. You are given that event A has occurred. What probabilities do you need to know to find the probability that event B will also occur?

50. In your own words, explain the difference between independent and dependent events.

51. Is it possible for two mutually exclusive events to be independent?

52. Give an example of two events A and B such that $P(A \mid B) = P(B \mid A) \neq 0$.

9.4 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. Tree diagrams get their name because of the way they _____.
2. In general, the total number of ways several events can occur in a given order is found by _____ the number of outcomes of each event together.
3. A permutation is an _____ of elements of a set where the order matters.
4. For any positive integer n, the factorial, denoted as $n!$, is the product of all positive integers from _____ through _____.
5. A group of n elements can be arranged in _____ ways.
6. The symbol ${}_nP_r$ denotes the number of permutations of _____ elements taken _____ at a time.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. For two events, if there are m possible outcomes for the first event and n possible outcomes for the second event, then there are $m + n$ ways for the two events to occur in the given order.
8. The product of the positive integers from 5 to 1 can be represented by 5!.
9. Each ordering of a set of elements if called a factorial.
10. 0! is undefined.

Practice

Use the fundamental counting principle to answer the questions. See Example 1.

1. To prepare for a game, Alan needs to make several red, yellow, and green cards with numbers 1 through 9 written on them. He needs one card for each color and number combination. How many cards should Alan make?
2. When rolling a 6-sided die 3 times, how many different sequences of outcomes can be observed?
3. Peter is furnishing his office and can choose from 3 desks, 5 chairs, and 3 bookcases. How many different options does he have for a furniture set made of a desk, a chair, and a bookcase?

4. Christmas is approaching fast, and Susan decides to decorate her Christmas tree. She has 5 different sets of glass ornaments, 3 types of tinsel, 2 different garlands, and 4 tree-toppers. If she wants to use one set of glass ornament, one type of tinsel, one garland, and a tree-topper to decorate the tree, how many different styles can she create?

5. A toy set allows a child to create toy soldiers from different body parts. There are 3 options for the body of the toy (with legs attached), 5 options for the right arm, 5 options for the left arm, and 6 options for the head. How many different toy soldiers can be created from these parts?

6. A test consists of 5 questions, each with 4 answer choices. If each question requires only one answer, how many different ways are there to answer the test?

7. A restaurant menu has 5 types of salads, 4 soups, 7 main dishes, 3 desserts, and 6 drinks options. If you want to order one item from each category, how many different meals can be made?

8. A frame shop has 10 different prints for sale by a certain artist. Each print can fit in 5 different frames, with 3 glass options, and 10 different color cardstock border options. If you want to buy a custom-framed print by this artist, how many different options do you have to choose from?

Evaluate the given factorial. See Example 2.

9. 2!

10. 5!

11. 9!

12. 13!

13. 6!

14. 10!

Evaluate the given permutation.

15. ${}_6P_2$

16. ${}_8P_3$

17. ${}_{14}P_1$

18. P_5^6

19. P_3^{11}

20. P_7^7

21. $P(9,4)$

22. $P(17,0)$

Applications

Solve. See Examples 3 and 4.

23. A combination lock has 3 dials with the digits 0 through 9 inscribed on them. How many different combinations are there for such a lock

a. if we assume that the digits in a combination must not be repeated, and

b. if the digits may be repeated?

24. A bowl holds 6 different types of fruit. How many ways are there to distribute these fruits among five children

a. if no child may have the same type of fruit, and

b. if there are no restrictions on the fruit selection?

25. Having bought 4 new houseplants, Melissa is choosing which pots to replant them into. If there are 6 pot designs, how many different options does she have

a. if she does not want to place two plants in the same type of pot, and

b. if the pot designs can be repeated?

26. In a memory training game, several lights flash in a sequence and the player needs to remember this sequence to repeat it later. There is a total of 9 lights, and on a certain difficulty level, each sequence consists of 4 lights flashing. How many such sequences exist

a. if the lights in the sequence may not be repeated, and

b. if the lights may be repeated?

27. Five friends decide to try their luck in horse betting. In a race featuring 8 horses, each of them selects a horse that they hope will win. How many different betting options do the friends have

a. if each friend must bet on a different horse, and

b. if more than one bet may be placed on a horse?

28. Before taking their children to an amusement park, parents examine the activities available there. They see that the park has 9 different activities, and they want to visit 5 of them before lunch. How many different schedules can they create

a. if they want all five activities to be different, and

b. if the activities may be repeated?

29. A piece of line art prepared for coloring has places for 5 colors. If Sharon has 10 different crayons, how many ways does she have to color the art

a. if all 5 colors must be different, and

b. if the colors may be repeated?

30. A bag contains 6 balls with different symbols inscribed on them. We pull one ball from the bag at a time and write down the respective symbol. After repeating this process 6 times, how many different symbol sequences can we obtain

a. if we do not return the balls to the bag, and

b. if we return the pulled ball into the bag before pulling again?

31. Ten students compete for the positions of president and vice president of a class. How many possible choices can be made to fill these positions?

32. After their cat gave birth to three kittens, the family decided to give these kittens to their friends. If six friends expressed the desire to take one of the kittens in, how many ways do the family have to distribute the kittens?

33. Sixteen employees take part in the company's lunchtime table tennis competition. How many ways can the top three winners of the competition be decided?

34. Suppose you are given a list of ten recently released movies and asked to name the four that you think are the best. How many different answers can you give?

35. For a group photo, it was decided that 4 people should sit in front and 5 people should stand behind them. How many different ways are there to organize the people sitting for the photo?

36. Mark and Shelly are on vacation. The tour manager at the hotel provides them with a description of 15 possible activities they can take part in. Each activity takes plenty of time, so they are advised not to plan two activities on the same day. If Mark and Shelly have 4 days and they want to participate in something new every day, how many different schedules can they make?

37. In a group strategy game, there are 10 unique roles, each of which can be assigned to only one player. If five people play the game, how many ways are there for these roles to be assigned?

38. An exhibition hall dedicated to a famous artist can hold five paintings. If the gallery owns 15 paintings by this artist, how many ways do they have to exhibit them in the hall?

39. Before a relay race, each of the 6 participating teams have to choose the color of the ribbon they will wear around their heads. If organizers provide ribbons of 8 different colors and the colors of all teams should be different, in how many ways could the colors be assigned to the teams?

40. The heads of a nine-headed hydra are arguing about the order in which they will speak at a conference. How many different orders can they speak in, if each head is going to speak once?

Writing & Thinking

41. Is it possible to calculate ${}_nP_r$, when $n < r$? Explain why or why not.

42. In your own words, explain when you should use the fundamental counting principle and when you should use the permutation formula to find the number of sequences that can be constructed from the elements of a set.

43. If you have two groups of movies, and someone lists the best three movies in each group, how can you find the number of different pairs of lists that you can receive?

6. A Senate committee of 8 members must be chosen with 4 Democrats and 4 Republicans. The senators eligible to be on this committee are 12 Democrats and 9 Republicans. How many possible ways can such a committee be formed?

Example 6 Calculating Combinations

A Senate committee of 6 members must be chosen with 3 Democrats and 3 Republicans. The senators eligible to be on this committee are 10 Democrats and 8 Republicans. How many possible ways can such a committee be formed?

Solution

First we need to determine the number of possible ways Democrats can be chosen and the number of possible ways the Republicans can be chosen.

Possible groups of 3 Democrats: $${}_{10}C_3 = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \cdot 2 \cdot 1 \cdot 7!} = 120$$

Possible groups of 3 Republicans: $${}_{8}C_3 = \frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!} = 56$$

Now we can find the total possible ways the Senate committee can be formed using the fundamental counting principle.

$${}_{10}C_3 \cdot {}_{8}C_3 = 120 \cdot 56 = 6720$$

There are 6720 possible ways that this Senate committee can be formed.

Now work margin exercise 6.

Margin Exercise Answers

1. a. Since the place in the race affects the size of the prize, the order is important. This is a permutation problem. **b.** An order in which the family puts the bags into the trunk is not important. This is a combination problem. **c.** Since it matters which of your friends will receive which gift, the order is important. This is a permutation problem. **2.** 630 **3.** 2,035,800 **4.** 495 **5. a.** 3024 **b.** 126 **6.** 62,370

9.5 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentence using information found in this chapter.

1. For the letters a, b, and c, the _____ abc is different from the _____ cab.

2. For the letters a, b, and c, the _____ abc is the same as the _____ cab.

3. Each combination of r elements has _____ corresponding permutations.

4. Determining how many different 4-topping pizzas you can order is a _____ problem.

5. Determining how many passwords you can create from 8 characters is a _____ problem.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. For permutations, the order of the elements is important. In a combination, the order doesn't matter.

7. The number of combinations of n elements taken r at a time is greater than the number of permutations of n elements taken r at a time.

8. The total number of combinations of n elements taken r at a time is symbolized ${}_rC_n$.

9. Determining how many ways a hand of cards can be dealt is a combination problem.

Practice

Determine whether each situation describes a combination or a permutation. See Example 1.

1. Ten people will be randomly chosen as winners from 2000 lottery participants. How many different winners can be selected?

2. George came to a store to buy some juice. He sees that the store sells 8 different types of juice of his favorite brand. If he wants to buy 3 different bottles of juice, how many options does he have?

3. When presented with a choice of multiple delicious-looking dishes, some people find it hard to decide what to try first. If you see 6 different dishes and want to try all of them once, in how many orders can you try the dishes?

4. To get to the next destination, a group of tourists must split into groups to travel in several cars. If the first car has six passenger seats, how many ways are there to choose the people who will sit in this car?

5. In an outdoor scavenger hunt, all participants must visit 8 of the 10 check points and complete the respective task, before going to the finish line. The order in which the check points are visited is decided by the participants. How many different routes can the participants choose from?

6. The panel on a lock has ten buttons numbered from 0 to 9. To open the lock, you need to press three of the buttons simultaneously. How many different combinations can be set for this type of lock?

7. Jessica decided to spend Saturday night watching horror movies with her two friends. If she plans to watch 3 different movies from her collection of 15 movies, how many different schedules can she create?

8. A survey was conducted among the regular customers of a cafeteria asking them to name their favorite and second favorite sandwiches from the menu. How many different answers can the customers give?

9. On a drawing lesson, children are asked to choose three different colors from the palette and mix them together. How many different choices can the children make?

10. To pass the assessment, a student must complete 10 out of 12 tasks given. How many ways are there for the student to choose these 10 tasks?

Evaluate the given combination.

11. ${}_6C_2$

12. ${}_7C_3$

13. ${}_{10}C_2$

14. ${}_{12}C_9$

15. ${}_{15}C_1$

16. ${}_{10}C_{10}$

17. ${}_{17}C_{16}$

18. ${}_{20}C_0$

Applications

Solve.

19. An ice cream shop sells 9 different flavors of ice cream. How many different two-scoop ice creams can be bought at this shop, if the order of the scoops does not matter?

20. Before to moving into a new apartment, Steven decided to change the wallpaper in 6 of its rooms. After starting to hang new wallpaper, he discovered that the amount of wallpaper he purchased is only enough to cover the walls in 4 rooms. How many ways can he choose the 4 rooms he will finish first, before going to buy more wallpaper?

21. A farmer wants to choose 3 sheep to take to a livestock show. If he has 10 sheep that are worthy of being exhibited at the show, how many different groups of sheep can he choose?

22. Irene wants to try folding several origami models to decorate her table. She has origami paper of 12 different colors, and she chooses 4 of the colors for her work. How many different color choices can she make?

23. David inherited some money and decided to use it to buy shares in a company. After consulting with an investment advisor, he received a list of 15 companies whose shares are expected to bring the most dividends in the near future. If David decides to invest in 3 of the companies from the list, how many different options would he have to choose from?

24. Allison wants to buy bagels for her office this morning. The local bagel store has 16 different varieties of bagels. If there are 12 people on her team and everyone would want a different type of bagel, how many ways can she select 12 bagels?

25. After finishing an escape room, you are given a choice of 20 identical chests, each with a different reward. You earned 4 keys from the escape room puzzles that can open any of the reward chests. How many different sets of rewards can you get by using the keys to open four chests?

26. To set up camp during a hiking trip, a group of hikers select 6 people to be responsible for pitching the tents. If there are 20 people in the group, in how many ways can the 6 people be selected?

27. There are 4 players in each game in a mahjong competition. If 50 people participate in the competition, in how many ways can the participants of the final game be chosen from them?

28. A tradition calls for each decorated cart during the harvest festival to be pulled by 6 horses. If a community owns 25 horses, how many ways do they have to choose the horses for their cart?

29. At the beginning of one type of domino game, each player draws a hand of 7 domino tiles from the shuffled set of 28 tiles. How many different starting hands are there?

30. As an employee of a certain company, Helen can visit the local swimming pool at half price 10 times a month. In how many ways can she choose 10 days in September to use this discount? Note that September has 30 days.

31. A windowsill in Angela's office is currently empty and is long enough to hold 4 flowerpots. After scouring the nearby shop, she finds 10 plants that she likes, and wants to buy 4 of them.

a. Determine the number of different options she can choose from.

b. While choosing the four out of ten plants, Angela imagines their arrangement on the sill. Determine the number of different arrangements she can choose from.

32. A person new to streaming video game plans a small giveaway with 3 cash prizes among his 50 subscribers.

a. Determine the number of possible outcomes for the giveaway if all prizes are different.

b. Determine the number of possible outcomes if all prizes are the same.

33. Before being put into a storage locker, 4 boxes of files are each labeled with a unique capital English letter.

a. Determine the number of ways to choose the four letters to label the boxes if the boxes contain the same type of information.

b. If each box contains a different type of information and the letter identifies the contents within the box, determine the number of ways to label the boxes.

34. Five students are selected to attend a conference from a 30-student debate class.

a. Determine the number of ways to choose the representatives if all of them have equal status at the conference.

b. Determine the number of ways to choose the representatives if each of them has a unique role to perform at the conference.

35. In a biathlon mixed relay, each team is composed of two women and two men. The national team consists of 9 women and 10 men, all of whom are skilled in both the cross-country skiing and the rifles shooting portions of the race. How many ways can the coaching staff select the participants for the relay?

36. Thomas wants to pack 4 T-shirts and 2 pairs of pants to take on vacation. In his wardrobe, he has 9 different T-shirts and 5 pairs of pants. How many different options does he have to choose from?

37. Amanda has decided to plant three types of berry bushes and two different fruit trees in her yard. She can choose from 7 different types of bushes and 8 types of fruit trees. How many different options can she consider?

38. A puppeteer is selecting 3 male puppets and 4 female puppets for a puppet show at a children's party. If she has 12 male puppets and 16 female puppets, how many different puppet combinations does she have to choose from?

39. For a game night with his friends, Kevin decided to order 5 different pizzas and 3 different salads from local pizza parlor. If the pizza parlor offers 15 types of pizza and 10 different salads, how many different orders could he make?

40. Jennifer has a collection of 15 different teacups and 20 different teaspoons. Three of her friends plan to join her for afternoon tea, so she chooses 4 teacups and 4 teaspoons to use. How many options does she have to choose from?

41. The school board announces that three best students of each class in the elementary school will be given an award. Consider two classes from this school. If one class has 25 students and the other has 30 students, in how many ways can the group of students receiving the rewards be selected from these two classes?

42. To play the local lottery, you must select 4 even numbers and 4 odd numbers from 1 to 50. You win if the selected numbers match the numbers drawn during the evening news. How many different selections can be made to play the lottery?

43. Two rival schools compete over which has the best swimmers. Each school selects 5 people from its swim team for a competition. If the swim team of the first school has 20 people and the swim team of the second school has 22 people, how many different ways can the participants of the competition be chosen?

44. At the beginning of a tabletop game, each player draws 4 cards from each of the two specialized decks of cards. If there are 20 cards in the first deck and 25 cards in the second deck, how many different hands can be drawn by the first player?

45. A teacher is creating a test and wants to include two assigned exercises from each section of the most recent chapter. If the chapter consists of three sections and she assigned 10 exercises from the first section, 8 from the second section, and 11 from the third section, how many different tests can she make? Assume that the order of questions in the test does not matter.

46. For an exhibit on ancient sculpture styles, a museum wants to display 3 Greek sculptures, 5 Roman sculptures, and 4 Egyptian sculptures. The museum currently has 12 sculptures from Greece, 15 from Rome, and 13 from Egypt. How many different exhibits can they create?

Writing & Thinking

47. Suppose you have two groups of things. You select three different things from the first group and pair them with three different things from the second group, thus obtaining three pairs. How can you find the number of different sets of three pairs that can be obtained in such a way?

Margin Exercise Answers

1. $\frac{1}{151,200}$ **2.** $\frac{1}{143}$ **3.** $\frac{108}{715}$ **4.** $\frac{1176}{4199}$

9.6 Exercises

Applications

Solve and write your answer in reduced fraction form.

1. A newly created soccer team chooses three different colors for the team jersey. If they randomly choose from 20 different colors, what is the probability that the jersey will be black with blue sleeves and a green team logo?

2. Peter chooses a random password containing four different digits, from 0 to 9, for his smartphone. What is the probability that his password will be 1984?

3. Monica selects four of the seven rainbow colors to paint stripes on the walls in her room. If she wants all of the colors to be different, what is the probability that she will paint the walls, from top to bottom, in blue, orange, indigo, and red?

4. Five friends stand in a row for a group photo. If all of them are different heights, what is the probability that they will stand in either ascending height order or descending height order?

5. A family is planning a trip to Europe. There are seven countries they would like to visit, including Italy. However, they only have enough time to visit three countries. If the family randomly selects three countries to visit, what is the probability that the first selected country will be Italy?

6. A new five-digit password is automatically generated every month to provide account security. In the password, each digit is unique. What is the probability that the password generated in July is the same as the password generated in June?

7. Alex generates a name for his podcast by randomly picking four different letters and writing them down. What is the probability that name of the podcast will be “Alex”?

8. An inexperienced tour manager of a rock band plans a tour of 10 cities by randomly picking the order that the cities will be visited. If there is only one tour plan that would be the least expensive, what is the probability that the tour will be planned in the least expensive way?

9. A creative writing teacher needs to choose 4 short story prompts to assign during the semester. He can choose from one prompt of each of the following genres: self-help, biography, romance, autobiography, science fiction, fantasy,

mystery, and historical fiction. What is the probability that the chosen books will be assigned in the order fantasy, mystery, self-help, and autobiography?

10. A hacker knows that the password he wants to break contains five different English letters. If the password is not case sensitive, what is the probability that the hacker will correctly guess it in the first three attempts?

11. A journalist decides which of his four interviews and two investigations are worthy to be published. If he is going to publish only two articles, what is the probability that these articles will be interviews?

12. The exit from an escape room opens if you press the correct three out of nine buttons at the same time. What is the probability that three randomly pressed buttons will open the exit on the first attempt?

13. A box contains 7 fruit-flavored candies and 3 caramels. If you randomly pick 4 candies from the box, what is the probability that all of them will be fruit flavored?

14. A car dealership had 9 silver cars and 7 blue cars at the beginning of the week. If 4 cars were sold during the week, what is the probability that all of them were silver?

15. The menu of a small pizzeria consists of 9 pizzas and 6 pastas. John chooses 5 different dishes for his birthday party. What is the probability that he will choose only pizzas?

16. Two classes in a high school hold an essay competition. Six students from each class chooses to participate. If there are only 4 winners, what is the probability that they are all from the same class?

17. At the end of a festival, the organizers hold a small lottery with 3 prizes. If you bought two of the 50 lottery tickets, what is the probability that you will win two of the three prizes?

18. Two groups of 10 cards are mixed together. If you randomly select 10 cards from the mix, what is the probability that they will all be from the first group?

19. If you randomly draw 4 cards from a standard deck of 52 cards, what is the probability that they all will be spades? (Recall that each deck has 13 spades, 13 hearts, 13 clubs, and 13 diamonds.)

20. Justin will get a 25% off discount code for an online shop if three people register at the shop using his affiliate link. He shares the link with his colleagues, 10 of which are men and 15 are women, and three of them register. What is the probability that the three people who registered through the link were women?

21. In preparation for a hike, two people are chosen to cook meals during stops. If the group consists of 5 men and 6 women, what is the probability that 1 man and 1 woman will be chosen to cook?

22. Anna and Zoe decided to each list their top 10 movies and then watch 3 movies from the combined list. If all the movies on their lists are different, what is the probability that they will choose 2 of Anna's movies and 1 of Zoe's movie?

23. Ten comedies and seven dramas have been sent to a film festival. If five films are to be demonstrated on the final day of the festival, what is the probability that there will be only one drama among them?

24. A scientist selects two different letters from 26 lowercase and 26 uppercase English letters as the names of two groups in an experiment. What is the probability that she will choose one lowercase and one uppercase letter?

25. If you choose 3 cards from a standard deck of 52 cards, what is the probability that exactly one of them will be a king? (Recall that a standard deck of cards has 4 kings.)

26. A student selects three out of five different types of sandwiches to take with him for lunch. If only one type of sandwich contains chicken, what is the probability that this type will be among the selected?

27. Ten students from the humanities department and eight students from the social science department take part in a race. If we assume that all of the students have the same running skills, what is the probability that only one student from the social science department will be one of the top three winners?

28. Eight junior computer programmers and five senior computer programmers are eligible to be on the team for a regional hackathon. If the team consists of 5 members, what is the probability that it will include 1 junior programmer and 4 senior programmers?

29. A child picks five toys from a bag containing five toys of each of five different shapes. What is the probability that the selected toys will all be of different shapes?

30. Out of a hundred pens in a box, ten have red ink. If you randomly choose three pens from the box, what is the probability that exactly one of them will have red ink?

31. A city is divided in two parts by a large river. The owner of a restaurant chain wants to open 6 new restaurants in the city, and she finds 6 potential locations on each side of the river. What is the probability that she will open 4 restaurants on the left side of the river and 2 on the right side?

32. A Doctor of Political Science has two popular lectures. He has 6 invitations to give the first lecture and 4 invitations to give the second lecture, all at different universities. Unfortunately, he only has time for 5 lectures. What is the probability that he will choose to read the first lecture 3 times and the second lecture 2 times?

33. A small shop sells 7 styles of T-shirts and 8 styles of sweaters. If a customer bought 5 different items of clothing, what is the probability that he bought 2 T-shirts and 3 sweaters?

34. A poet had 6 copies of his poetry collection left from a previous printing. He printed 10 new copies in preparation for a performance. If 4 books were sold after the performance, what is the probability that 2 were old copies and 2 were new copies?

35. A photographer selects 12 photographs for an exhibition. If he has 8 rural photographs and 6 city photographs, what is the probability that he will select 6 photographs of each type?

36. The list of questions at the end of a chapter consists of 13 questions, and a student can answer 7 of them. If a test includes 11 questions from the list, what is the probability that the student will be able to answer exactly 6 of them?

37. In a school known for its mathematics education, 9 students from class A and 11 students from class B compete for seven places on the team going to the national mathematics competition. What is the probability that the team will consist of 5 students from class A and 2 students form class B?

38. The menu of a fast food restaurant consists of 9 chicken sandwiches and 8 fish sandwiches. If there is an order for 7 different sandwiches, what is the probability that 4 of them are with chicken?

39. To celebrate the career of a famous writer born in the city, the local museum prepares an exhibit dedicated to the life of the writer. The museum owns 9 personal belongings of the writer and 9 drafts of his works. If they plan to display 8 items what is the probability that 5 of them will be personal belongings?

40. After the annual reviews, company management decides to promote 9 employees. Of the 20 people participating in the review, 7 have more than 5 years of work experience. What is the probability that all of the employees with more than 5 years of work experience all will be promoted?

Writing & Thinking

41. Suppose you are choosing several objects from a set of different objects. Which of the two probabilities is greater: the probability of choosing certain objects in a specific order or the probability of choosing the same objects, but in a random order?

42. Is it helpful to know that characters in a password do not repeat when trying to guess it? Explain why.

43. What conditions should be met when using permutations and combinations to find the probability?

44. Assume you have 3 yellow and 2 green balls in a bag. In the event of drawing three balls from the bag, how can you find the probability that not all of them are yellow?

e. All seniors take the senior capstone course. There are 25 sections of the course. Eight of the 25 sections are randomly selected and all seniors in those sections are asked about their favorite food.

Solution

a. This is systematic sampling as every n^{th} person is selected.

b. This is convenience sampling as the selection of the sample is convenient to you, the researcher.

c. This is random sampling as all seniors have an equally likely chance of being selected.

d. This is stratified sampling as the seniors are separated into groups based on a similar characteristic (academic major), and then seniors are randomly selected from each group.

e. This is cluster sampling as the seniors are separated into sections of the course (clusters), and then entire clusters are randomly selected.

Now work margin exercise 2.

Margin exercise answers

1. a. Population: local junior high teachers, Sample: 35 teachers surveyed, 60% is a sample statistic as it only refers to the teachers surveyed. **b.** Population: bowlers in a Western state, Sample: 600 bowlers surveyed, 42% is a population parameter as it refers to all bowlers in the Western state. The population parameter is based on sample statistics that are not reported. **2. a.** This is cluster sampling as the employees are separated into departments in the factory (clusters), and then entire clusters are randomly selected. **b.** This is systematic sampling as every n^{th} person is selected. **c.** This is random sampling as all student workers have an equally likely chance of being selected. **d.** This is convenience sampling as the selection of the sample is convenient to the researcher. **e.** This is stratified sampling as the employees are separated into groups based on a similar characteristic (years of service), and then employees are randomly selected from each group.

10.1 Exercises

Concept Check

Fill-in-the-blank. Complete the sentences using information found in this section.

1. A _____ is the particular group of interest being studied.

2. A _____ involves collecting data from every member of the population.

3. A subset of a population is called a _____.

4. A number that describes a particular characteristic of a population is called a _____.

5. A number that describes a particular characteristic of a sample is called a _____ _____.

6. A _____ _____ reflects the population by having the same characteristics and does not favor any group over another.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A census collects data from every member of a sample.

8. A sample statistic gives information about the entire population.

9. In a study that interviews all high school students at the local school to find out what percentage of American teenagers have an iPhone, American teenagers are the population.

10. Asking everyone in your homeroom class what type of soda is their favorite is an example of stratified sampling.

Practice

For the following scenarios, identify the population, sample, and whether the results represent a population parameter or sample statistic. See Example 1.

1. A school board recently read a report indicating that unhealthy eating habits were increasing in teenagers. As a result, the school board surveyed the local high school students. Of the 300 students surveyed, 62% indicated that they ate unhealthy foods at least once a day.

2. A local youth organization interviewed 1000 adults across a Midwest state about their views on teenagers' involvement in youth sports. The resulting report stated that approximately 73% of the adults in the Midwest state are in favor of increased teenager involvement in youth sports.

3. The local parent-teacher organization interviewed 850 parents across a large metropolitan area regarding their views on homework in middle school. The report the organization released stated that approximately 54% of the parents in the large metropolitan area believed there is too much homework in middle school.

4. A small athletic conference read a report indicating that students participating in high school sports were more likely to have negative body images. As a result, the conference surveyed the local high school athletes. Of the 400 athletes surveyed, 38% indicated that they had a negative body image.

Identify the sampling technique used to obtain each sample. See Example 2.

5. The school board divides the city by school boundaries and then all student test scores are used from three randomly selected school districts.

6. The names of all the students are placed in a hat and 20 names are selected without looking. Data are collected from those 20 students.

7. A study of sleep habits classifies participants according to age using groupings of 20–29, 30–39, and 40–49. Random samples of participants are taken from each group.

8. A big box store company has multiple stores in 50 large cities. Ten of the 50 large cities are selected and all stores in each of the 10 cities are checked for cleanliness.

9. Every 100^{th} music box on an assembly line is given a reliability test.

10. A vision study uses participants who are classified by eye color (blue, brown, hazel, etc.). A random sample of participants are taken from each group.

11. The first 50 people leaving a hospital are asked how much money they spend per day on lunch in the hospital cafeteria.

12. Students who are registered for mathematics classes are classified according to their academic major and then random samples of students from each major are selected.

13. Every 20^{th} cell phone coming off an assembly line is checked for defects.

14. The last 25 people leaving the cinema are asked for their opinion of the movie.

15. Every 8^{th} person leaving the amusement park are asked about their favorite ride.

16. Illinois is divided into 102 counties and then 13 individual counties are randomly selected and data from all residents of those counties are collected.

17. The names of all members of a population are entered into a database. A computer uses a random number generator to select members of the population.

18. A group of people are classified according to race and then random samples are taken from each group.

19. The Registrar's Office puts all 2000 students' names in a list and randomly generates 125 names to gather information.

20. Students at the local high school are classified according to their class (fr., so., etc.). Then a random sample of 20 students from each class is selected.

21. All students are required to take a mathematics course. There are 10 sections of the course. Three of the 10 sections are randomly selected, and data are collected from all students in those sections.

22. Every 10^{th} person walking through a shopping mall is surveyed about their spending habits.

23. Twenty people are assigned numbers 1–20. An icosahedron (20-sided die) is rolled and the person corresponding to the number is selected.

24. A taste test for potato chips is given at the end cap of a snack aisle in a grocery store.

25. Every 5^{th} car in a parking garage is checked for the appropriate parking pass.

26. A group of local runners are surveyed as to the best marathon training program.

27. A state is divided by area codes and then 4 area codes are randomly selected and data from all members of those area codes are collected.

28. The "best restaurant in town" is determined by the responses from local readers who voluntarily mail in a survey printed in the newspaper.

29. The names of all 250 employees are placed in a bin and 5 names are selected without looking.

30. The last 35 people leaving the Homecoming talent show are asked for their opinion of the show.

Writing & Thinking

31. Give one example for each of the five sampling techniques.

32. Explain, in your own words, what a representative sample is.

do not add up to 100%. This is not the proper use of a circle graph. This sort of information should be presented as a bar graph.

Margin exercise answers

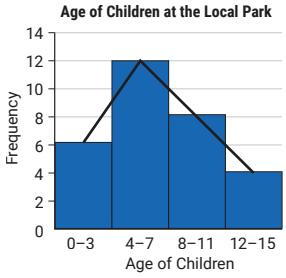
1. a. Pennsylvania; **b.** 4 more institutions

3.

Number of Jobs Applied For	Frequency
6–11	3
12–17	4
18–23	11
24–29	7
30–35	9
36–41	2
42–47	2
48–53	4

4. a. Approximately 500 people surveyed; **b.** 340 people used between 5 and 16 gigabytes of data; **c.** 225 people use 8 gigabytes of data or less.

5.



6.

Stem	Leaf
6	0 2 4 5
7	0 0 1 3 4 4 5
8	0 0 1 4 4 4 5 5 5 6 6 7 8 9 9
9	0 0 4 8

2.

Number of Jobs Applied For	Frequency
6	1
10	2
12	2
15	2
18	2
20	2
21	2
22	5
24	2
25	1
27	2
28	1
29	1
30	1
32	3
34	3
35	1
36	1
37	1
45	3
48	3
50	1

10.2 Exercises

Concept Check

Fill-in-the-blank. Complete the sentences using information found in this section.

1. A _____ _____ is the simplest way to view each observed data value and how often it appears in a set of data.

2. A grouped frequency distribution is created by splitting a frequency distribution into _____.

3. In a grouped frequency distribution, the number of data points in a class is called the _____ _____

4. In a grouped frequency distribution, it's important that the data is split so that all the classes are the _____ _____.

5. A _____ is a special form of a bar graph that is used to display the frequency distribution of numerical classes.

6. A _____ _____ can be constructed from a histogram.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. In a grouped frequency distribution, classes may overlap.

8. In a histogram, the bars do not touch similar to a bar graph.

9. A frequency distribution is a table in its simplest form that indicates the number of times a data value occurs.

10. In a grouped frequency distribution or histogram, all the individual data values are able to be seen.

Practice

1. The following frequency distribution shows the most common majors in the College of Arts and Sciences at a local university. Use the frequency distribution to answer the questions below.

Major	Frequency
Biology	80
Chemistry	65
Communication	55
Psychology	45
Pre-Law	40

   a. What is the most common major?

   b. How many more biology majors are there than chemistry majors?

2. Students at a local middle school were surveyed as to what is their favorite pet. Use the frequency distribution to answer the questions below.

Favorite Pet	Frequency
Cat	6
Dog	8
Fish	5
Hamster	5
Rabbit	2

   a. What was the favorite pet of students?

   b. How many students were surveyed?

   c. How many students prefer dogs to cats?

3. Students at a local elementary school were surveyed as to what is their birth month. Use the frequency distribution to answer the questions below.

Birth Month	Frequency
January	8
February	12
March	10
May	9
August	5
November	6

a. What was the most popular birth month?

b. How many students were surveyed?

c. How many more students were born in March than November?

4. The following grouped frequency distribution shows the number of cell phones sold in a week by local salespeople. Use the grouped frequency distribution to answer the questions below.

Number of Cell Phones Sold	Frequency
1–10	30
11–20	40
21–30	25
31–40	20
41–50	25
51–60	15

a. Data were collected from how many salespeople?

b. How many salespeople sold 30 or fewer cell phones in a week?

c. How many salespeople sold more than 40 cell phones in a week?

5. The following grouped frequency distribution shows the number miles ran by running club members in a week. Use the grouped frequency distribution to answer the questions below.

Number of Miles Ran	Frequency
1–12	6
13–24	12
25–36	10
37–49	4

a. How many members are there in the running club?

b. How many members ran 36 miles or fewer in a week?

c. How many members ran 25 or more miles in a week?

6. A group of students at a local elementary school were surveyed as to how many pets they had in their house. Construct a frequency distribution of the data collected.

0 2 1 0 0 2 2 1 3 4 1 2 5 0

0 1 2 1 2 0 2 3 1 3 1 4 5 1

7. A group of teachers at a local school were surveyed as to how many cups of coffee they drank each day. Construct a frequency distribution of the data collected.

6 2 0 1 0 3 1 1 3 1 4 2 4 2 6 5

2 4 3 1 3 0 5 3 2 4 2 4 2 5 2 3

8. The weights of newborn babies were gathered from a local hospital. Construct a grouped frequency distribution of the data collected with a starting class of 1–3.

7 9 2 3 4 8 10 6 5

1 7 8 4 1 9 10 5 12

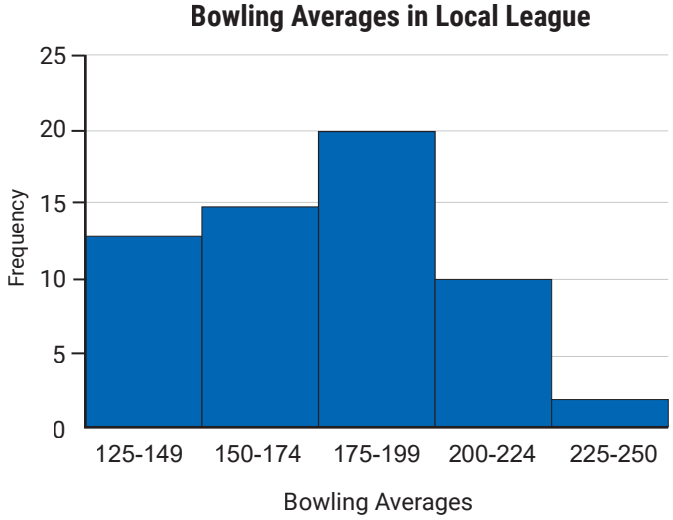
9. The grades from a recent statistics exam were collected. Construct a grouped frequency distribution of the data collected with a starting class of 51–60.

51 94 95 53 55 61 84 91 85 63 65

67 71 74 75 80 81 81 71 72 83 86 87

89 90 91 73 74 91 92 93 94 96 82

10. The following histogram displays data collected from a local bowling league and shows the average of the bowlers. Use the histogram to answer the following questions.

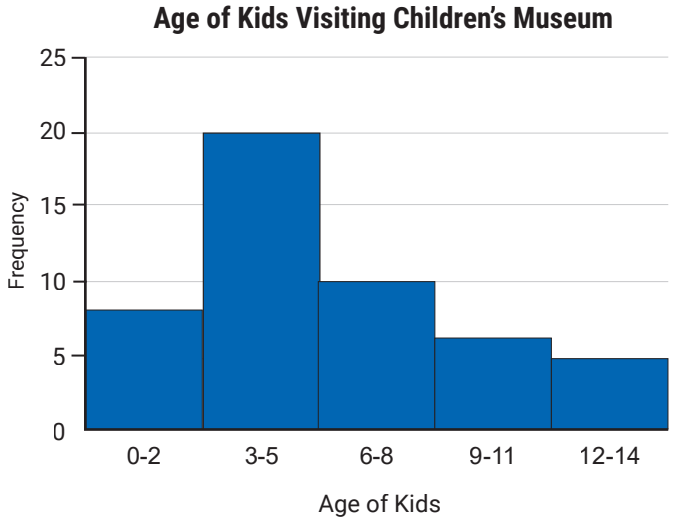


a. How many bowlers are in the local league?

b. How many bowlers have an average greater than or equal to 200?

c. How many bowlers have an average less than 175?

11. The following histogram displays the age range of kids at a local children's museum on a Saturday. Use the histogram to answer the following questions.

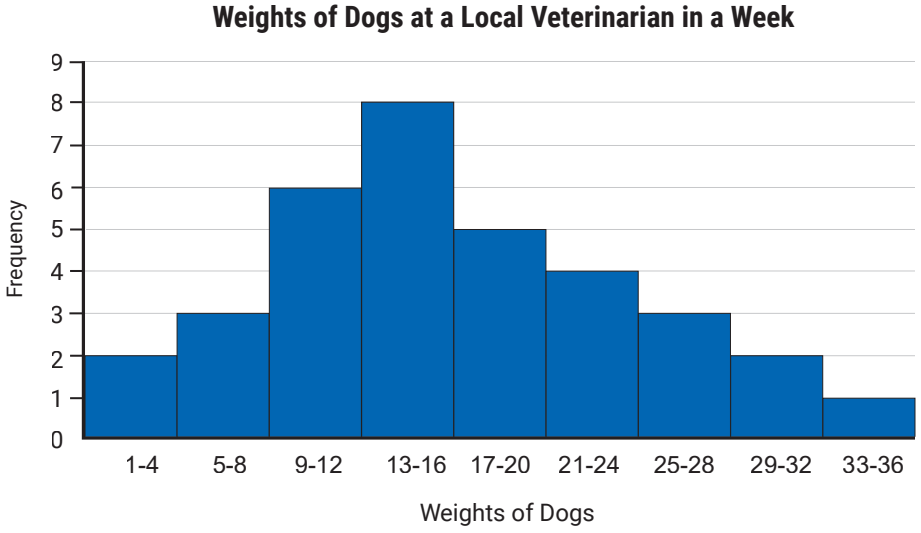


a. How many kids visited the children's museum?

b. How many kids were younger than 6?

c. How many kids were 9 or older?

12. The following histogram displays the weights (in pounds) of dogs at a local veterinarian's office in the past week. Use the histogram to answer the following questions.

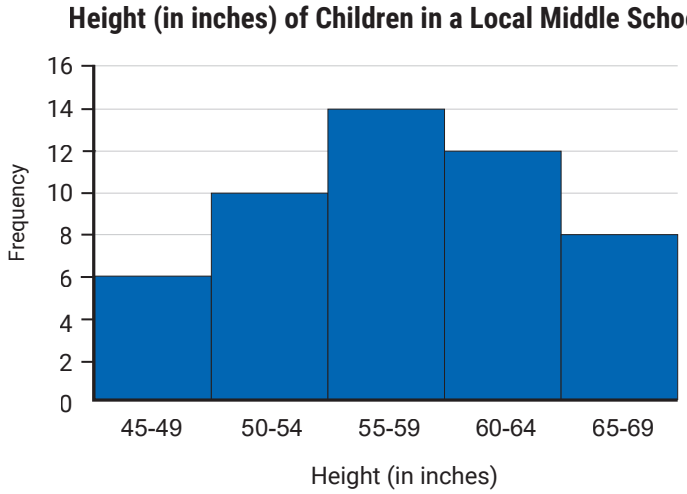


a. How many dogs went to the veterinarian in the past week?

b. How many dogs weigh less than 9 pounds?

c. How many dogs weight 21 pounds or more?

13. The following histogram displays the heights (in inches) of children in a local middle school. Use the histogram to answer the following questions.



a. How many children's heights were recorded?

b. How many children are taller than 54 inches?

c. How many children are 64 inches or shorter?

14. Given the following grouped frequency distribution of batting averages in a local slow-pitch softball league, construct a histogram based on the data.

Batting Average	Frequency
.150–.184	6
.185–.219	10
.220–.254	15
.255–.289	25
.290–.324	30
.325–.359	25

15. Given the following grouped frequency distribution of cars sold in a week by salespeople, construct a histogram based on the data.

Cars Sold	Frequency
1–5	8
6–10	6
11–15	4
16–20	0
21–25	2

16. Given the following grouped frequency distribution of the heights (in inches) of kids in a local basketball league, construct a histogram based on the data.

Height	Frequency
48–51	2
52–55	3
56–59	4
60–63	7
64–67	9
68–71	6

17. Given the following grouped frequency distribution of the weights (in pounds) of luggage on a recent flight, construct a histogram based on the data.

Weight	Frequency
25–30	18
31–36	28
37–42	25
43–48	20
49–54	3

18. Given the following grouped frequency distribution of exam scores, construct a histogram and corresponding frequency polygon on the same set of axes.

Exam Scores	Frequency
42–48	1
49–55	1
56–62	2
63–69	3
70–76	8
77–83	6
84–90	7
91–97	5

19. Given the following grouped frequency distribution of miles rode by a local cycling club, construct a histogram and corresponding frequency polygon on the same set of axes.

Number of Miles Cycled	Frequency
40–59	3
60–79	6
80–99	8
100–119	12
120–139	15
140–159	10

20. Given the following grouped frequency distribution of the number of text messages sent to a group chat by members of the group each week, construct a histogram and corresponding frequency polygon on the same set of axes.

Number of Messages Sent	Frequency
1–9	3
10–18	8
19–27	12
28–36	10
37–45	16
46–54	9
55–63	4

21. Use the following exam scores from a recent psychology exam to create a stem-and-leaf plot.

94 70 90 91 71 72 80 80 82 84 88 94

89 89 90 72 83 84 74 91 92 92 93 75

22. Use the following resting pulse rates to create a stem-and-leaf plot.

48 49 62 64 49 80 69 70 84 50 51 52 52 53

55 58 62 72 67 81 82 68 71 83 84 71 72 65

23. Use the following ages in a local running club to create a stem-and-leaf plot.

24 31 28 42 54 56 58 41 23 27 29 34 36 45

26 33 43 29 30 25 38 32 37 48 62 68 59 68

24. The following stem-and-leaf plot contains mileage ran by a local running club during a week. Use the plot to answer the questions below.

Stem	Leaf
1	5 5 5 8 9
2	0 2 2 5 5 5 8
3	0 0 5 5 8 8 9
4	0 0 5 6 8

a. How many runners ran 25 miles for the week?

b. How many runners ran 30 or more miles during the week?

c. How many runners are in the local running club?

25. The following stem-and-leaf plot contains scores from a recent chemistry exam. Use the plot to answer the questions below.

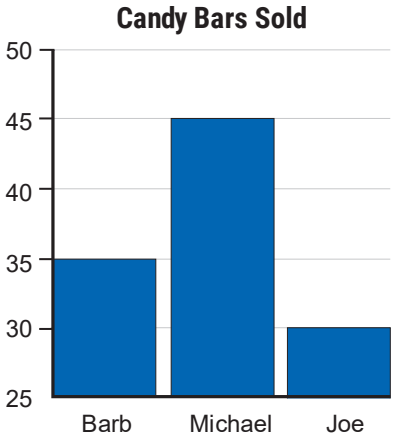
Stem	Leaf
5	2 2 8
6	0 1 1 5 7 8
7	0 2 3 3 5 6 8 8 8 9
8	1 2 4 5 6 6 8 9
9	0 2 6 8 9

a. How many students scored 78?

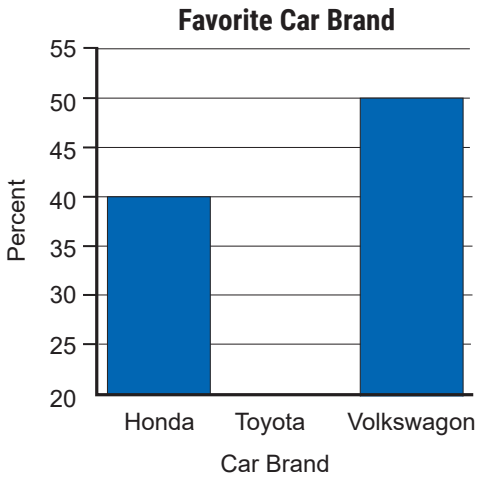
b. How many students scored 82 or more?

c. How many students took the exam?

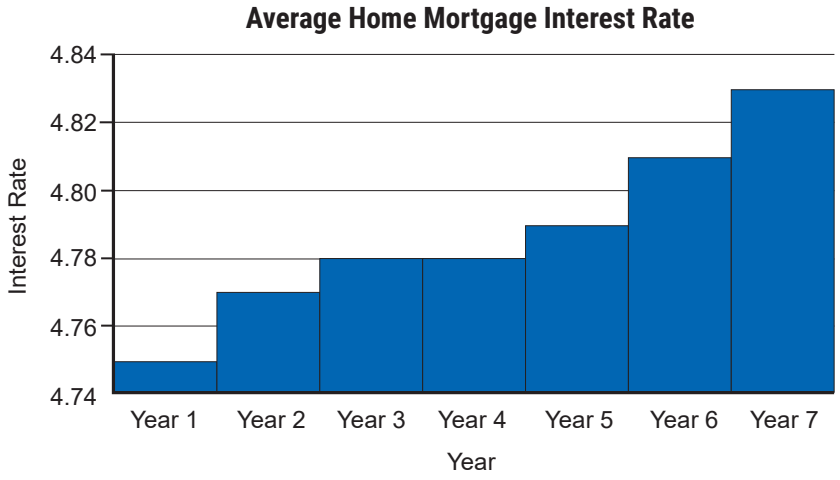
26. The following bar graph displays the number of candy bars sold by students for a local school fundraiser. In what way is the bar graph misleading?



27. The following bar graph displays the results of a survey where people were asked to identify their favorite car brand. In what way is the bar graph misleading?



28. The following bar graph displays the average home mortgage interest rate over a seven year period. In what way is the bar graph misleading?



Writing & Thinking

29. Explain how to convert a frequency distribution to a grouped frequency distribution.

30. Describe a stem-and-leaf plot in your own words.

Solution

a. As there are only three categories, we will add the grade from each category and divide by three.

$$\frac{0.95+0.85+0.87}{3}=\frac{2.67}{3}=0.89$$

Therefore, your grade based on the arithmetic mean of the three categories is 89%.

b. Note that the weights add to 100%. To determine the weighted mean, we must multiply the assigned weights for each category by each category's grade.

$$(0.10)(0.95)+(0.30)(0.85)+(0.60)(0.87)=0.872$$

Therefore, your grade based on the weighted mean is 87.2%.

Note that the weighted mean is different than the arithmetic mean.

Now work margin exercise 9.

Margin Exercise Answers
1. 82.25 **2.** 50.875 **3.** 4 **4.** 3 **5.** 4 **6.** multimodal: 32 and 45 **7.** There is no mode. **8.** mean: 4.5; median: 5; mode: 5 **9.** 83%

10.3 Exercises

Concept Check

Fill-in-the-blank. Complete the sentences using information found in this section.

1. A measure of _____ _____ describes what a typical value in a data set looks like.

2. The _____ is the sum of all data values divided by the number of data values.

3. The middle value in a ranked data set is called the _____.

4. A data set that has two modes is said to be _____.

5. A data set is multimodal if there are more than _____ data values that qualify as the mode.

6. A _____ _____ is one in which some data values contribute more to the mean than others.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The median is the data value that occurs most often in a data set.

8. The mean is also called the arithmetic average.

9. The mean must be a value in the data set.

10. There can be more than one median.

Practice

Find the mean, median, and mode for each of the following data sets.

1. 15, 16, 12, 18, 23, 21, 20, 19, 14, 28

2. 43, 49, 49, 41, 43, 45, 46, 48, 46, 42, 44, 45, 47, 49

3. \$6, \$8, \$4, \$9, \$6, \$5, \$7, \$6, \$5, \$9, \$12

4. 3, 3, 3, 3, 3, 3, 3, 3, 3, 3

5. 82, 84, 86, 84, 86, 84, 86, 82

6. 110, 120, 115, 112, 114, 120, 115, 110, 112, 114

7. Use the following statistics regarding yearly salaries at Steppingstone Incorporated.

   Mean: \$110,000 Median: \$78,000 Mode: \$62,000

   a. What is the most common salary?

   b. What salary did half the employees exceed?

8. Suppose you have earned scores 78, 84, 68, and 88 on exams in one of your classes.

   a. What is the mean of the scores?

   b. What score would you need on the next exam to have an overall mean of 80?

   c. If the maximum score on an exam is 100, is it possible to have a mean of 85 after the fifth exam? Explain.

Find the mean, median, and mode for each of the following scenarios.

9. A random sample of the weights of passengers' carry-on luggage was collected. The weights (in pounds) of the luggage pieces were as follows.

   36 44 40 31 33 40 37 40 38 41

10. A random sample of the heights of basketball players at a local tournament was collected. The heights (in inches) of the players were as follows.

    62 78 72 71 74 69 73 74 75 77

11. The number of pacifiers lost each day by babies at the local daycare are as follows.

3 2 0 2 0 2 0 3 4 2 1 2

12. Bowling scores by bowlers in a local recreational league are as follows.

164 185 152 160 163 165 195 185 182 167

13. Golf scores by golfers in a local recreational league are as follows.

82 78 84 98 72 91 77 78 86 90

Find the weighted mean. See Example 9.

14. A teacher computes a student's average using a weighted mean where quizzes count 25%, homework 15%, and exams 60%. If a student's quiz grade is 82%, homework 90%, and exams 88%, what is the student's average?

15. A teacher computes a student's average using a weighted mean where quizzes count 15%, homework 10%, and exams 75%. If a student's quiz grade is 90%, homework 92%, and exams 85%, what is the student's average?

16. A savvy consumer looking to buy a new camera decided to give weighs to the areas she thought were important in the shopping process. She decides to give image quality 50%, build quality 20%, and performance 30%. She has narrowed her decision down to two choices. Camera A scores 8 for image quality, 6 for build quality, and 9 for performance. Camera B scores 7 for image quality, 9 for build quality, and 8 for performance. What is the weighted mean for each camera, and based on the scores, what camera should she purchase?

17. In a local golf tournament, the first and second round score counts for 25% of the average score, and the third round counts for 50%. Lydia scores 76 in the first round, 73 in the second round, and 75 in the third round while Adah scores 78 in the first round, 76 in the second round, and 68 in the third round What is the weighted mean for Lydia and Adah, and based on the scores, who wins the tournament? (In golf, the lower score wins.)

For each frequency distribution, determine the mean, median, and mode of the data set. See Example 8.

18.

Text Messages Sent in a Day	Number Reporting Texts Sent (Frequency)
5	8
6	10
7	8
8	6
9	4
10	4

19.

Number of Siblings in a Family	Frequency
0	2
1	10
2	12
3	5
4	2
5	1

20.

Number of Pets in a Household	Frequency
0	4
1	8
2	12
3	6
4	5
5	5

21.

Number of Work Emails Per Day	Frequency
25	3
26	5
27	6
28	1
29	2
30	8

Find the mode for each of the following scenarios. State if the data set is unimodal, bimodal, multimodal, or has no mode. See Examples 5 through 7.

22. The number of bottles used each day by babies at a local daycare are as follows.

6 4 5 6 5 8 7 10 6 5 9 10

23. A sample of batting averages by players in a local softball recreational league are as follows.

.250 .285 .267 .290 .310 .260

.335 .301 .315 .325 .279 .283

24. The number of free throws taken by team members during a recent basketball game are as follows.

3 2 0 4 6 5 2 8 2 7 1 3 1 4 3

25. The number of pieces of junk mail received each day are as follows.

4 1 3 2 4 2 5 1 2 3 5 1 3 5 4

26. The number of movies seen in a cinema each month are as follows.

8 1 4 3 5 2 3 5 4 7 3

27. The number of pencils lost by teachers during a week are as follows.

1 0 4 3 5 0 2 4 6 2 4 3 2 1 3 5 7

Writing & Thinking

28. Eliana is trying to average 6 made field goals per basketball game for the month. Thus far, she has had made field goals of 8, 4, 9, 5, 7, 6, 8, and 2 over the first eight 8 games. She has two games left. What must she average over the remaining two games to end up with a mean of 6 made field goals per basketball game for the month?

29. Give an example of when it would be better to use the mode than median.

35 38 30 25 33 34 30 40 41 42 41 39 36 40

Solution

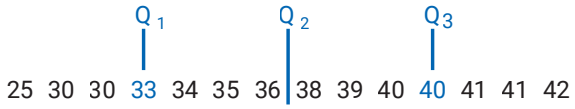
The data set is not in ranked order. To find the quartiles, the data must be rearranged into ascending order.

25 30 30 33 34 35 36 38 39 40 40 41 41 42

There are 14 data values, so the second quartile, or median, is the mean of the seventh and eighth values (36 and 38) in the ranked data set. The second quartile of the data set is 37.

To find the first quartile, we need to find the middle of the first half of the data, in other words the median of that set. There are 7 values in the first half of the data so the fourth value is the median. Thus, the first quartile is 33.

To find the third quartile, we need to find the middle of the second half of the data, in other words the median of that set. There are 7 values in the second half of the data so the fourth value is the median. Therefore, the third quartile is 40.



First Quartile = 33

Second Quartile (Median) = 37

Third Quartile = 40

Now work margin exercise 5.

Margin Exercise Answers

1. 26 **2.** 9 **3.** Approximately 7.76 points **4.** Alejandro's score is not good enough to get into his top college choice. **5.** First Quartile = 60; Second Quartile (Median) = 71; Third Quartile = 74

10.4 Exercises

Concept Check

Fill-in-the-blank. Complete the sentences using information found in this section.

1. The _____ is the difference between the largest value and smallest value in a data set.

2. A _____ _____ _____ describes the "spread" of a data set.

3. The _____ _____ indicates how much we would expect a data value to differ from the mean.

4. _____ divide the data set up into 100 equal parts and indicates approximately what percentage of the data lies at or below the data value

5. A measure of _____ _____ describes the position of a specific data value in a data set compared to the rest of the values in the set.

6. When percentiles divide a data set into four equal parts the parts are called _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The standard deviation takes only two data values into consideration.

8. The median is the second quartile.

9. Quartiles divide a data set into eight equal parts.

10. The larger the standard deviation the more the data values are spread out

Practice

Find the range and standard deviation for each of the following data sets. Round the standard deviation to the nearest hundredth. See Examples 1 through 3.

1. 14, 18, 16, 15, 18, 19, 12, 14, 16, 19

2. 42, 45, 44, 43, 46, 48, 47, 48, 49, 42, 43, 44, 46, 48

3. $20, $22, $24, $29, $26, $25, $23, $26, $25, $29, $32, $28

4. 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7

5. 21, 23, 22, 23, 26, 24, 20, 25

6. 6, 8, 9, 8, 5, 7, 10, 5, 8, 7

7. Use the following statistics regarding yearly salaries at a local corporation.

   Q_1: $64,000 Q_2: $78,000 Q_3: $82,000 80^{th} Percentile: $84,500

   a. What salary did half the employees exceed?

   b. About what percentage of employees' salaries exceeded $82,000?

   c. About what percentage of the employees earned $64,000 or less?

   d. About what percentage of the employees earned more than $84,500?

8. Use the following statistics regarding monthly salaries at a local insurance company.

   First Quartile: $38,000 Second Quartile: $51,000

   Third Quartile: $76,000 84^{th} Percentile: $87,500

   a. What salary did half the employees exceed?

b. About what percent of the employees' salaries exceeded $38,000?

c. About what percent of the employees' salaries was less than $76,000?

d. About what percent of the employees' salaries was more than $87,500?

9. Use the following statistics regarding the hourly wage for staff at a local company.

First Quartile: $8.00/hour Second Quartile: $16.00/hour

Third Quartile: $22.00/hour 68th Percentile: $19.00/hour

a. What hourly wage did half the employees exceed?

b. About what percent of the employees' hourly wage exceeded $19.00/hour?

c. About what percent of the employees' hourly wage was less than $16.00/hour?

d. About what percent of the employees' hourly wage was less than $22.00/hour?

10. A random sample of the weights of passengers' carry-on luggage was collected. The weights (in pounds) of the luggage pieces were as follows.

36 44 40 31 33 40 37 40 38 41

11. A random sample of the heights of basketball players at a local tournament was collected. The heights (in inches) of the players were as follows.

62 78 72 71 74 69 73 74 75 77

12. The number of pacifiers lost each day by babies at the local daycare is as follows.

3 2 0 2 0 2 0 3 4 2 1 2

13. A sample of bowling scores by bowlers in a local recreational league are as follows.

164 185 152 160 163 165 195 185 182 167

14. Golf scores by golfers in a local recreational league are as follows.

82 78 84 98 72 91 77 78 86 90

15. The number of rebounds gathered by players in a recent basketball game are as follows.

4 8 6 7 5 4 8 9 12 6

Find the quartiles for each of the following scenarios. See Example 5

16. A sample of golf scores by golfers in a local tournament are as follows.

82 78 68 69 74 71 73 72 80 81

17. A sample of assists by point guards in a local basketball tournament are as follows.

8 2 3 4 6 2 5 8 12 5 4 7 9

18. A sample of the number of text messages sent each day by teenagers is as follows.

12 15 11 10 6 20 25 18 22 16 15 20 21 23

19. The number of emails sent per day are as follows.

34 36 22 38 40 28 20 21 32 30 12 41

20. The number of text messages sent per day are as follows.

8 15 4 20 45 3 10 22 30 10 15 25

21. The number if diapers changed per day are as follows.

12 15 8 6 12 10 14 13 8 10 15 14 9

Applications

Solve.

22. Brenda is applying to medial schools that only accept applicants who score in the top 10% on the MCAT. Brenda receives her MCAT score that indicates that she scored at the 80^{th} percentile. Will Brenda meet the criteria for the medical schools she is applying to?

23. On a recent standardized test, Carlos scored at the 80^{th} percentile, Alessandro scored at the third quartile, and Isabella scored at the 82^{nd} percentile. Which of the three had the best test score?

24. Two English classes took the same exam. Both classes had a mean score of 85, but one class had a standard deviation of 3 while the other had a standard deviation of 5. What conclusion can be drawn about the exam scores of the two classes?

25. A statistics class recently took an exam that resulted in a mean of 82. If the standard deviation for the exam scores is 3.5, what does that indicate?

Writing & Thinking

26. Given the following data set, what would be a better measure of dispersion, the range or standard deviation? Why?

45 48 46 45 21 47 44 43 48 49 59 43

27. Give an example of two data sets where the range is the same but the standard deviation is different.

28. Explain, in your own words, what standard deviation indicates about a data set.

To find Sally's z-score, substitute the data value 1280, the mean 1000, and the standard deviation 140 into the formula and simplify.

$$\text{Sally: } z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{1280 - 1000}{140} = 2$$

To find Janet's z-score, substitute the data value 1440, the mean 1200, and the standard deviation 160 into the formula and simplify.

$$\text{Janet: } z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{1440 - 1200}{160} = 1.5$$

Now compare the two z-scores. Sally has a z-score of 2, which means her score is 2 standard deviations above the mean. Janet has a z-score of 1.5, which means her score is 1.5 standard deviations above the mean. While both scores are above the mean, Sally's z-score is farther from the mean than Janet's, which means that Sally performed better.

Now work margin exercise 5.

Margin Exercise Answers

1. a. 100; **b.** 50 **2. a.** Approximately 95%; **b.** Approximately 68% **3. a.** Approximately 68%; **b.** Approximately 95%; **c.** Approximately 84%; **d.** Approximately 16% **4. a.** 2.4; **b.** −1.5; **c.** 2.65 **5.** Brian has a z-score of 1.5 while Michael has a z-score of 1.25, meaning Brian performed better.

10.5 Exercises

Concept Check

Fill-in-the-blank. Complete the sentences using information found in this section.

1. A _____ _____ is one in which the distribution is bell-shaped with most of the data points clustered in the middle and tapered on the ends and is symmetrical.
2. The normal distribution is completely defined by its mean and _____ _____.
3. In a normal distribution, the _____, _____, and _____ are all the same.
4. When the distribution of data is bell-shaped it is possible to estimate the percentage of data values that fall within a few standard deviations using the _____ _____.
5. The _____ tells us how many standard deviations a data value is from the mean.
6. If a data value is below the mean, then the z-score is _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If a z-score is positive, then the data value is above the mean.

8. According to the Empirical Rule, approximately 95% of the data values fall within one standard deviation above and below the mean.

9. The total area under the normal curve is 1.

10. The normal distribution is bimodal.

Practice

1. Given $\mu = 60$ and $\sigma = 3$. Find the following.

a. The data value that is two standard deviations above the mean.

b. The data value that is three standard deviations above the mean.

c. The data value that is one standard deviation below the mean.

d. The data value that is two standard deviations below the mean.

2. Given $\mu = 45$ and $\sigma = 2.5$. Find the following.

a. The data value that is one standard deviation above the mean.

b. The data value that is two standard deviations above the mean.

c. The data value that is two standard deviations below the mean.

d. The data value that is three standard deviations below the mean.

3. Given $\mu = 30$ and $\sigma = 2$. Find the following.

a. The data value that is two standard deviations above the mean.

b. The data value that is three standard deviations above the mean.

c. The data value that is one standard deviation below the mean.

d. The data value that is three standard deviations below the mean.

Find the z-score for each given value.

4. $\mu = 50$ and $\sigma = 2$

a. $x = 45$

b. $x = 54.5$

c. $x = 51$

5. $\mu = 1200$ and $\sigma = 75$

a. $x = 1357.5$

b. $x = 1102.5$

c. $x = 1290.75$

6. $\mu = 80$ and $\sigma = 4.5$

a. $x = 69.875$

b. $x = 89.45$

c. $x = 68.3$

7. $\mu = 10$ and $\sigma = 0.5$

a. $x = 10.82$

b. $x = 9.15$

c. $x = 11.2$

Use the Empirical Rule to answer the following questions.

8. The lifetime of projector light bulbs are approximately normally distributed with a mean of 1000 hours and a standard deviation of 60.

a. What percentage of the light bulbs last between 940 and 1060 hours?

b. Identify the range of hours that 95% of the projector light bulbs last.

9. Exam scores were approximately normally distributed with a mean of 70 and a standard deviation of 4.

a. What percentage of the exam scores were between 62 and 78 hours?

b. Identify the range of exam scores that 68% of students are likely to earn.

10. The lifetime of car batteries are approximately normally distributed with a mean of 40,000 miles and a standard deviation of 2,000 miles.

a. What percentage of the car batteries last between 38,000 and 42,000 miles?

b. Identify the range of miles that 95% of the car batteries last.

11. The heights of female basketball players are approximately normally distributed with a mean of 72 inches and a standard deviation of 2 inches.

a. What percentage of the female basketball players have heights between 68 inches and 76 inches?

b. Identify the range of heights that includes 99.7% of the female basketball players.

12. The lifetime of car tires are approximately normally distributed with a mean of 14,000 miles and a standard deviation of 600 miles.

a. What percentage of the car tires last between 12,200 and 15,800 miles?

b. Identify the range of miles that 68% of the car tires last.

Use the empirical rule to answer the following questions.

A placement exam is given to all entering students at a local university. The scores are approximately normally distributed with a mean of 50 and a standard deviation of 4.

13. What percent of students scored between 46 and 54?

14. What percent of students scored between 42 and 58?

15. What percent of students scored between 38 and 62?

16. What percent of students scored less than 54?

17. What percent of students scored less than 58?

18. What percent of students scored less than 46?

19. What percent of students scored less than 42?

20. What percent of students scored more than 38?

21. What percent of students scored more than 46?

22. What percent of students scored more than 54?

23. What percent of students scored more than 58?

24. What percent of students scored more than 62?

The battery charge length of the most recent cell phones is approximately normally distributed with a mean of 12 hours and a standard deviation of 1 hour.

25. What percent of batteries lasted between 11 and 13 hours?

26. What percent of batteries lasted between 10 and 14 hours?

27. What percent of batteries lasted between 9 and 15 hours?

28. What percent of batteries lasted less than 11 hours?

29. What percent of batteries lasted less than 14 hours?

30. What percent of batteries lasted less than 9 hours?

31. What percent of batteries lasted more than 11 hours?

32. What percent of batteries lasted more than 14 hours?

33. What percent of batteries lasted more than 15 hours?

34. What percent of batteries lasted more than 13 hours?

Applications

Solve.

35. Jordan and Alicia each took a standardized test for their respective areas for graduate school. Although they took different tests they want to compare their performances. Jordan scored 800, and his test had a mean of 600 with a standard deviation of 80. Alicia scored 1200, and her test had a mean of 1000 with a standard deviation of 100. Both tests' scores are approximately normally distributed. Who performed better?

36. Willow and Lydia are both taking biology but with different teachers. Although they took different tests they want to compare their performances. Willow scored 86, and her test had a mean of 80 with a standard deviation of 4. Lydia scored 82, and her test had a mean of 78 with a standard deviation of 2. Both tests' scores are approximately normally distributed. Who performed better?

37. Jameson and Marie each took a standardized test for their respective areas for graduate school. Although they took different tests they want to compare their performances. Jameson scored 176, and his test had a mean of 151 with a standard deviation of 10. Marie scored 334, and her test had a mean of 300 with a standard deviation of 17. Both tests' scores are approximately normally distributed. Who performed better?

38. Brian and Isabella are both taking psychology at their respective universities. Although they took different final exams they want to compare their performances. Brian scored 85, and his test had a mean of 80 with a standard deviation of 4. Isabella scored 76, and her test had a mean of 68 with a standard deviation of 4. Both exams' scores are approximately normally distributed. Who performed better?

Writing & Thinking

39. Explain the characteristics of a normal distribution.

40. In your own words, explain the empirical rule.

Therefore, we have $CD = [-61]$.

b. D is a 4×1 matrix and C is a 1×4 matrix. Because the number of columns in D is equal to the number of rows in C, we can multiply D by C. The product DC will be a 4×4 matrix.

Instead of writing out all 16 steps, we will do the work inside of a matrix. (This is possible since each entry is the product of only two numbers. This can get complicated if each entry is the sum of multiple products.)

$$DC = \begin{bmatrix} 5 \\ 7 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & -9 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5(-1) & 5(-9) & 5(-1) & 5 \cdot 2 \\ 7(-1) & 7(-9) & 7(-1) & 7 \cdot 2 \\ -1(-1) & -1(-9) & -1(-1) & -1 \cdot 2 \\ 3(-1) & 3(-9) & 3(-1) & 3 \cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -45 & -5 & 10 \\ -7 & -63 & -7 & 14 \\ 1 & 9 & 1 & -2 \\ -3 & -27 & -3 & 6 \end{bmatrix}$$

Notice that while we can find both products CD and DC, the resulting matrices are not the same.

Now work margin exercise 7.

Margin Exercise Answers

1. a. 3×3 matrix; **b.** 2×5 matrix; **c.** 3×4 matrix **2. a.** $\begin{bmatrix} 6 & -4 \\ -7 & -5 \end{bmatrix}$ **b.** $\begin{bmatrix} 7 & 4 \\ 15 & 8 \\ 13 & 15 \end{bmatrix}$

3. a. The matrices cannot be subtracted; **b.** $\begin{bmatrix} -6 & 19 \\ 3 & -5 \\ 1 & 3 \\ -9 & -2 \end{bmatrix}$ **4. a.** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b.** $\begin{bmatrix} -56 & 56 \\ -32 & -24 \\ -24 & -48 \end{bmatrix}$

5. a. cannot be found **b.** can be found; 4×5 **6.** $AB = \begin{bmatrix} -1 & 37 \\ -8 & -14 \\ 15 & -5 \end{bmatrix}$ **7. a.** $CD = [35]$

b. $DC = \begin{bmatrix} 24 & -12 & 8 & 32 \\ -30 & 15 & -10 & -40 \\ 12 & -6 & 4 & 16 \\ -6 & 3 & -2 & -8 \end{bmatrix}$

A.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A rectangular array of numbers is called a _____ (plural _____).

2. Matrices are usually named with _____ _____, and each number in the matrix is called an _____.

3. Notation with a smaller number to the right and below the variable is called _____ _____.

4. Two matrices A and B can be added as long as they have the _____ _____.

5. Multiplication of a matrix by a _____ _____, or _____, can be compared to the multiplication of a polynomial by a constant.

6. A restriction with multiplying two matrix A by matrix B is that the number of _____ in A must be equal to the number of _____ in B.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The double subscript notation b_{43} is read "b four three".

8. When adding or subtracting two matrices, the matrices must have the same dimensions.

9. The dimensions of a matrix are written as the number of columns by the number of rows.

10. The number of columns in A need to match the number of rows in B in order for the product AB to be found.

Practice

Determine the dimensions of the given matrix. See Example 1.

1. $\begin{bmatrix} -2 & 5 \\ 1 & 3 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -6 & 0 \\ 5 & 3 & -7 \end{bmatrix}$

3. $\begin{bmatrix} 4 & 1 \\ 0 & 9 \\ -3 & 2 \end{bmatrix}$

4. $\begin{bmatrix} 6 & 9 & 3 \end{bmatrix}$

5. $\begin{bmatrix} -6 & 3 & -7 & 9 \\ 4 & 7 & -2 & 3 \end{bmatrix}$

6. $\begin{bmatrix} -2 \\ 0 \\ 6 \\ 1 \\ -8 \end{bmatrix}$

7. $\begin{bmatrix} 0 & -6 & 7 & 4 & 3 \\ 9 & 1 & 0 & 8 & 6 \\ 2 & 4 & 9 & 6 & 4 \end{bmatrix}$

8. $\begin{bmatrix} 1 & -8 & 2 & 4 \\ 0 & 6 & -3 & 1 \\ 0 & 0 & -2 & 9 \\ 0 & 0 & 0 & 7 \end{bmatrix}$

Add, if possible. See Example 2.

9. $A+B=\begin{bmatrix}4 & -3\\ 1 & 7\end{bmatrix}+\begin{bmatrix}-1 & 0\\ 5 & 2\end{bmatrix}$

10. $A+B=\begin{bmatrix}-9\\ 4\end{bmatrix}+\begin{bmatrix}6\\ 5\end{bmatrix}$

11. $C+D=\begin{bmatrix}0 & 3\\ 5 & -4\end{bmatrix}+\begin{bmatrix}-2 & 2\\ 1 & -3\end{bmatrix}$

12. $X+Y=\begin{bmatrix}-8\\ 9\end{bmatrix}+\begin{bmatrix}5 & -7\\ 3 & 4\end{bmatrix}$

13. $X+Y=\begin{bmatrix}4 & -6 & 1\\ -3 & 0 & -5\end{bmatrix}+\begin{bmatrix}0 & 1 & 4\\ 3 & 2 & 6\end{bmatrix}$

14. $C+D=\begin{bmatrix}7 & 3 & 4\\ -2 & 0 & 5\end{bmatrix}+\begin{bmatrix}1 & 6\\ -7 & 9\\ 8 & 8\end{bmatrix}$

15. $A+B=\begin{bmatrix}1 & 0\\ 5 & -3\\ -2 & 7\end{bmatrix}+\begin{bmatrix}-4 & 5\\ 3 & -1\\ -2 & -5\end{bmatrix}$

16. $C+D=\begin{bmatrix}0 & 2 & 8\\ -7 & 1 & -3\\ 4 & 0 & 6\end{bmatrix}+\begin{bmatrix}5 & 0 & -1\\ 4 & 4 & 0\\ 0 & -1 & 3\end{bmatrix}$

Subtract, if possible. See Example 3.

17. $A-B=\begin{bmatrix}2 & 1\\ 5 & 6\end{bmatrix}-\begin{bmatrix}1 & 3\\ 0 & 2\end{bmatrix}$

18. $C-D=\begin{bmatrix}-4\\ 1\end{bmatrix}-\begin{bmatrix}3\\ 6\end{bmatrix}$

19. $X-Y=\begin{bmatrix}9\\ 0\end{bmatrix}-\begin{bmatrix}1 & 2\\ 5 & 0\end{bmatrix}$

20. $A-B=\begin{bmatrix}7 & -2\\ 4 & 1\end{bmatrix}-\begin{bmatrix}-1 & 5\\ 3 & 6\end{bmatrix}$

21. $A-B=\begin{bmatrix}8 & -2 & 4\end{bmatrix}-\begin{bmatrix}0 & 6 & 1\end{bmatrix}$

22. $X-Y=\begin{bmatrix}9 & 2 & 7\\ -3 & 5 & -7\end{bmatrix}-\begin{bmatrix}5 & -4 & 1\\ -6 & 3 & 1\end{bmatrix}$

23. $C-D=\begin{bmatrix}-2 & 0 & 1\\ 5 & -9 & 4\end{bmatrix}-\begin{bmatrix}7 & -3 & 1\end{bmatrix}$

24. $X-Y=\begin{bmatrix}-3 & 0 & 6\\ 5 & -4 & 9\\ -2 & 3 & 1\end{bmatrix}-\begin{bmatrix}1 & -9 & 4\\ 1 & 0 & 8\\ 6 & 5 & 1\end{bmatrix}$

Find the product. See Example 4.

25. $3A=3\begin{bmatrix}4 & 1\\ -2 & 3\end{bmatrix}$

26. $-4B=-4\begin{bmatrix}7 & 2\\ 0 & -1\end{bmatrix}$

27. $9X = 9\begin{bmatrix} 2 \\ -3 \\ -6 \\ 7 \end{bmatrix}$

28. $2Y = 2\begin{bmatrix} 1 & -6 & 5 & 2 \\ 0 & 7 & -4 & 0 \end{bmatrix}$

29. $-6C = -6\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & -1 \end{bmatrix}$

30. $-7A = -7\begin{bmatrix} 7 & -2 & 5 & -9 & -3 \end{bmatrix}$

31. $5X = 5\begin{bmatrix} 0 & 3 & -4 \\ 6 & 1 & 5 \\ -7 & 2 & -3 \end{bmatrix}$

32. $-1B = -1\begin{bmatrix} 7 & -8 & 3 & -6 \\ 1 & -6 & 4 & -6 \\ -4 & 0 & 1 & 2 \\ 9 & -2 & -1 & 3 \end{bmatrix}$

33. Let A be a 3×2 matrix and B be a 2×4 matrix. Determine if the given products can be found. If the product can be found, state the dimensions of the resulting matrix.

a. AB

b. BA

34. Let X be a 5×3 matrix and Y be a 6×5 matrix. Determine if the given products can be found. If the product can be found, state the dimensions of the resulting matrix.

a. XY

b. YX

35. Let C be a 3×4 matrix and D be a 4×3 matrix. Determine if the given products can be found. If the product can be found, state the dimensions of the resulting matrix.

a. CD

b. DC

36. Let A be a 7×2 matrix and B be a 2×3 matrix. Determine if the given products can be found. If the product can be found, state the dimensions of the resulting matrix.

a. AB

b. BA

Find the product AB using the given matrices. See Example 6.

37. $A = \begin{bmatrix} 2 & 7 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 1 \\ 4 & 5 \end{bmatrix}$

38. $A = \begin{bmatrix} -3 & 4 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

39. $A = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \end{bmatrix}$

40. $A=\begin{bmatrix}-6 & 7 & 0\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2\\ 0 & 5\\ -1 & 0\end{bmatrix}$

41. $A=\begin{bmatrix}4 & 1 & -7\\ 8 & 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-4 & 8\\ 1 & 1\\ -5 & 3\end{bmatrix}$

42. $A=\begin{bmatrix}1 & 0\\ -3 & 6\\ 2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}4 & -2\\ 3 & 5\end{bmatrix}$

43. $A=\begin{bmatrix}-1 & 3\\ -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}9 & -8 & 0 & 3\\ -2 & 1 & 7 & 0\end{bmatrix}$

44. $A=\begin{bmatrix}-4 & 1\\ 3 & -2\\ 0 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 5 & 3 & -1\\ 2 & 0 & 6 & 4\end{bmatrix}$

Using the given matrices, find the indicated product, if possible. See Example 7.

$A=\begin{bmatrix}5 & -2\\ -2 & 1\\ 0 & -4\end{bmatrix}$ $B=\begin{bmatrix}6 & 0 & -2\\ 4 & 5 & -2\end{bmatrix}$ $C=\begin{bmatrix}5\\ 3\end{bmatrix}$ $D=\begin{bmatrix}6 & 1 & -8\end{bmatrix}$

45. AB

46. BA

47. AC

48. CD

49. DC

50. DA

Writing & Thinking

51. Describe matrices that cannot be added or subtracted.

52. Is it possible to have to matrices that cannot be added but can be multiplied? Why or why not?

53. For Matrices A and B, what must be true for both AB and BA to exist?

- For the exponential function $y = b^x$,

  the domain is all real x, and

  the range is all $y > 0$. (The graph is above the x-axis.)

  There is a horizontal asymptote at $y = 0$.
- For the logarithmic function $y = \log_b x \left(\text{or } x = b^y\right)$,

  the domain is all $x > 0$, and (The graph is to the right of the y-axis.)

  the range is all real y.

  There is a vertical asymptote at $x = 0$.

Margin Exercise Answers

1. a. $\log_4 4 = 1$ **b.** $\log_4 64 = 3$ **c.** $\log_4\left(\frac{1}{4}\right) = -1$ **d.** $3^2 = 9$ **e.** $3^4 = 81$ **f.** $3^{-1} = \frac{1}{3}$ **2. a.** 0 **b.** 1 **c.** 30 **d.** 6 **e.** −3 **3.** 4 **4.** $\frac{5}{2}$

A.2 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. The function $x = b^y$ is equivalent to $y =$ _____.

2. The line $y = 0$ is the _____ asymptote of $y = b^x$.

3. The inverse of an exponential function is a/an _____ function.

4. Regardless of the base, the logarithm of 1 is _____.

5. The graph of a logarithmic function can be found by _____ the corresponding exponential function across the line $y = x$.

6. The points on the graph of the inverse function can be found by _____ the coordinates of the ordered pairs.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so that the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Exponential functions of the form $y = b^x$ are one-to-one functions and have inverses.

8. The exponent of an exponential function is the base of its inverse logarithmic function.

9. Exponents are logarithms.

10. The logarithm of the base is always 1.

Practice

Express each equation in logarithmic form. See Example 1.

1. $7^2 = 49$

2. $3^3 = 27$

3. $5^{-2} = \frac{1}{25}$

4. $2^{-5} = \frac{1}{32}$

5. $1 = \pi^0$

6. $6^0 = 1$

7. $10^2 = 100$

8. $10^1 = 10$

9. $10^k = 23$

10. $4^k = 11.6$

11. $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

12. $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

Express each equation in exponential form. See Example 1.

13. $\log_3 9 = 2$

14. $\log_5 125 = 3$

15. $\log_9 3 = \frac{1}{2}$

16. $\log_b 4 = \frac{2}{3}$

17. $\log_7 \frac{1}{7} = -1$

18. $\log_{1/2} 8 = -3$

19. $\log_{10} N = 1.74$

20. $\log_2 42.3 = x$

21. $\log_b 18 = 4$

22. $\log_b 39 = 10$

23. $\log_n y^2 = x$

24. $\log_b a = x^2$

Solve by first changing each equation to exponential form. See Examples 3 and 4.

25. $\log_4 x = 2$

26. $\log_3 x = 4$

27. $\log_{14} 196 = x$

28. $\log_{25} 125 = x$

29. $\log_5 \frac{1}{125} = x$

30. $\log_3 \frac{1}{9} = x$

31. $\log_{36} x = -\frac{1}{2}$

32. $\log_{81} x = -\frac{3}{4}$

33. $\log_x 32 = 5$

34. $\log_x 121 = 2$

35. $\log_8 x = \frac{5}{3}$

36. $\log_{16} x = \frac{3}{4}$

37. $\log_8 8^{3.7} = x$

38. $\log_{10} 10^{1.52} = x$

39. $\log_5 5^{\log_5 25} = x$

40. $\log_4 4^{\log_2 8} = x$

Graph each function and its inverse on the same set of axes. Label two points on each graph.

41. $f(x) = 6^x$

42. $f(x) = 2^x$

43. $y = \left(\frac{2}{3}\right)^x$

44. $y = \left(\frac{1}{4}\right)^x$

45. $f(x) = \log_4 x$

46. $f(x) = \log_5 x$

47. $y = \log_{1/2} x$

48. $y = \log_{1/3} x$

49. $y = \log_8 x$

50. $y = \log_7 x$

51. Consider the function $y = c\left(3^x\right)$ where c is a constant greater than zero. List the following:

a. The domain of the function.

b. The range of the function.

c. Any asymptotes of the graph of the function.

d. Give c two different values and sketch the graphs of both functions.

52. Consider the function $y = c\left(3^{-x}\right)$ where c is a constant greater than zero. List the following:

a. The domain of the function.

b. The range of the function.

c. Any asymptotes of the graph of the function.

d. Give c two different values and sketch the graphs of both functions.

Writing & Thinking

53. Discuss, in your own words, the symmetrical relationship of the graphs of the two functions $y = 10^x$ and $y = \log_{10} x$.

54. Discuss, in your own words, the symmetrical relationship of the graphs of the two logarithmic functions $y = \log_{10} x$ and $y = -\log_{10} x$.

Since $T = 120°$ when $t = 5$, substituting these values allows us to find k.

$$\begin{aligned} 120 &= 80e^{-k(5)} + 70 \\ 50 &= 80e^{-5k} \\ \frac{50}{80} &= e^{-5k} \\ \ln\frac{5}{8} &= \ln e^{-5k} \qquad \text{Take the natural log of both sides.} \\ \ln 0.625 &= -5k \\ k &= \frac{\ln 0.625}{-5} \approx \frac{-0.4700}{-5} \approx 0.0940 \end{aligned}$$

The formula can now be written as $T = 80e^{-0.0940t} + 70$.

With all the constants in the formula known, we can find t when $T = 100°$.

$$\begin{aligned} 100 &= 80e^{-0.0940t} + 70 \\ 30 &= 80e^{-0.0940t} \\ \frac{30}{80} &= e^{-0.0940t} \\ \ln\frac{3}{8} &= \ln e^{-0.0940t} \qquad \text{Take the natural log of both sides.} \\ \ln 0.375 &= -0.0940t \\ t &= \frac{\ln 0.375}{-0.0940} \\ &\approx \frac{-0.9808}{-0.0940} \approx 10.43 \text{ minutes} \end{aligned}$$

The tea will cool to 100° in about 10.43 minutes.

Now work margin exercise 6.

Margin Exercise Answers

1. 2.31 days **2.** 27.73 years **3.** $I = 10^{8.2}$ **4.** 2.51 **5.** $y = 12e^{-0.00002876t}$; 9.00 grams
6. $T = 120e^{-0.0811t} + 65$; 8.55 minutes

A.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The formula $A = A_0e^{-0.04t}$ is for the _____ of radium where t is in _____.

2. The formula $A = A_0 2^{-\frac{t}{5600}}$ is used for carbon-14 dating to determine the age of _____, where t is measured in _____.

3. The magnitude of an earthquake is measured on the _____ scale.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

4. In Newton's law of cooling, the variable C is the constant temperature of the medium surrounding the cooling object.

5. The formula $A = A_0 e^{-0.1t}$ is used for skin healing, where t is measured in hours.

Applications

Solve. If necessary, round answers to the nearest hundredth (unless otherwise specified).

1. If Kim invests \$2000 at a rate of 7% compounded continuously, what will be her balance after 10 years?

2. Find the amount of money that will be accumulated in a savings account if \$3200 is invested at 6.5% for 6 years and the interest is compounded continuously.

3. Four thousand dollars is invested at 6% compounded continuously. How long will it take for the balance to be \$8000?

4. How long does it take \$1000 to double if it is invested at 5% compounded continuously?

5. The reliability of a certain type of flashlight battery is given by $f = e^{-0.03x}$, where f is the fractional part of the batteries produced that last x hours. What fraction of the batteries produced are good after 40 hours of use?

6. From Exercise 5, how long will at least one-half of the batteries last?

7. The concentration of a drug in the blood stream is given by $C = C_0 e^{-0.8t}$, where C_0 is the initial dosage and t is the time in hours elapsed after administering the dose. If 20 mg of the drug is given, how much time elapses until 5 mg of the drug remains?

8. Using the formula in Exercise 7, determine the amount of the drug present after 3 hours if 0.60 mg is given.

9. One law for skin healing is $A = A_0 e^{-0.1t}$, where A is the number of cm^2 of unhealed area after t days and A_0 is the number of cm^2 of the original wound. Find the number of days needed to reduce the wound to one-third the original size.

10. A swarm of bees grows according to the formula $P = P_0 e^{0.35t}$, where P_0 is the number present initially and t is the time in days. How many bees will be present in 6 days if there were 1000 present initially? (Round to the nearest integer.)

11. If inversion of raw sugar is given by $A = A_0 e^{-0.03t}$, where A_0 is the initial amount and t is the time in hours, how long will it take for 1000 lb of raw sugar to be reduced to 800 lb?

12. Atmospheric pressure P is related to the altitude, h by the formula $P = P_0 e^{-0.00004h}$, where P_0 the pressure at sea level, is approximately 15 lb per in.^{2} Determine the pressure at 5000 in.

13. A radioactive substance decays according to $A = A_0 e^{-0.0002t}$, where A_0 is the initial amount and t is the time in years. If $A_0 = 640$ grams, find the time for A to decay to 400 grams.

14. A substance decays according to $A = A_0 e^{-0.045t}$, where t is in hours and A_0 is the initial amount. Determine the half-life of the substance.

15. An employee is learning to assemble remote-control units. The number of units per day he can assemble after t days of intensive training is given by $N = 80\left(1 - e^{-0.3t}\right)$. How many days of training will be needed before the employee is able to assemble 40 units per day?

16. A scientist collects a lava sample and measures that its temperature is 1650°. To safely analyze the sample, it must be no warmer than 500°. The scientist stores the sample in a cooling chamber with a temperature of 50° and finds that in 2 hours, the lava has cooled to 1000°. When will the lava sample be safe to analyze?

17. The temperature of a carrot cake is 350° when it is removed from the oven. The temperature in the room is 72°. In 10 minutes, the cake cools to 280°. How long will it take for the cake to cool to 160°?

18. How long does it take \$10,000 to double if it is invested at 8% compounded quarterly?

19. If \$1000 is deposited at 6% compounded monthly, how long before the balance is \$1520?

20. The value V of a machine at the end of t years is given by $V = C\left(1 - r\right)^t$, where C is the original cost of the machine and r is the rate of depreciation. A machine that originally cost \$12,000 is now valued at \$3800. How old is the machine if $r = 0.12$?

21. The formula $A = A_0 2^{-\frac{t}{5600}}$ is used for carbon–14 dating to determine the age of fossils where t is measured in years. Determine the half-life of carbon–14.

22. Radioactive iodine has a half-life of 60 days. If an accident occurs at a nuclear plant and 30 grams of radioactive iodine are present, in how many days will 1 gram be present? (Round k to at least 7 decimal places.)

23. If a principal P is doubled, then $A = 2P$. Use the formula for continuously compounded interest to find the time it takes the principal to double in value if the rate of interest is **a.** 5% **b.** 10% (Note that the time for doubling the principal is completely independent of the principal itself.)

24. If a principal P is tripled, then $A = 3P$. Use the formula for continuously compounded interest to find the time it takes the principal to triple in value if the interest rate is **a.** 4% **b.** 8% (Note that the time for tripling the principal is completely independent of the principal itself.)

25. The 1906 earthquake in San Francisco measured 8.6 on the Richter scale. In 1971, an earthquake in the San Fernando Valley measured 6.6 on the Richter scale. How many times greater was the 1906 earthquake than the 1971 earthquake? (See Example 4.)

26. In 1985, an earthquake in Mexico measured 8.1 on the Richter scale. How many times greater was this earthquake than the one in Landers, California in 1992 that measured 7.3 on the Richter scale? (See Example 4.)

27. Population does not generally grow in a linear fashion. In fact, the population of many species grows exponentially, at least for a limited time. Using the exponential model $y = y_0 e^{kt}$ for population growth, estimate the population of a state in 2020 if the population was 5 million in 1990 and 6 million in 2000. (Assume that t is measured in years and $t = 0$ corresponds to 1990.)

28. Suppose that a lake is stocked with 500 fish, and biologists predict that the population of these fish will be approximated by the function $P(t) = 500\ln(2t + e)$ where t is measured in years. What will the fish population be in 3 years? in 5 years? in 10 years? (Round answers to the nearest integer.)

29. Sales representatives of a new type of computer predict that sales can be approximated by the function $S(t) = 1000 + 500\ln(3t + e)$ where t is measured in years. What are the predicted sales in 2 years? in 5 years? in 10 years? Round to the nearest integer.

30. In chemistry, the pH of a solution is a measure of the acidity or alkalinity of a solution. Water has a pH of 7 and, in general, acids have a pH less than 7 and alkaline solutions have a pH greater than 7. The model for pH is $\text{pH} = -\log\left[\text{H}^+\right]$ where $\left[\text{H}^+\right]$ is the hydrogen ion concentration in moles per liter of a solution.

a. Find the pH of a solution with a hydrogen ion concentration of 8.6×10^{-7}.

b. Find the hydrogen ion concentration $\left[\text{H}^+\right]$ of a solution if the pH of the solution is 4.5. Write the answer in scientific notation.

31. Sound levels: A decibel (abbreviated dB) is a unit used to measure the loudness of sound. The decibel level D of a sound of intensity I is measured by comparing it to a barely audible sound of intensity I_0 with the following formula: $D = 10\log\left(\frac{I}{I_0}\right)$. Sounds measuring over 85 dB are not considered safe.

a. Find the decibel level of a rock concert with an intensity of $6.24 \times 10^{11} I_0$.

b. What is the intensity level of 85 dB?

c. What is the intensity level of 60 dB (normal conversation)?