Since $T = 120°$ when $t = 5$, substituting these values allows us to find k.

$$\begin{aligned} 120 &= 80e^{-k(5)} + 70 \\ 50 &= 80e^{-5k} \\ \frac{50}{80} &= e^{-5k} \\ \ln\frac{5}{8} &= \ln e^{-5k} \qquad \text{Take the natural log of both sides.} \\ \ln 0.625 &= -5k \\ k &= \frac{\ln 0.625}{-5} \approx \frac{-0.4700}{-5} \approx 0.0940 \end{aligned}$$

The formula can now be written as $T = 80e^{-0.0940t} + 70$.

With all the constants in the formula known, we can find t when $T = 100°$.

$$\begin{aligned} 100 &= 80e^{-0.0940t} + 70 \\ 30 &= 80e^{-0.0940t} \\ \frac{30}{80} &= e^{-0.0940t} \\ \ln\frac{3}{8} &= \ln e^{-0.0940t} \qquad \text{Take the natural log of both sides.} \\ \ln 0.375 &= -0.0940t \\ t &= \frac{\ln 0.375}{-0.0940} \\ &\approx \frac{-0.9808}{-0.0940} \approx 10.43 \text{ minutes} \end{aligned}$$

The tea will cool to 100° in about 10.43 minutes.

Now work margin exercise 6.

Margin Exercise Answers

1. 2.31 days **2.** 27.73 years **3.** $I = 10^{8.2}$ **4.** 2.51 **5.** $y = 12e^{-0.00002876t}$; 9.00 grams
6. $T = 120e^{-0.0811t} + 65$; 8.55 minutes

A.3 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The formula $A = A_0e^{-0.04t}$ is for the _____ of radium where t is in _____.

2. The formula $A = A_0 2^{-\frac{t}{5600}}$ is used for carbon-14 dating to determine the age of _____, where t is measured in _____.

3. The magnitude of an earthquake is measured on the _____ scale.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

4. In Newton's law of cooling, the variable C is the constant temperature of the medium surrounding the cooling object.

5. The formula $A = A_0 e^{-0.1t}$ is used for skin healing, where t is measured in hours.

Applications

Solve. If necessary, round answers to the nearest hundredth (unless otherwise specified).

1. If Kim invests \$2000 at a rate of 7% compounded continuously, what will be her balance after 10 years?

2. Find the amount of money that will be accumulated in a savings account if \$3200 is invested at 6.5% for 6 years and the interest is compounded continuously.

3. Four thousand dollars is invested at 6% compounded continuously. How long will it take for the balance to be \$8000?

4. How long does it take \$1000 to double if it is invested at 5% compounded continuously?

5. The reliability of a certain type of flashlight battery is given by $f = e^{-0.03x}$, where f is the fractional part of the batteries produced that last x hours. What fraction of the batteries produced are good after 40 hours of use?

6. From Exercise 5, how long will at least one-half of the batteries last?

7. The concentration of a drug in the blood stream is given by $C = C_0 e^{-0.8t}$, where C_0 is the initial dosage and t is the time in hours elapsed after administering the dose. If 20 mg of the drug is given, how much time elapses until 5 mg of the drug remains?

8. Using the formula in Exercise 7, determine the amount of the drug present after 3 hours if 0.60 mg is given.

9. One law for skin healing is $A = A_0 e^{-0.1t}$, where A is the number of cm^2 of unhealed area after t days and A_0 is the number of cm^2 of the original wound. Find the number of days needed to reduce the wound to one-third the original size.

10. A swarm of bees grows according to the formula $P = P_0 e^{0.35t}$, where P_0 is the number present initially and t is the time in days. How many bees will be present in 6 days if there were 1000 present initially? (Round to the nearest integer.)

11. If inversion of raw sugar is given by $A = A_0 e^{-0.03t}$, where A_0 is the initial amount and t is the time in hours, how long will it take for 1000 lb of raw sugar to be reduced to 800 lb?

12. Atmospheric pressure P is related to the altitude, h by the formula $P = P_0 e^{-0.00004h}$, where P_0 the pressure at sea level, is approximately 15 lb per in.^{2} Determine the pressure at 5000 in.

13. A radioactive substance decays according to $A = A_0 e^{-0.0002t}$, where A_0 is the initial amount and t is the time in years. If $A_0 = 640$ grams, find the time for A to decay to 400 grams.

14. A substance decays according to $A = A_0 e^{-0.045t}$, where t is in hours and A_0 is the initial amount. Determine the half-life of the substance.

15. An employee is learning to assemble remote-control units. The number of units per day he can assemble after t days of intensive training is given by $N = 80\left(1 - e^{-0.3t}\right)$. How many days of training will be needed before the employee is able to assemble 40 units per day?

16. A scientist collects a lava sample and measures that its temperature is 1650°. To safely analyze the sample, it must be no warmer than 500°. The scientist stores the sample in a cooling chamber with a temperature of 50° and finds that in 2 hours, the lava has cooled to 1000°. When will the lava sample be safe to analyze?

17. The temperature of a carrot cake is 350° when it is removed from the oven. The temperature in the room is 72°. In 10 minutes, the cake cools to 280°. How long will it take for the cake to cool to 160°?

18. How long does it take \$10,000 to double if it is invested at 8% compounded quarterly?

19. If \$1000 is deposited at 6% compounded monthly, how long before the balance is \$1520?

20. The value V of a machine at the end of t years is given by $V = C\left(1 - r\right)^t$, where C is the original cost of the machine and r is the rate of depreciation. A machine that originally cost \$12,000 is now valued at \$3800. How old is the machine if $r = 0.12$?

21. The formula $A = A_0 2^{-\frac{t}{5600}}$ is used for carbon–14 dating to determine the age of fossils where t is measured in years. Determine the half-life of carbon–14.

22. Radioactive iodine has a half-life of 60 days. If an accident occurs at a nuclear plant and 30 grams of radioactive iodine are present, in how many days will 1 gram be present? (Round k to at least 7 decimal places.)

23. If a principal P is doubled, then $A = 2P$. Use the formula for continuously compounded interest to find the time it takes the principal to double in value if the rate of interest is **a.** 5% **b.** 10% (Note that the time for doubling the principal is completely independent of the principal itself.)

24. If a principal P is tripled, then $A = 3P$. Use the formula for continuously compounded interest to find the time it takes the principal to triple in value if the interest rate is **a.** 4% **b.** 8% (Note that the time for tripling the principal is completely independent of the principal itself.)

25. The 1906 earthquake in San Francisco measured 8.6 on the Richter scale. In 1971, an earthquake in the San Fernando Valley measured 6.6 on the Richter scale. How many times greater was the 1906 earthquake than the 1971 earthquake? (See Example 4.)

26. In 1985, an earthquake in Mexico measured 8.1 on the Richter scale. How many times greater was this earthquake than the one in Landers, California in 1992 that measured 7.3 on the Richter scale? (See Example 4.)

27. Population does not generally grow in a linear fashion. In fact, the population of many species grows exponentially, at least for a limited time. Using the exponential model $y = y_0 e^{kt}$ for population growth, estimate the population of a state in 2020 if the population was 5 million in 1990 and 6 million in 2000. (Assume that t is measured in years and $t = 0$ corresponds to 1990.)

28. Suppose that a lake is stocked with 500 fish, and biologists predict that the population of these fish will be approximated by the function $P(t) = 500\ln(2t + e)$ where t is measured in years. What will the fish population be in 3 years? in 5 years? in 10 years? (Round answers to the nearest integer.)

29. Sales representatives of a new type of computer predict that sales can be approximated by the function $S(t) = 1000 + 500\ln(3t + e)$ where t is measured in years. What are the predicted sales in 2 years? in 5 years? in 10 years? Round to the nearest integer.

30. In chemistry, the pH of a solution is a measure of the acidity or alkalinity of a solution. Water has a pH of 7 and, in general, acids have a pH less than 7 and alkaline solutions have a pH greater than 7. The model for pH is $\text{pH} = -\log\left[\text{H}^+\right]$ where $\left[\text{H}^+\right]$ is the hydrogen ion concentration in moles per liter of a solution.

a. Find the pH of a solution with a hydrogen ion concentration of 8.6×10^{-7}.

b. Find the hydrogen ion concentration $\left[\text{H}^+\right]$ of a solution if the pH of the solution is 4.5. Write the answer in scientific notation.

31. Sound levels: A decibel (abbreviated dB) is a unit used to measure the loudness of sound. The decibel level D of a sound of intensity I is measured by comparing it to a barely audible sound of intensity I_0 with the following formula: $D = 10\log\left(\frac{I}{I_0}\right)$. Sounds measuring over 85 dB are not considered safe.

a. Find the decibel level of a rock concert with an intensity of $6.24 \times 10^{11} I_0$.

b. What is the intensity level of 85 dB?

c. What is the intensity level of 60 dB (normal conversation)?