

Therefore, we have $CD = [-61]$.

- b. D is a 4×1 matrix and C is a 1×4 matrix. Because the number of columns in D is equal to the number of rows in C , we can multiply D by C . The product DC will be a 4×4 matrix.

Instead of writing out all 16 steps, we will do the work inside of a matrix. (This is possible since each entry is the product of only two numbers. This can get complicated if each entry is the sum of multiple products.)

$$\begin{aligned} DC &= \begin{bmatrix} 5 \\ 7 \\ -1 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & -9 & -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 5(-1) & 5(-9) & 5(-1) & 5 \cdot 2 \\ 7(-1) & 7(-9) & 7(-1) & 7 \cdot 2 \\ -1(-1) & -1(-9) & -1(-1) & -1 \cdot 2 \\ 3(-1) & 3(-9) & 3(-1) & 3 \cdot 2 \end{bmatrix} \\ &= \begin{bmatrix} -5 & -45 & -5 & 10 \\ -7 & -63 & -7 & 14 \\ 1 & 9 & 1 & -2 \\ -3 & -27 & -3 & 6 \end{bmatrix} \end{aligned}$$

Notice that while we can find both products CD and DC , the resulting matrices are not the same.

Now work margin exercise 7.

Margin Exercise Answers

1. a. 3×3 matrix; b. 2×5 matrix; c. 3×4 matrix 2. a. $\begin{bmatrix} 6 & -4 \\ -7 & -5 \end{bmatrix}$ b. $\begin{bmatrix} 7 & 4 \\ 15 & 8 \\ 13 & 15 \end{bmatrix}$

3. a. The matrices cannot be subtracted; b. $\begin{bmatrix} -6 & 19 \\ 3 & -5 \\ 1 & 3 \\ -9 & -2 \end{bmatrix}$ 4. a. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} -56 & 56 \\ -32 & -24 \\ -24 & -48 \end{bmatrix}$

5. a. cannot be found b. can be found; 4×5 6. $AB = \begin{bmatrix} -1 & 37 \\ -8 & -14 \\ 15 & -5 \end{bmatrix}$ 7. a. $CD = [35]$

b. $DC = \begin{bmatrix} 24 & -12 & 8 & 32 \\ -30 & 15 & -10 & -40 \\ 12 & -6 & 4 & 16 \\ -6 & 3 & -2 & -8 \end{bmatrix}$

A.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A rectangular array of numbers is called a _____ (plural _____).

2. Matrices are usually named with _____, and each number in the matrix is called an _____.
3. Notation with a smaller number to the right and below the variable is called _____.
4. Two matrices A and B can be added as long as they have the _____.
5. Multiplication of a matrix by a _____, or _____, can be compared to the multiplication of a polynomial by a constant.
6. A restriction with multiplying two matrix A by matrix B is that the number of _____ in A must be equal to the number of _____ in B .

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The double subscript notation b_{43} is read “ b four three”.
8. When adding or subtracting two matrices, the matrices must have the same dimensions.
9. The dimensions of a matrix are written as the number of columns by the number of rows.
10. The number of columns in A need to match the number of rows in B in order for the product AB to be found.

Practice

Determine the dimensions of the given matrix. See Example 1.

1. $\begin{bmatrix} -2 & 5 \\ 1 & 3 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -6 & 0 \\ 5 & 3 & -7 \end{bmatrix}$

3. $\begin{bmatrix} 4 & 1 \\ 0 & 9 \\ -3 & 2 \end{bmatrix}$

4. $[6 \ 9 \ 3]$

5. $\begin{bmatrix} -6 & 3 & -7 & 9 \\ 4 & 7 & -2 & 3 \end{bmatrix}$

6. $\begin{bmatrix} -2 \\ 0 \\ 6 \\ 1 \\ -8 \end{bmatrix}$

7. $\begin{bmatrix} 0 & -6 & 7 & 4 & 3 \\ 9 & 1 & 0 & 8 & 6 \\ 2 & 4 & 9 & 6 & 4 \end{bmatrix}$

8. $\begin{bmatrix} 1 & -8 & 2 & 4 \\ 0 & 6 & -3 & 1 \\ 0 & 0 & -2 & 9 \\ 0 & 0 & 0 & 7 \end{bmatrix}$

Add, if possible. See Example 2.

$$9. A+B = \begin{bmatrix} 4 & -3 \\ 1 & 7 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 5 & 2 \end{bmatrix}$$

$$10. A+B = \begin{bmatrix} -9 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$11. C+D = \begin{bmatrix} 0 & 3 \\ 5 & -4 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix}$$

$$12. X+Y = \begin{bmatrix} -8 \\ 9 \end{bmatrix} + \begin{bmatrix} 5 & -7 \\ 3 & 4 \end{bmatrix}$$

$$13. X+Y = \begin{bmatrix} 4 & -6 & 1 \\ -3 & 0 & -5 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 4 \\ 3 & 2 & 6 \end{bmatrix}$$

$$14. C+D = \begin{bmatrix} 7 & 3 & 4 \\ -2 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -7 & 9 \\ 8 & 8 \end{bmatrix}$$

$$15. A+B = \begin{bmatrix} 1 & 0 \\ 5 & -3 \\ -2 & 7 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 3 & -1 \\ -2 & -5 \end{bmatrix}$$

$$16. C+D = \begin{bmatrix} 0 & 2 & 8 \\ -7 & 1 & -3 \\ 4 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 5 & 0 & -1 \\ 4 & 4 & 0 \\ 0 & -1 & 3 \end{bmatrix}$$

Subtract, if possible. See Example 3.

$$17. A-B = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$$

$$18. C-D = \begin{bmatrix} -4 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$19. X-Y = \begin{bmatrix} 9 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}$$

$$20. A-B = \begin{bmatrix} 7 & -2 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 5 \\ 3 & 6 \end{bmatrix}$$

$$21. A-B = [8 \ -2 \ 4] - [0 \ 6 \ 1]$$

$$22. X-Y = \begin{bmatrix} 9 & 2 & 7 \\ -3 & 5 & -7 \end{bmatrix} - \begin{bmatrix} 5 & -4 & 1 \\ -6 & 3 & 1 \end{bmatrix}$$

$$23. C-D = \begin{bmatrix} -2 & 0 & 1 \\ 5 & -9 & 4 \end{bmatrix} - [7 \ -3 \ 1]$$

$$24. X-Y = \begin{bmatrix} -3 & 0 & 6 \\ 5 & -4 & 9 \\ -2 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -9 & 4 \\ 1 & 0 & 8 \\ 6 & 5 & 1 \end{bmatrix}$$

Find the product. See Example 4.

$$25. 3A = 3 \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}$$

$$26. -4B = -4 \begin{bmatrix} 7 & 2 \\ 0 & -1 \end{bmatrix}$$

$$27. 9X = 9 \begin{bmatrix} 2 \\ -3 \\ -6 \\ 7 \end{bmatrix}$$

$$28. 2Y = 2 \begin{bmatrix} 1 & -6 & 5 & 2 \\ 0 & 7 & -4 & 0 \end{bmatrix}$$

$$29. -6C = -6 \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & -1 \end{bmatrix}$$

$$30. -7A = -7[7 \quad -2 \quad 5 \quad -9 \quad -3]$$

$$31. 5X = 5 \begin{bmatrix} 0 & 3 & -4 \\ 6 & 1 & 5 \\ -7 & 2 & -3 \end{bmatrix}$$

$$32. -1B = -1 \begin{bmatrix} 7 & -8 & 3 & -6 \\ 1 & -6 & 4 & -6 \\ -4 & 0 & 1 & 2 \\ 9 & -2 & -1 & 3 \end{bmatrix}$$

33. Let A be a 3×2 matrix and B be a 2×4 matrix. Determine if the given products can be found. If the product can be found, state the dimensions of the resulting matrix.

a. AB

b. BA

34. Let X be a 5×3 matrix and Y be a 6×5 matrix. Determine if the given products can be found. If the product can be found, state the dimensions of the resulting matrix.

a. XY

b. YX

35. Let C be a 3×4 matrix and D be a 4×3 matrix. Determine if the given products can be found. If the product can be found, state the dimensions of the resulting matrix.

a. CD

b. DC

36. Let A be a 7×2 matrix and B be a 2×3 matrix. Determine if the given products can be found. If the product can be found, state the dimensions of the resulting matrix.

a. AB

b. BA

Find the product AB using the given matrices. See Example 6.

$$37. A = \begin{bmatrix} 2 & 7 \\ -1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 1 \\ 4 & 5 \end{bmatrix}$$

$$38. A = \begin{bmatrix} -3 & 4 \\ 0 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$39. A = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \text{ and } B = [4 \quad 2]$$

$$40. A = \begin{bmatrix} -6 & 7 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -2 \\ 0 & 5 \\ -1 & 0 \end{bmatrix}$$

$$41. A = \begin{bmatrix} 4 & 1 & -7 \\ 8 & 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 & 8 \\ 1 & 1 \\ -5 & 3 \end{bmatrix}$$

$$42. A = \begin{bmatrix} 1 & 0 \\ -3 & 6 \\ 2 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & -2 \\ 3 & 5 \end{bmatrix}$$

$$43. A = \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 9 & -8 & 0 & 3 \\ -2 & 1 & 7 & 0 \end{bmatrix}$$

$$44. A = \begin{bmatrix} -4 & 1 \\ 3 & -2 \\ 0 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 5 & 3 & -1 \\ 2 & 0 & 6 & 4 \end{bmatrix}$$

Using the given matrices, find the indicated product, if possible. See Example 7.

$$A = \begin{bmatrix} 5 & -2 \\ -2 & 1 \\ 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 0 & -2 \\ 4 & 5 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \quad D = \begin{bmatrix} 6 & 1 & -8 \end{bmatrix}$$

$$45. AB$$

$$48. CD$$

$$46. BA$$

$$49. DC$$

$$47. AC$$

$$50. DA$$

Writing & Thinking

51. Describe matrices that cannot be added or subtracted.

52. Is it possible to have two matrices that cannot be added but can be multiplied? Why or why not?

53. For Matrices A and B , what must be true for both AB and BA to exist?