6. A Senate committee of 8 members must be chosen with 4 Democrats and 4 Republicans. The senators eligible to be on this committee are 12 Democrats and 9 Republicans. How many possible ways can such a committee be formed?

Example 6 Calculating Combinations

A Senate committee of 6 members must be chosen with 3 Democrats and 3 Republicans. The senators eligible to be on this committee are 10 Democrats and 8 Republicans. How many possible ways can such a committee be formed?

Solution

First we need to determine the number of possible ways Democrats can be chosen and the number of possible ways the Republicans can be chosen.

Possible groups of 3 Democrats: $${}_{10}C_3 = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \cdot 2 \cdot 1 \cdot 7!} = 120$$

Possible groups of 3 Republicans: $${}_{8}C_3 = \frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!} = 56$$

Now we can find the total possible ways the Senate committee can be formed using the fundamental counting principle.

$${}_{10}C_3 \cdot {}_{8}C_3 = 120 \cdot 56 = 6720$$

There are 6720 possible ways that this Senate committee can be formed.

Now work margin exercise 6.

Margin Exercise Answers

1. a. Since the place in the race affects the size of the prize, the order is important. This is a permutation problem. **b.** An order in which the family puts the bags into the trunk is not important. This is a combination problem. **c.** Since it matters which of your friends will receive which gift, the order is important. This is a permutation problem. **2.** 630 **3.** 2,035,800 **4.** 495 **5. a.** 3024 **b.** 126 **6.** 62,370

9.5 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentence using information found in this chapter.

1. For the letters a, b, and c, the _____ abc is different from the _____ cab.

2. For the letters a, b, and c, the _____ abc is the same as the _____ cab.

3. Each combination of r elements has _____ corresponding permutations.

4. Determining how many different 4-topping pizzas you can order is a _____ problem.

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5. Determining how many passwords you can create from 8 characters is a _____ problem.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. For permutations, the order of the elements is important. In a combination, the order doesn't matter.

7. The number of combinations of n elements taken r at a time is greater than the number of permutations of n elements taken r at a time.

8. The total number of combinations of n elements taken r at a time is symbolized ${}_rC_n$.

9. Determining how many ways a hand of cards can be dealt is a combination problem.

Practice

Determine whether each situation describes a combination or a permutation. See Example 1.

1. Ten people will be randomly chosen as winners from 2000 lottery participants. How many different winners can be selected?

2. George came to a store to buy some juice. He sees that the store sells 8 different types of juice of his favorite brand. If he wants to buy 3 different bottles of juice, how many options does he have?

3. When presented with a choice of multiple delicious-looking dishes, some people find it hard to decide what to try first. If you see 6 different dishes and want to try all of them once, in how many orders can you try the dishes?

4. To get to the next destination, a group of tourists must split into groups to travel in several cars. If the first car has six passenger seats, how many ways are there to choose the people who will sit in this car?

5. In an outdoor scavenger hunt, all participants must visit 8 of the 10 check points and complete the respective task, before going to the finish line. The order in which the check points are visited is decided by the participants. How many different routes can the participants choose from?

6. The panel on a lock has ten buttons numbered from 0 to 9. To open the lock, you need to press three of the buttons simultaneously. How many different combinations can be set for this type of lock?

7. Jessica decided to spend Saturday night watching horror movies with her two friends. If she plans to watch 3 different movies from her collection of 15 movies, how many different schedules can she create?

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8. A survey was conducted among the regular customers of a cafeteria asking them to name their favorite and second favorite sandwiches from the menu. How many different answers can the customers give?

9. On a drawing lesson, children are asked to choose three different colors from the palette and mix them together. How many different choices can the children make?

10. To pass the assessment, a student must complete 10 out of 12 tasks given. How many ways are there for the student to choose these 10 tasks?

Evaluate the given combination.

11. ${}_6C_2$

12. ${}_7C_3$

13. ${}_{10}C_2$

14. ${}_{12}C_9$

15. ${}_{15}C_1$

16. ${}_{10}C_{10}$

17. ${}_{17}C_{16}$

18. ${}_{20}C_0$

Applications

Solve.

19. An ice cream shop sells 9 different flavors of ice cream. How many different two-scoop ice creams can be bought at this shop, if the order of the scoops does not matter?

20. Before to moving into a new apartment, Steven decided to change the wallpaper in 6 of its rooms. After starting to hang new wallpaper, he discovered that the amount of wallpaper he purchased is only enough to cover the walls in 4 rooms. How many ways can he choose the 4 rooms he will finish first, before going to buy more wallpaper?

21. A farmer wants to choose 3 sheep to take to a livestock show. If he has 10 sheep that are worthy of being exhibited at the show, how many different groups of sheep can he choose?

22. Irene wants to try folding several origami models to decorate her table. She has origami paper of 12 different colors, and she chooses 4 of the colors for her work. How many different color choices can she make?

23. David inherited some money and decided to use it to buy shares in a company. After consulting with an investment advisor, he received a list of 15 companies whose shares are expected to bring the most dividends in the near future. If David decides to invest in 3 of the companies from the list, how many different options would he have to choose from?

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24. Allison wants to buy bagels for her office this morning. The local bagel store has 16 different varieties of bagels. If there are 12 people on her team and everyone would want a different type of bagel, how many ways can she select 12 bagels?

25. After finishing an escape room, you are given a choice of 20 identical chests, each with a different reward. You earned 4 keys from the escape room puzzles that can open any of the reward chests. How many different sets of rewards can you get by using the keys to open four chests?

26. To set up camp during a hiking trip, a group of hikers select 6 people to be responsible for pitching the tents. If there are 20 people in the group, in how many ways can the 6 people be selected?

27. There are 4 players in each game in a mahjong competition. If 50 people participate in the competition, in how many ways can the participants of the final game be chosen from them?

28. A tradition calls for each decorated cart during the harvest festival to be pulled by 6 horses. If a community owns 25 horses, how many ways do they have to choose the horses for their cart?

29. At the beginning of one type of domino game, each player draws a hand of 7 domino tiles from the shuffled set of 28 tiles. How many different starting hands are there?

30. As an employee of a certain company, Helen can visit the local swimming pool at half price 10 times a month. In how many ways can she choose 10 days in September to use this discount? Note that September has 30 days.

31. A windowsill in Angela's office is currently empty and is long enough to hold 4 flowerpots. After scouring the nearby shop, she finds 10 plants that she likes, and wants to buy 4 of them.

a. Determine the number of different options she can choose from.

b. While choosing the four out of ten plants, Angela imagines their arrangement on the sill. Determine the number of different arrangements she can choose from.

32. A person new to streaming video game plans a small giveaway with 3 cash prizes among his 50 subscribers.

a. Determine the number of possible outcomes for the giveaway if all prizes are different.

b. Determine the number of possible outcomes if all prizes are the same.

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33. Before being put into a storage locker, 4 boxes of files are each labeled with a unique capital English letter.

a. Determine the number of ways to choose the four letters to label the boxes if the boxes contain the same type of information.

b. If each box contains a different type of information and the letter identifies the contents within the box, determine the number of ways to label the boxes.

34. Five students are selected to attend a conference from a 30-student debate class.

a. Determine the number of ways to choose the representatives if all of them have equal status at the conference.

b. Determine the number of ways to choose the representatives if each of them has a unique role to perform at the conference.

35. In a biathlon mixed relay, each team is composed of two women and two men. The national team consists of 9 women and 10 men, all of whom are skilled in both the cross-country skiing and the rifles shooting portions of the race. How many ways can the coaching staff select the participants for the relay?

36. Thomas wants to pack 4 T-shirts and 2 pairs of pants to take on vacation. In his wardrobe, he has 9 different T-shirts and 5 pairs of pants. How many different options does he have to choose from?

37. Amanda has decided to plant three types of berry bushes and two different fruit trees in her yard. She can choose from 7 different types of bushes and 8 types of fruit trees. How many different options can she consider?

38. A puppeteer is selecting 3 male puppets and 4 female puppets for a puppet show at a children's party. If she has 12 male puppets and 16 female puppets, how many different puppet combinations does she have to choose from?

39. For a game night with his friends, Kevin decided to order 5 different pizzas and 3 different salads from local pizza parlor. If the pizza parlor offers 15 types of pizza and 10 different salads, how many different orders could he make?

40. Jennifer has a collection of 15 different teacups and 20 different teaspoons. Three of her friends plan to join her for afternoon tea, so she chooses 4 teacups and 4 teaspoons to use. How many options does she have to choose from?

41. The school board announces that three best students of each class in the elementary school will be given an award. Consider two classes from this school. If one class has 25 students and the other has 30 students, in how many ways can the group of students receiving the rewards be selected from these two classes?

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42. To play the local lottery, you must select 4 even numbers and 4 odd numbers from 1 to 50. You win if the selected numbers match the numbers drawn during the evening news. How many different selections can be made to play the lottery?

43. Two rival schools compete over which has the best swimmers. Each school selects 5 people from its swim team for a competition. If the swim team of the first school has 20 people and the swim team of the second school has 22 people, how many different ways can the participants of the competition be chosen?

44. At the beginning of a tabletop game, each player draws 4 cards from each of the two specialized decks of cards. If there are 20 cards in the first deck and 25 cards in the second deck, how many different hands can be drawn by the first player?

45. A teacher is creating a test and wants to include two assigned exercises from each section of the most recent chapter. If the chapter consists of three sections and she assigned 10 exercises from the first section, 8 from the second section, and 11 from the third section, how many different tests can she make? Assume that the order of questions in the test does not matter.

46. For an exhibit on ancient sculpture styles, a museum wants to display 3 Greek sculptures, 5 Roman sculptures, and 4 Egyptian sculptures. The museum currently has 12 sculptures from Greece, 15 from Rome, and 13 from Egypt. How many different exhibits can they create?

Writing & Thinking

47. Suppose you have two groups of things. You select three different things from the first group and pair them with three different things from the second group, thus obtaining three pairs. How can you find the number of different sets of three pairs that can be obtained in such a way?

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