9.4 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. Tree diagrams get their name because of the way they _____.
2. In general, the total number of ways several events can occur in a given order is found by _____ the number of outcomes of each event together.
3. A permutation is an _____ of elements of a set where the order matters.
4. For any positive integer n, the factorial, denoted as $n!$, is the product of all positive integers from _____ through _____.
5. A group of n elements can be arranged in _____ ways.
6. The symbol ${}_nP_r$ denotes the number of permutations of _____ elements taken _____ at a time.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. For two events, if there are m possible outcomes for the first event and n possible outcomes for the second event, then there are $m + n$ ways for the two events to occur in the given order.
8. The product of the positive integers from 5 to 1 can be represented by 5!.
9. Each ordering of a set of elements if called a factorial.
10. 0! is undefined.

Practice

Use the fundamental counting principle to answer the questions. See Example 1.

1. To prepare for a game, Alan needs to make several red, yellow, and green cards with numbers 1 through 9 written on them. He needs one card for each color and number combination. How many cards should Alan make?
2. When rolling a 6-sided die 3 times, how many different sequences of outcomes can be observed?
3. Peter is furnishing his office and can choose from 3 desks, 5 chairs, and 3 bookcases. How many different options does he have for a furniture set made of a desk, a chair, and a bookcase?

4. Christmas is approaching fast, and Susan decides to decorate her Christmas tree. She has 5 different sets of glass ornaments, 3 types of tinsel, 2 different garlands, and 4 tree-toppers. If she wants to use one set of glass ornament, one type of tinsel, one garland, and a tree-topper to decorate the tree, how many different styles can she create?

5. A toy set allows a child to create toy soldiers from different body parts. There are 3 options for the body of the toy (with legs attached), 5 options for the right arm, 5 options for the left arm, and 6 options for the head. How many different toy soldiers can be created from these parts?

6. A test consists of 5 questions, each with 4 answer choices. If each question requires only one answer, how many different ways are there to answer the test?

7. A restaurant menu has 5 types of salads, 4 soups, 7 main dishes, 3 desserts, and 6 drinks options. If you want to order one item from each category, how many different meals can be made?

8. A frame shop has 10 different prints for sale by a certain artist. Each print can fit in 5 different frames, with 3 glass options, and 10 different color cardstock border options. If you want to buy a custom-framed print by this artist, how many different options do you have to choose from?

Evaluate the given factorial. See Example 2.

9. 2!

10. 5!

11. 9!

12. 13!

13. 6!

14. 10!

Evaluate the given permutation.

15. ${}_6P_2$

16. ${}_8P_3$

17. ${}_{14}P_1$

18. P_5^6

19. P_3^{11}

20. P_7^7

21. $P(9,4)$

22. $P(17,0)$

Applications

Solve. See Examples 3 and 4.

23. A combination lock has 3 dials with the digits 0 through 9 inscribed on them. How many different combinations are there for such a lock

a. if we assume that the digits in a combination must not be repeated, and

b. if the digits may be repeated?

24. A bowl holds 6 different types of fruit. How many ways are there to distribute these fruits among five children

a. if no child may have the same type of fruit, and

b. if there are no restrictions on the fruit selection?

25. Having bought 4 new houseplants, Melissa is choosing which pots to replant them into. If there are 6 pot designs, how many different options does she have

a. if she does not want to place two plants in the same type of pot, and

b. if the pot designs can be repeated?

26. In a memory training game, several lights flash in a sequence and the player needs to remember this sequence to repeat it later. There is a total of 9 lights, and on a certain difficulty level, each sequence consists of 4 lights flashing. How many such sequences exist

a. if the lights in the sequence may not be repeated, and

b. if the lights may be repeated?

27. Five friends decide to try their luck in horse betting. In a race featuring 8 horses, each of them selects a horse that they hope will win. How many different betting options do the friends have

a. if each friend must bet on a different horse, and

b. if more than one bet may be placed on a horse?

28. Before taking their children to an amusement park, parents examine the activities available there. They see that the park has 9 different activities, and they want to visit 5 of them before lunch. How many different schedules can they create

a. if they want all five activities to be different, and

b. if the activities may be repeated?

29. A piece of line art prepared for coloring has places for 5 colors. If Sharon has 10 different crayons, how many ways does she have to color the art

a. if all 5 colors must be different, and

b. if the colors may be repeated?

30. A bag contains 6 balls with different symbols inscribed on them. We pull one ball from the bag at a time and write down the respective symbol. After repeating this process 6 times, how many different symbol sequences can we obtain

a. if we do not return the balls to the bag, and

b. if we return the pulled ball into the bag before pulling again?

31. Ten students compete for the positions of president and vice president of a class. How many possible choices can be made to fill these positions?

32. After their cat gave birth to three kittens, the family decided to give these kittens to their friends. If six friends expressed the desire to take one of the kittens in, how many ways do the family have to distribute the kittens?

33. Sixteen employees take part in the company's lunchtime table tennis competition. How many ways can the top three winners of the competition be decided?

34. Suppose you are given a list of ten recently released movies and asked to name the four that you think are the best. How many different answers can you give?

35. For a group photo, it was decided that 4 people should sit in front and 5 people should stand behind them. How many different ways are there to organize the people sitting for the photo?

36. Mark and Shelly are on vacation. The tour manager at the hotel provides them with a description of 15 possible activities they can take part in. Each activity takes plenty of time, so they are advised not to plan two activities on the same day. If Mark and Shelly have 4 days and they want to participate in something new every day, how many different schedules can they make?

37. In a group strategy game, there are 10 unique roles, each of which can be assigned to only one player. If five people play the game, how many ways are there for these roles to be assigned?

38. An exhibition hall dedicated to a famous artist can hold five paintings. If the gallery owns 15 paintings by this artist, how many ways do they have to exhibit them in the hall?

39. Before a relay race, each of the 6 participating teams have to choose the color of the ribbon they will wear around their heads. If organizers provide ribbons of 8 different colors and the colors of all teams should be different, in how many ways could the colors be assigned to the teams?

40. The heads of a nine-headed hydra are arguing about the order in which they will speak at a conference. How many different orders can they speak in, if each head is going to speak once?

Writing & Thinking

41. Is it possible to calculate ${}_nP_r$, when $n < r$? Explain why or why not.

42. In your own words, explain when you should use the fundamental counting principle and when you should use the permutation formula to find the number of sequences that can be constructed from the elements of a set.

43. If you have two groups of movies, and someone lists the best three movies in each group, how can you find the number of different pairs of lists that you can receive?