

## Multiplication Rule for Dependent Events

If two events,  $A$  and  $B$ , are dependent, then

$$P(A \cap B) = P(A) \cdot P(B | A) = P(B) \cdot P(A | B).$$

FORMULA

9. Assume that there are 15 juniors and 18 seniors in a computer science club. Two members are chosen at random to participate in a new project. What is the probability that the first member chosen is a junior and the second member chosen is a senior?

### Example 9 Using the Multiplication Rule for Dependent Events

Assume that there are 17 men and 24 women in the Lions Club. Two members are chosen at random each year to serve on the hospitality committee. What is the probability of choosing two members at random where the first member chosen is a man and the second is a woman?

#### Solution

Note that since we are choosing two members, the first choice will influence the probability for the second choice, assuming we do not want to choose the same member twice. This means we are dealing with dependent events. We want to find  $P(\text{man} \cap \text{woman})$  which, according to the multiplication rule for dependent events, equals  $P(\text{man}) \cdot P(\text{woman} | \text{man})$ . When the first member is picked, there are 17 men out of 41 members. When the second member is picked, we assume that we have already picked a man, so that leaves all 24 women, but only 40 remaining members. The calculation is as follows.

$$P(\text{man} \cap \text{woman}) = P(\text{man}) \cdot P(\text{woman} | \text{man}) = \frac{17}{41} \cdot \frac{24}{40} \approx 0.2488$$

#### Now work margin exercise 9.

#### Margin Exercise Answers

1.  $\frac{83}{253} \approx 0.3281$  2. a.  $\frac{79}{156} \approx 0.5064$  b.  $\frac{79}{96} \approx 0.8229$  3. Yes 4. a.  $\frac{1}{8}$  b.  $\frac{1}{8}$  5.  $\frac{1}{78} \approx 0.0128$   
6.  $\frac{1}{208} \approx 0.0048$  7. a. 0.1176 b. 0.9784 8.  $\frac{1}{204} \approx 0.0049$  9.  $\frac{45}{176} \approx 0.2557$

## 9.3 Exercises

### Concept Check

**Fill-in-the-Blank.** Complete the sentences using information found in this chapter.


- The probability that an event will occur given that some other event has already occurred is called \_\_\_\_\_ probability.
- The notation  $P(A | B)$  is read as “the probability of \_\_\_\_\_ given the occurrence of \_\_\_\_\_.”

3. The vertical bar within a probability statement will always mean \_\_\_\_\_.
4. Finding the conditional probability of  $B$  given that  $A$  has occurred implies that  $P(A)$  \_\_\_\_\_.
5. The multiplication rule for independent elements states that the probability that mutually independent events occur at the same time is the \_\_\_\_\_ of their probabilities.
6. When two events are not independent, the occurrence of one \_\_\_\_\_ the occurrence of the other.

**True/False.** Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The probability of an event  $B$ , given that event  $A$  has occurred, is always equal to the probability of an event  $A$ , given that event  $B$  has occurred.
8. Two events are said to be independent if knowing that one event occurred does not provide information regarding whether the other event has occurred.
9. The probability that mutually dependent events occur at the same time is the product of their probabilities.
10. Flipping a coin and rolling a die are independent events.

## Practice

 Use the data tables to find the indicated probabilities. Round your answer to the nearest ten thousandth, if necessary. See Example 1.

A restaurant manager asks customers what they think about the restaurant's new menu. Use the table below to answer the following questions.

	Like	Dislike	Neutral/ Undecided	Total
Soups	39	8	7	54
Salads	28	3	8	39
Main courses	33	5	9	47
Total	100	16	24	140

1. If a customer orders a soup, what is the probability that he or she will like it?
2. If a customer likes a dish, what is the probability that this dish is a soup?
3. If a customer orders a salad, what is the probability that he or she will not dislike it?
4. If a customer does not dislike a dish, what is the probability that this dish is a salad?

An airline collects information about a drink preference during a flight depending on the age of a passenger. Use the table below to answer the following questions.

Age	Water	Juice	Soft Drinks	Total
Below 18	50	43	53	146
18-30	43	31	46	120
31-50	121	203	115	439
Over 50	150	105	40	295
Total	364	382	254	1000

5. If a passenger is over 50 years old, what is the probability that he or she will choose a juice?
6. If a passenger chooses a juice, what is the probability that he or she is over 50 years old?
7. If a passenger is between 18 and 30 years old, what is the probability that he or she will not choose water?
8. If a passenger does not choose water, what is the probability that he or she is between 18 and 30 years old?

The results of a final exam for each year of study are given in the table below. Use the table to answer the following questions.

Year of study	A	B	C	D	Total
Freshman	95	65	12	5	177
Sophomore	85	70	6	3	164
Junior	77	67	15	2	161
Senior	72	74	5	1	152
Total	329	276	38	11	654

9. If a student is a sophomore, what is the probability that he or she will get C?
10. If a student gets C, what is the probability that he or she is a sophomore?
11. If a student is a senior, what is the probability that he or she will get either A or B?
12. If a student gets either A or B, what is the probability that he or she is a senior?

Determine the indicated conditional probability. Round your answer to the nearest ten thousandth, if necessary. See Example 2.

13. The best 8 of 32 teams go on to the playoffs. Of the teams going on to the playoffs, six participated in last year's tournament.
  - a. What is the probability that a randomly chosen team participated in last year's tournament, given that the team goes on to the playoffs?

- b. If 22 of the teams participated in last year's tournament, what is the probability that a randomly chosen team goes to the playoffs, given that the team participated in last year's tournament?
14. Twelve out of 50 employees were offered a promotion during the last review session. Five of them had worked at the company for more than 5 years.
- a. What is the probability that a randomly chosen employee has worked at the company for more than 5 years, given that he or she is offered a promotion?
- b. If 16 employees have worked at the company for more than 5 years, what is the probability that a randomly chosen employee is offered a promotion, given that he or she has worked at the company for more than 5 years?
15. Out of 100 students, 85 say that they are satisfied with their specialization choice. Of those satisfied, 15 specialize in mathematics.
- a. What is the probability that a randomly chosen student specializes in mathematics, given that he or she is satisfied with the choice?
- b. If 21 students specialize in mathematics, what is the probability that a randomly chosen student is satisfied with the specialization choice, given that he or she specializes in mathematics?
16. Out of 150 university staff members, 96 are graduates of the university. Of the graduates, 12 are graduates of this year.
- a. What is the probability that a random staff member is a graduate of this year, given that he or she graduated from the university?
- b. If 18 university staff members are graduates of this year, what is the probability that a random university staff member graduated from the university, given that he or she is a graduate of this year?
17. Out of 500 new companies, 240 made a profit last year. Of the profitable ones, 105 companies were related to the IT industry.
- a. Determine the probability that a new company is related to the IT industry, given that it made a profit last year?
- b. If 202 new companies are related to the IT industry, what is the probability that a new company made a profit last year, given that the company is related to the IT industry?
18. A study involving 250 people was conducted to determine the effects of a new medicine. Of the participants, 150 received the medicine and the rest were given a placebo. Out of the people who took the medicine, 102 felt their health improving. Of all the participants, 114 people felt the improvement.
- a. What is the probability that a randomly chosen participant felt the improvement, given that he or she took the medicine?

- b. What is the probability that a randomly chosen participant took the medicine, given that he or she felt the improvement?

Determine whether the given events are independent. See Example 3.

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19. A coin is tossed and a card is drawn from a standard deck. Are the events  $A = \{\text{observing a tail}\}$  and  $B = \{\text{drawing a spade}\}$  independent?
20. Two games of American roulette are played. Consider the two events  
 $A = \{\text{a ball landing on red in the first game}\}$  and  
 $B = \{\text{a ball landing on red in the second game}\}$ .  
Are these two events independent?
21. A 6-sided die is rolled. Are the events  $\{\text{rolling a number smaller than 4}\}$  and  $\{\text{rolling an even number}\}$  independent?
22. A coin is tossed two times. Consider the two events  
 $A = \{\text{observing a head on the first toss}\}$  and  
 $B = \{\text{observing the same outcome on both tosses}\}$ .  
Are these two events independent?
23. When being randomly assigned into groups, are the events “getting into the first group” and “getting into the second group” independent?
24. Are the events of correctly guessing the first, second, and third digits in an arbitrary 3-digit code independent?

Find the probability of the given event using **a.** a tree diagram and **b.** the multiplication rule for independent events. See Example 4.

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25. When playing against a computer, a chess player has equal chances to win and to lose. Find the probability that the player will win two games in a row.
26. Two people play rock-paper-scissors. Find the probability of the first player showing rock and the second player showing paper.
27. Olivia has 6 pencils and 4 erasers. Ann likes 2 of the pencils and 3 of the erasers. Before the game, Olivia randomly gives a pencil and an eraser to each of her four playmates, including Ann. What is the probability that Ann will receive a pencil and an eraser that she likes?
28. A small test consists of three questions. Each question has three answer choices, only one of which is correct. If a person randomly chooses an answer for each question, what is the probability that he or she will give correct answers for the first two questions and an incorrect answer for the last question?

Use the multiplication rule for independent events to find the probability of the given event. Round your answer to the nearest ten thousandth, if necessary. See Examples 5 and 6.

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29. A coin is tossed and a card is drawn from a standard deck of 52 cards. Find the probability of getting a head on the coin and a ten from the deck of cards.
30. Two 6-sided dice are rolled. Find the probability of getting a 5 or a 6 on both dice.
31. Find the probability that the first digit of a random whole number from 10 to 99 is even and the second digit is odd.
32. Four differently sized coins are tossed. Find the probability of getting a head on the largest two coins.
33. Suppose a playlist contains 15 songs, and each song played is selected completely at random. If we choose one song from the playlist, what is the probability that this song will be played twice in a row at the start?
34. A bag contains 9 balls numbered from 1 to 9. We draw a ball from the bag, look at its number, and then return it into the bag. If we repeat this process three times, what is the probability that the ball number each time will be divisible by 4?

Use the multiplication rule for independent events to find the probability of the given event. Round your answer to the nearest ten thousandth, if necessary. See Example 7.

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35. Three cards are drawn with replacement from a standard deck of 52 cards.
  - a. What is the probability that all three cards are not diamonds?
  - b. What is the probability that at least one card is not a diamond?
36. When installing new software, the provider guarantees three months of free consultations. The probabilities that a client will ask for consultations in each of the first three months are estimated as  $P(I) = 0.6$ ,  $P(II) = 0.4$ , and  $P(III) = 0.2$ .
  - a. Find the probability that a client will not use free consultations.
  - b. Find the probability that a client will not use free consultations in at least one of the three months.
37. According to market analysts, the probabilities that the company's profit for each of the next four years will be greater than planned are  $P(I) = 0.7$ ,  $P(II) = 0.8$ ,  $P(III) = 0.9$ , and  $P(IV) = 0.95$ .
  - a. What is the probability that the company will fail to exceed the plan in each of the four years?
  - b. What is the probability that the company will fail to exceed the plan at least once?

38. A car is sold as a base model with four possible additional options ( $A, B, C, D$ ). The probabilities that a customer will install each of these options are estimated as  $P(A) = 0.2$ ,  $P(B) = 0.35$ ,  $P(C) = 0.16$ , and  $P(D) = 0.3$ .
- What is the probability that a customer will not choose any options while buying the car?
  - What is the probability that a customer will choose some, but not all, of the possible options?

Use the multiplication rule for dependent events to find the probability of the given event. Round your answer to the nearest ten thousandth, if necessary. See Examples 8 and 9.

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39. What is the probability of drawing two face cards from a standard deck of 52 cards, if the cards are drawn without replacement?
40. The host of a game holds 10 sticks, one of which is shorter than the rest. Players randomly draw a stick until the short one is drawn out. What is the probability that the second player will draw the short stick?
41. What is the probability of drawing a jack, then a queen, and finally a king from a standard deck of 52 cards, if the cards are drawn without replacement?
42. A bowl contains 12 candies, three of which are chocolate. What is the probability that the first three candies randomly picked from the bowl are chocolate?
43. Basketball qualifying matches were held during two days, with eight teams selected on each day. Then, the selected teams are being randomly assigned into pairs. What is the probability that in the first pair, the first team qualified during the first day, and the second team qualified during the second day?
44. Zoe and her brother, along with 48 other people, participate in a marathon. Assuming all participants have an equal chance of winning, what is the probability that Zoe will take the second place and her brother will place first?
45. In a batch of 100 decals, four decals have defects. Two decals are randomly selected from the batch for testing. Determine the probability that the first decal has no defects and the second decal is defective.
46. King Arthur randomly chooses two knights from 12 Knights of the Round Table to set off on the quest for the Holy Grail. Determine the probability that King Arthur will first choose Lancelot and then choose either Gawain or Percival.
47. Assume that a hotel has 10 studio rooms, 20 single rooms, 15 twin rooms, and 10 double rooms. The hotel management plans to conduct a sanitary inspection in two randomly selected rooms. Determine the probability that the first room selected is a single room, and the second is a double room.

48. A restaurant menu has 9 soups, 5 salads, 17 main dishes, and 4 desserts. A restaurant critic usually chooses two dishes for his magazine review. What is the probability that he will first order a salad and then a main dish?

### Writing & Thinking

49. You are given that event  $A$  has occurred. What probabilities do you need to know to find the probability that event  $B$  will also occur?
50. In your own words, explain the difference between independent and dependent events.
51. Is it possible for two mutually exclusive events to be independent?
52. Give an example of two events  $A$  and  $B$  such that  $P(A|B) = P(B|A) \neq 0$ .