Finding Probability Given the Odds

If we are given the odds in favor of an event A as n to m, then the probability of event A can be calculated by

$$P(A) = \frac{n}{n+m}.$$

FORMULA

Using this formula, we can find the probability of Audible winning the Kentucky Derby.

$$P(\text{Audible wins Kentucky derby}) = \frac{1}{1+8} = \frac{1}{9} \approx 0.11111$$

The probability of Audible winning the Kentucky Derby is slightly more than 11%.

Example 12 Calculating Probability Given the Odds

Dominic sees that his college's soccer team has 3 : 2 odds in favor of winning their game this weekend. What is the probability implied by these odds?

Solution

Using the formula for finding probability given the odds, we have the following.

$$P(\text{wins game}) = \frac{3}{3+2} = \frac{3}{5} = 0.6$$

Thus, the soccer team has a 60% chance of winning their game this weekend.

Now work margin exercise 12.

12. Patricia sees that her college's baseball team has 5 : 3 odds in favor of winning their game this weekend. What is the probability implied by these odds?

Margin Exercise Answers

1. No **2.** Yes **3.** Yes **4.** Yes **5.** Yes **6.** $\frac{2}{3}$ **7.** $\frac{1}{2}$ **8.** $\frac{10}{13}$ **9.** $\frac{3}{5}$ **10.** 0.96 **11. a.** $\frac{1}{19}$ **b.** $\frac{18}{19}$ **c.** $\frac{1}{18}$ or 1 : 18 **12.** $\frac{5}{8} = 0.625$

9.2 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. Two events are mutually exclusive if they cannot _____ at the same time.

2. If two events are mutually exclusive, the probability of the outcome of the two events can be found by _____ their probabilities.

3. When calculating the probability of non-mutually exclusive events, the _____ of the two events needs to be subtracted.

4. The _____ of a set is all outcomes in the universal set that are not part of the set.

5. The sum of the probabilities of a set and its _____ equals 1.

6. The odds _____ an event occurring are calculated by the ratio of the probability of the event to the probability of the complement of the event.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If it is possible for two events to occur at the same time, then those two events are mutually exclusive.

8. If a certain team is given 1 to 3 odds of winning a game, this means that for every dollar bet on the team, a person would win $3 if the team wins.

9. An event A and its complement A' make up the entire sample space.

10. If you have 1 to 5 odds in favor of winning a game, then the probability of winning is $\frac{1}{5}$.

Practice

Determine whether the given events are mutually exclusive. See Examples 1 through 5.

1. Are the events $A = \{1, 3, 5, 7, 9, 11\}$ and $B = \{4, 8, 12\}$ mutually exclusive?

2. Are the events $A = \{1, 2, 3, 5, 8, 13\}$ and $B = \{3, 6, 9, 12\}$ mutually exclusive?

3. Consider the set of whole numbers 1 through 10. Are the events $A = \{\text{the whole numbers in the set divisible by } 5\}$ and $B = \{\text{the whole numbers in the set no less than } 5\}$ mutually exclusive?

4. Consider the set of whole numbers 10 through 20. Are the events $A = \{\text{the whole numbers in the set greater than } 18\}$ and $B = \{\text{the whole numbers in the set divisible by } 6\}$ mutually exclusive?

5. In one trial of drawing a card from a deck of 52 cards, are the events "drawing a 2 of hearts" and "drawing a 10 of spades" mutually exclusive?

6. In one trial of drawing two cards from a deck of 52 cards, are the events "drawing two queens" and "drawing two kings" mutually exclusive?

7. In a single trial of rolling a 6-sided die, are the events "getting a number less than 3" and "getting a number greater than 4" mutually exclusive?

8. In a single trial of rolling a 6-sided die, are the events "getting a number less than 5" and "getting an even number" mutually exclusive?

9. Are the events "the archer hits the target" and "the archer misses the target" mutually exclusive?

10. Are the events "the weather is cold" and "the weather is sunny" mutually exclusive?

11. Are the events "a swimmer takes first place in the competition" and "a swimmer takes second place in the competition" mutually exclusive?

12. Are the events "a cup is filled with black tea" and "a cup is made of porcelain" mutually exclusive?

Find the indicated probability. Round your answer to the nearest ten thousandth, if necessary. See Examples 6 and 7.

13. In one trial of rolling a 6-sided die, what is the probability of observing a number less than 3 or greater than 4?

14. When selecting a card from a deck of 52 cards, what is the probability of drawing a 7 of spades or a 9 of hearts?

15. A bag contains 12 balls; 4 of the balls are white, 2 are green, and 6 are black. What is the probability that a randomly chosen ball is white or black?

16. Three coins are tossed. Find the probability of observing either three heads or three tails.

17. If you randomly choose a whole number from 1 to 50 (inclusive), what is the probability that the number will be even or will end with 5?

18. Several bottles are placed on a table. The labels indicate that 6 bottles contain black tea, 5 bottles contain milk, 7 bottles contain coffee, and the remaining 8 bottles contain apple juice. If you randomly choose a bottle from the table, what is the probability that the bottle you chose contains coffee or milk?

19. An analyst estimates that the price of wheat will increase with a probability of $\frac{7}{12}$ or will remain at the same price with a probability of $\frac{1}{6}$. Find the probability that the price will not decrease.

20. The probabilities of getting each of the three main prizes in a lottery are $\frac{1}{10,000}$, $\frac{1}{500}$, and $\frac{1}{100}$, respectively. If each participant can win only one prize, what is the probability of getting one of the three main prizes?

Find the indicated probability. Round your answer to the nearest ten thousandth, if necessary. See Examples 8 and 9.

21. When rolling a 6-sided die, what is the probability of observing a number less than 5 or an odd number?

22. When selecting a card from a deck of 52 cards, what is the probability of getting a card of a black suit or an ace?

23. When tossing two coins, what is the probability of the coins landing on different faces or the first coin landing on tails?

24. When choosing a random whole number from 1 to 30 (inclusive), what is the probability that the number will be divisible by six or divisible by four?

25. At an animal shelter, 20% of cats have white fur, 25% have black spots, and 5% have white fur and black spots. If you randomly choose one cat from the shelter, what is the probability that it will be white or will have black spots?

26. In a bag of candies, 35% of candies are chocolate and 25% contain nuts. In addition, 10% of candies are chocolate with nuts. What is the probability that a randomly chosen candy will either be chocolate or contain nuts?

27. While choosing a melon, Kate notices that 17 out of 20 melons on a counter have no visible flaws, 14 are ripe, and 12 both have no flaws and are ripe. Had she randomly picked one, what would be the probability of getting a flawless and ripe melon?

28. A box contains 34 pieces of paper covered with strange colored symbols. There are 20 pieces with some of the symbols colored red, 18 pieces with some of the symbols colored green, and 10 pieces that have both red and green symbols. Find the probability that a randomly chosen piece have symbols that are colored red or green or both red and green.

Solve and round your answer to the nearest ten thousandth, if necessary. See Examples 10 and 11.

29. Thirty percent of applicants pass the interview for a job position. Determine the probability that an applicant will not pass the interview.

30. A study shows that 5% of people who register for an event end up not going to it. Determine the probability that a person will go to the event for which they registered.

31. An archer has an 89% chance of hitting the target. Determine the probability that the archer will miss the target.

32. In a certain country, 59.7% of households own at least one car. Determine the probability that a randomly chosen household does not own a car.

33. The statistical data show that police officers make up $\frac{5}{1000}$ of the total population of a country. Find the probability that a randomly chosen citizen is not a police officer.

34. When tossing a coin 4 times, the probability that the number of heads is equal to the number of tails is $\frac{3}{8}$. What is the probability that the number of heads is different from the number of tails?

35. A 20-sided die is rolled.

a. Calculate the probability of getting a number greater than 18.

b. Calculate the probability of getting a number that is not greater than 18.

c. Calculate the odds in favor of getting a number greater than 18.

36. In a game of chance, a player is shown a table with 6 rows and 6 columns. In the table, each cell hides the name of a prize and one of the cells hides the jackpot. The player chooses one cell and wins the prize hidden there.

a. Calculate the probability of getting the jackpot.

b. Calculate the probability of not getting the jackpot.

c. Calculate the odds in favor of getting the jackpot.

37. A recent quality test found that a machine produces 6 defective parts out of every 500 parts manufactured.

a. Calculate the probability of a component having defects.

b. Calculate the probability of a component having no defects.

c. Calculate the odds in favor of a component having no defects.

38. The results of a study show that a new drug stops the development of a certain disease for 10 out of 25 study participants.

a. Calculate the probability of the drug stopping the development of the disease.

b. Calculate the probability of the drug not stopping the development of the disease.

c. Calculate the odds in favor of the drug stopping the development of the disease.

39. A company assesses the reliability of a new car and states that 498 out of 500 cars will not need repairs within the first 10,000 miles.

a. Calculate the probability of a car driving the first 10,000 miles without needing repairs.

b. Calculate the probability of a car needing repairs within the first 10,000 miles.

c. Calculate the odds in favor of a car driving the first 10,000 miles without repairs.

40. An athlete completed the 100-meters sprint in less than 11 seconds in 32 of the last 76 attempts.

a. Calculate the probability of the athlete completing the next attempt in less than 11 seconds.

b. Calculate the probability of the athlete completing the next attempt in 11 seconds or more.

c. Calculate the odds in favor of the athlete completing the next attempt in less than 11 seconds.

Determine the probability implied by each of the following odds. Round your answer to the nearest ten thousandth, if necessary. See Example 12.

41. If we disregard the individual strength of the teams, any team that wins the quarterfinals has 1 : 3 odds in favor of winning the tournament.

42. For the car you have chosen to win the race, the odds in favor of placing in the top three are 3 : 17.

43. The odds of a certain disease worsening without proper treatment are 5 : 2.

44. The odds of a new power supply unit failing in the first two years are 3 : 22.

45. The odds of making a correct decision on the first try are 2 : 7.

46. With the cards you were dealt, the odds of winning the round of the game are 2 : 11.

47. The leader of the chess club has 12 : 13 odds in favor of getting the first place in the local tournament.

48. The odds of house prices falling in the next year are 2 : 9.

Writing & Thinking

49. In your own words, explain the difference between mutually exclusive and non-mutually exclusive events.

50. Give an example of the event with 1 : 1 odds.