c. $\{1, 2, 3, 4\} \subseteq A$. $\{1, 2, 3, 4\}$ contains all the elements of A. This means it is a subset of A, but not a proper subset of A.

Now work margin exercise 9.

Margin exercise answers

1. $W = \{0, 1, 2, 3\}$ **2.** $T = \{a, b, c, d, e, f, g, h\}$ **3.** $F = \{x \mid x \text{ is an NFL team}\}$ **4. a.** T is the set of Philadelphia professional sports teams **b.** $T = \{\text{Phillies, Eagles, 76ers, Flyers, Union}\}$ **c.** $T = \{x \mid x \text{ is a Philadelphia professional sports team}\}$ **5. a.** Florida $\in S$; **b.** Los Angeles $\notin S$ **6. a.** Paris $\in C$; **b.** London $\notin E$ **7. a.** and **c.** are empty sets. **8.** There are 7 proper subsets of B. $\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$. **9. a.** $\varnothing \subseteq$ and $\subset A$. **b.** No symbol can be inserted here. While bird $\in A$, bird is a not a subset of A. In order to be a *subset*, it is necessary to be a *set*. **c.** No symbol can be inserted here. The element in this set, camel, is not an element of A.

8.1 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. A _____ is a collection of items.

2. To write a set using _____ notation, list each member of the set, separating members by commas, and enclosing them in braces, { }

3. _____is method of writing a set that it uses the features of both the word description method and the roster method.

4. A set with no elements is called the _____ or the null set.

5. A _____ is a set that consists of elements of another set.

6. If there are n elements in a set, use the formula _____ to determine the number of proper subsets of that set.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. The only ways to write sets are with word descriptions and set-builder notation.

8. Given $A = \{1, 2, 3, 4, 5\}$, it is correct to write that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $\{5\} \subseteq A$.

9. A set cannot be considered a subset of itself.

10. The set of all negative natural numbers is the empty set.

Practice

Write a word description of these sets. There may be more than one correct answer.

1. {January, February, March, April, May, June, July, August, September, October, November, December}

2. $\{m, o, z, a, r, t\}$

3. $\{2, 4, 6, 8, 10\}$

4. $\{-9, -7, -5, -3, -1\}$

5. $\{x \mid x \text{ is a book written by J.R.R. Tolkien}\}$

6. $\{x \mid x \text{ is an appetizer on sale during happy hour}\}$

Write these sets using roster notation. See Examples 1 and 2.

7. The set of months of the year with less than 30 days.

8. The set of days in the week that begin with letter S.

9. The planets in our solar system that begin with the letter M.

10. $\{x \mid x \text{ is a letter in the word "Bookkeeper"}\}$

11. $\{x \mid x \text{ is a natural number and } 5 \le x < 9\}$

12. $\{x \mid x \text{ is a prime number between 4 and 20}\}$

Write these sets using set-builder notation. There may be more than one correct answer. See Example 3.

13. $\{\text{Winter, Spring, Summer, Autumn}\}$

14. $\{\text{red, yellow, blue}\}$

15. $\{j, u, l, y\}$

16. $\{p, i, c, k, l, e\}$

17. $\{1, 2, 3, 4, 5\}$

18. $\{-10, -8, -6, -4\}$

Fill in the blank with the correct symbol, $\in$ or $\notin$. See Examples 5 and 6.

$A = \{1, 3, 5, 7, 9\}$

19. 1 _____ A

20. 2 _____ A

21. $\{1\}$ _____ A

22. $\varnothing$ _____ A

Determine if the set is empty. See Example 7.

23. $\{x \mid x \text{ is an even prime number greater than } 4\}$

24. $\{x \mid x \text{ is a positive number and } x < 0\}$

25. The set of all crew members who journeyed to America with Christopher Columbus in 1492 who are alive today.

26. The set of all states of the United States.

Determine the number of proper subsets of the given set. See Example 8.

27. $A = \{1, 2, 3\}$

28. A is the set of the 7 Wonders of the Ancient World

29. $\{x \mid x \text{ is a day of the week that begins with the letter } T\}$

30. $\{x \mid x \text{ is a month of the year that begins with the letter } A\}$

List all the subsets of the given set. See Example 8.

31. $A = \{\text{Jack, Jill}\}$

32. $B = \{x \mid x \text{ is a letter in the word "cat"}\}$

33. $C = \varnothing$

34. $D = \{\text{North, South, East, West}\}$

Determine which symbol or symbols $(\subset, \subseteq)$, if any, can be placed in the blank to form a true statement. See Example 9.

35. $\{\text{warm}\}$ _____ $\{\text{cool, warm, hot}\}$

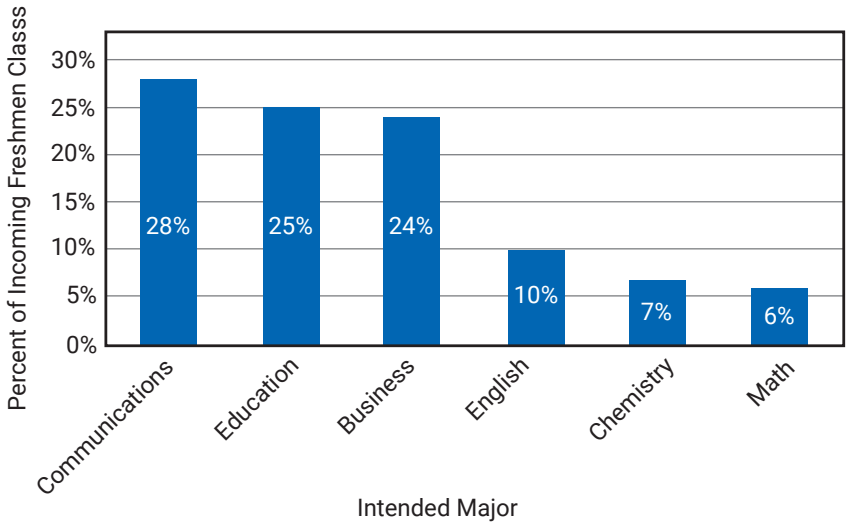
36. Mets _____ $\{\text{Mets, Yankees, Jets, Knicks, Rangers}\}$

37. $\{1, 2, 3\}$ _____ $\{3, 2, 1\}$

38. $\varnothing$ _____ $\{\text{elephant, camel}\}$

Applications

The bar graph below shows the intended major of the incoming freshmen class at a local university by percentage. Represent each set by roster notation or use appropriate notation to indicate the empty set.



39. Set of intended majors in which the percent of incoming freshmen exceeds 12%.

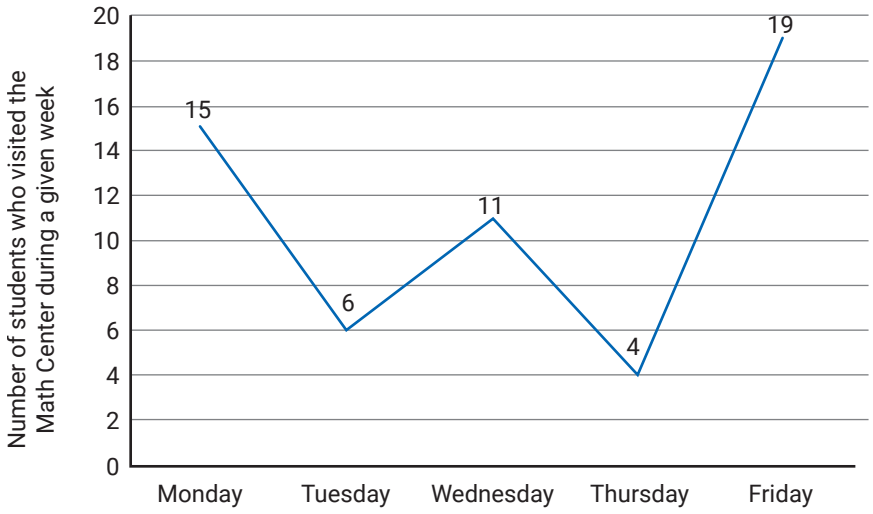
40. Set of intended majors in which the percent of incoming freshmen exceeds 30%.

41. Set of intended majors in which the percent of incoming freshmen is at least 9% and at most 26%.

42. $\{x \mid x \text{ is the intended major of at least 40\% of the incoming freshmen}\}$

43. $\{x \mid x \text{ is the intended major of between 23\% and 26\% of the incoming freshmen}\}$

The line graph below shows the number of students who visited the Math Center each weekday during a given week. Represent each set by roster notation or use appropriate notation to indicate the empty set.



44. The set of days in which between 5 and 12 students visited.

45. The set of days in which more than 10 students visited.

46. The set of days in which less than 3 students visited.

47. $\{x \mid x \text{ is a day in which an odd number of students visited}\}$

48. $\{x \mid x \text{ is day in which the number of students that visited is a prime number}\}$

Writing & Thinking

49. Do $\{\varnothing\}$ and $\varnothing$ represent the same thing? Explain your answer.

50. If a set has 15 proper subsets, how many elements are in the set?