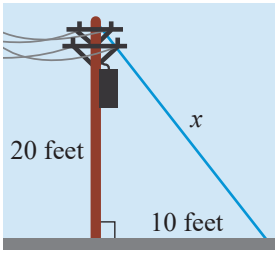
Example 11 Application: Finding the Length of the Hypotenuse

A guy wire is attached to the top of a telephone pole and anchored to the ground 10 feet from the base of the pole. If the pole is 20 feet high, what is the length of the guy wire?



Solution

Let x = the length of the guy wire.

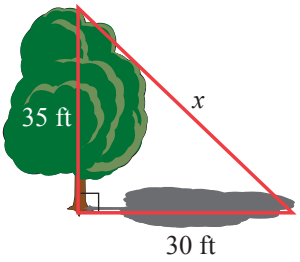
Then, by the Pythagorean Theorem,

$$x^2 = 10^2 + 20^2$$
$$x^2 = 100 + 400$$
$$x^2 = 500$$
$$x = \sqrt{500} \approx 22.36$$

The guy wire is about 22.36 feet long.

Now work margin exercise 11.

11. A 35-foot-tall tree casts a shadow which is 30 feet long. Find the distance from the top of the tree to the end of its shadow to the nearest hundredth.



Margin Exercise Answers

1. Scalene **2.** Yes **3. a.** 90° **b.** $\overline{RO}$ **c.** $\overline{BR}$ and $\overline{BO}$ **d.** Yes, because $m\angle R = 90°$. **4.** b **5.** 2.5 cm **6.** 27 feet **7.** Yes, by ASA **8.** No, $8^2 \neq 7^2 + 4^2$ **9.** 17 cm **10.** $\sqrt{32}$ yd or about 5.66 yd **11.** 46.10 ft

6.6 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. A _____ _____ consists of two points on a line and all the points between them.
2. Longer sides are _____ angles with larger measures.
3. In similar triangles, the _____ angles have the same measure.
4. If two triangles are congruent, the lengths of corresponding sides are _____.
5. Two triangles are congruent if they have the same _____ and the same _____.

6. The symbols ≅ is read "is _____ to."

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A scalene triangle has at least two sides of equal lengths.

8. An acute is a triangle where three angles are acute.

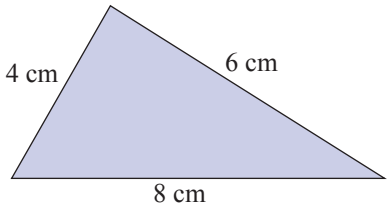
9. In similar triangles, the lengths of corresponding sides are equal.

10. The hypotenuse is the longest side of a triangle.

Practice

Classify each triangle in the most precise way possible, given the indicated lengths of its sides and/or measures of its angles. See Example 1.

1.



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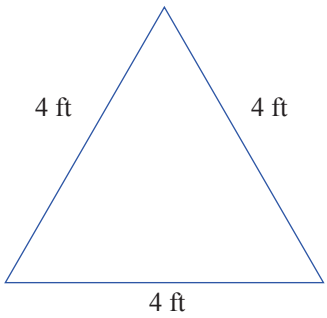

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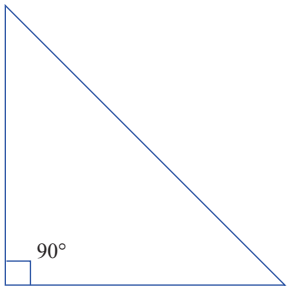

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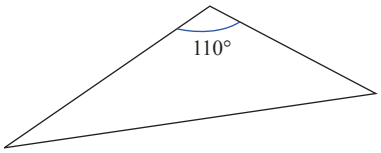

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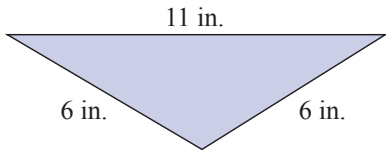

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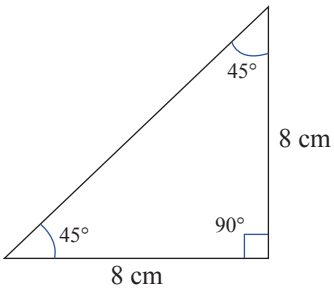

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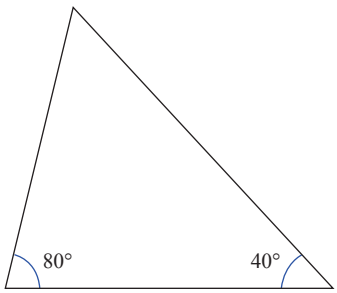

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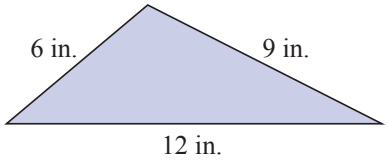
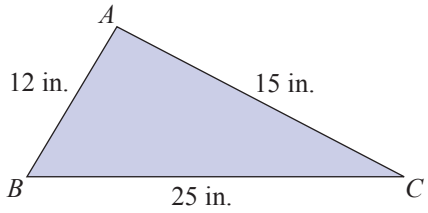
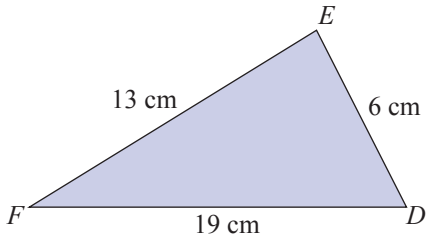


Solve.

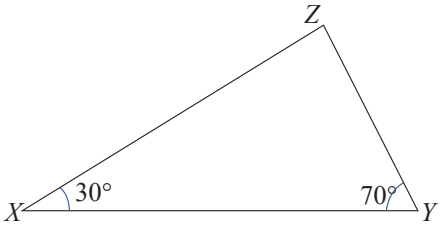
9. Suppose the lengths of the sides of $\triangle ABC$ are as shown in the figure. Is this possible? Explain your reasoning.



10. Suppose the lengths of the sides of $\triangle DEF$ are as shown in the figure. Is this possible? Explain your reasoning.



11. In the triangle shown, $m\angle X = 30°$ and $m\angle Y = 70°$.



a. What is $m\angle Z$?

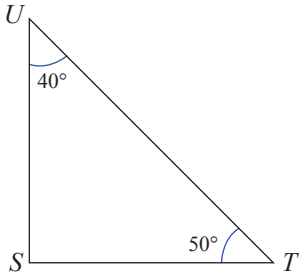
b. What kind of triangle is $\triangle XYZ$?

c. Which side is opposite $\angle X$?

d. Which sides include $\angle X$?

e. Is $\triangle XYZ$ a right triangle?

12. In the triangle shown, $m\angle T = 50°$ and $m\angle U = 40°$.



a. What is $m\angle S$?

b. What kind of triangle is $\triangle STU$?

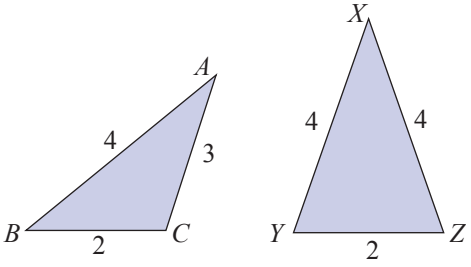
c. Which side is opposite $\angle T$?

d. Which sides include $\angle T$?

e. Is $\triangle STU$ a right triangle?

Determine whether each pair of triangles is similar. If the pair of triangles are similar, explain why and indicate the similarity by using the ~ symbol. See Example 4.

13.



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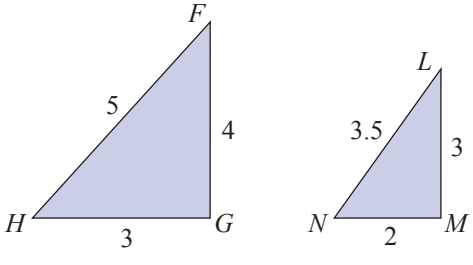

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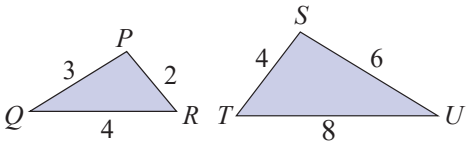

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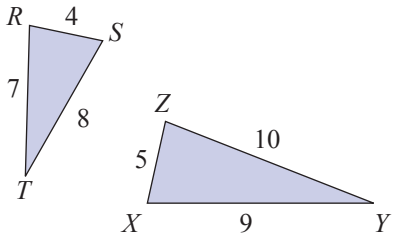

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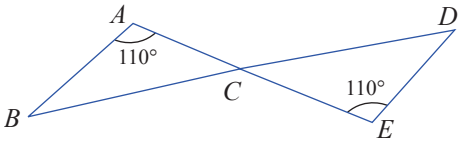

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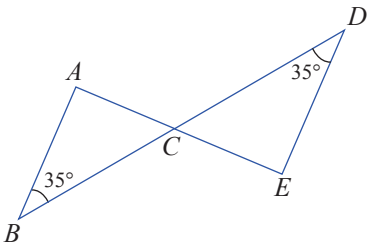
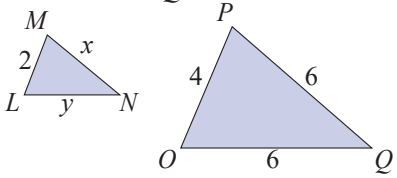
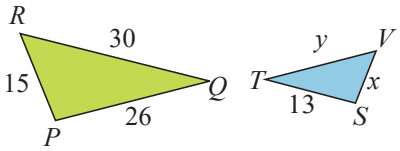


Find the values for x and y. See Example 5.

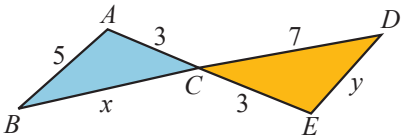
19. $\triangle LMN \sim \triangle OPQ$



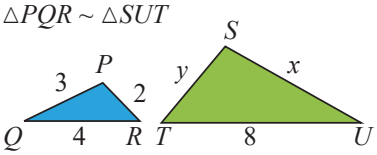
20. $\triangle PQR \sim \triangle STV$



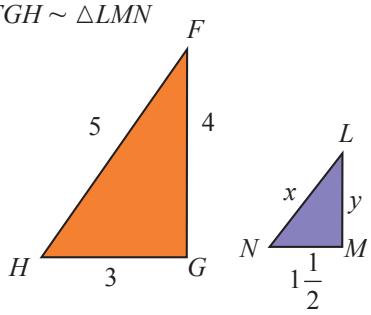
21. $\triangle ABC \sim \triangle EDC$



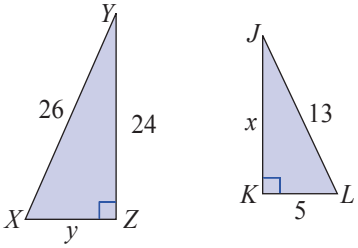
22. $\triangle PQR \sim \triangle SUT$



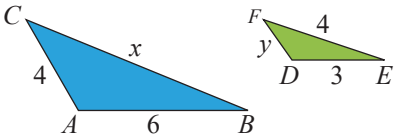
23. $\triangle FGH \sim \triangle LMN$



24. $\triangle XYZ \sim \triangle LJK$

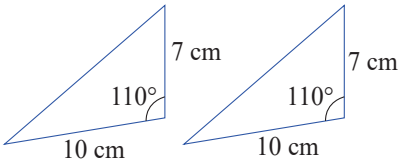


25. $\triangle ABC \sim \triangle DEF$



Determine whether each pair of triangles is congruent. If the pair of triangles is congruent, state the property that confirms that they are congruent. See Example 7.

26.



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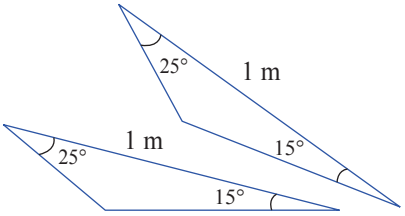

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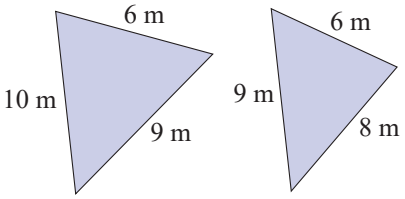

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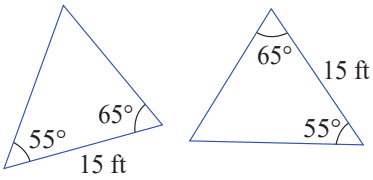

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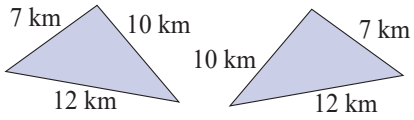

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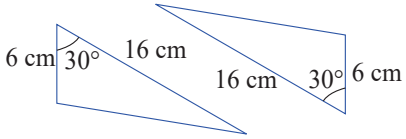

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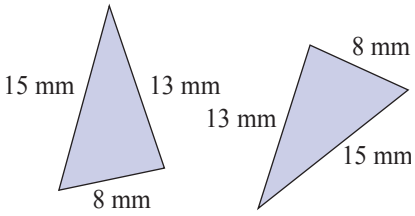

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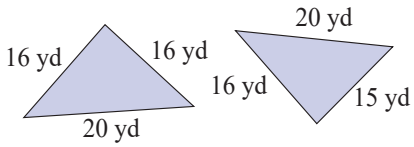
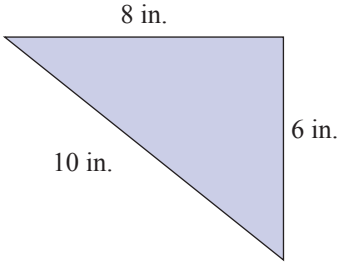


Use the Pythagorean Theorem to determine whether or not each triangle is a right triangle. See Example 8.

34.



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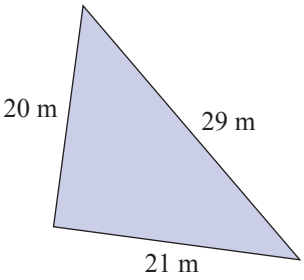

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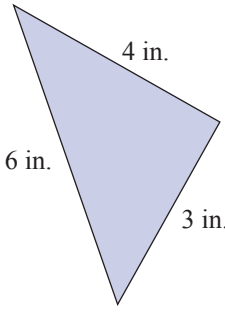

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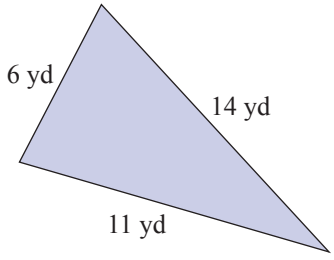

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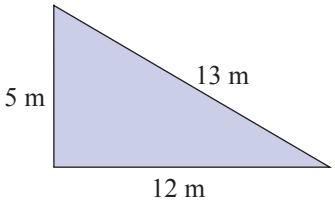

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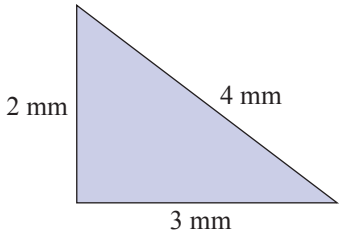
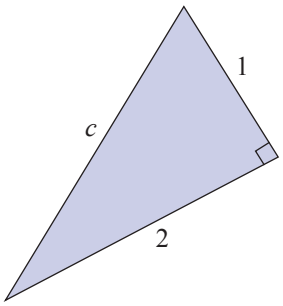


Find the hypotenuse for each right triangle accurate to the nearest hundredth. See Example 9.

40.



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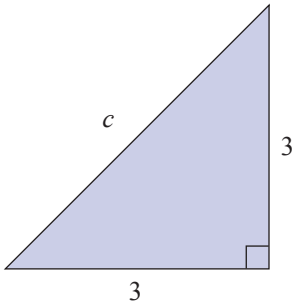

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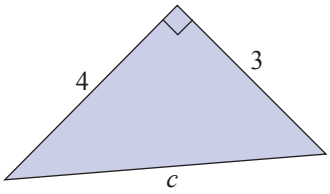

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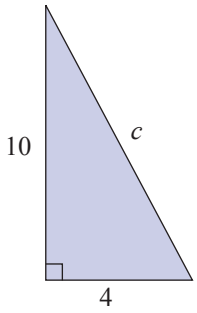

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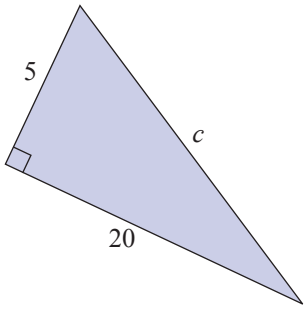

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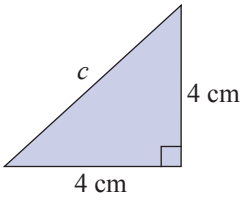

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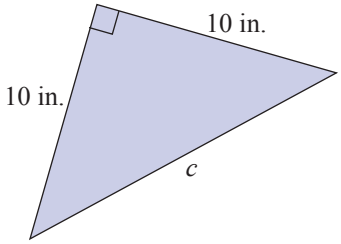

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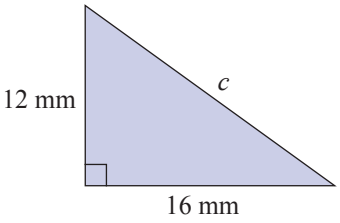
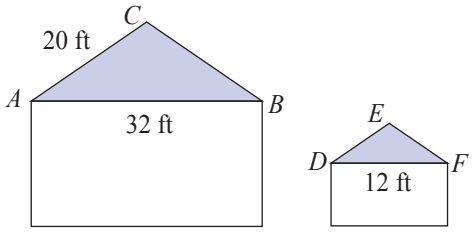


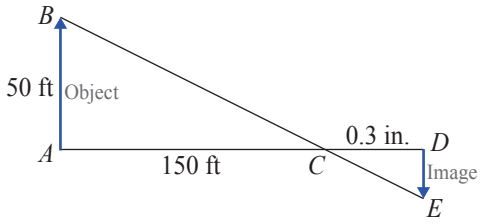
Applications

Solve.

48. A child's playhouse is built to look like a smaller version of the family house, where the ends of the roofs have similar proportions. The width of the main house (AB) is 32 feet and the length from the peak to the gutter of the roof for one of the sides is 20 feet. If the width of the playhouse (DF) is 12 feet, what is the length from the peak to the gutter (DE) of the playhouse roof?

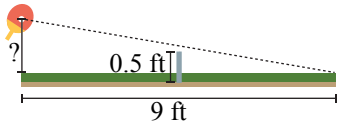


49. A camera uses a lens that will look at a properly focused object (such as a person or a tree) and then display an inverted image of this object on a screen or film which is on the opposite side of the lens as shown in the figure. If a picture of a 50-foot tall building (AB), which is 150 feet from the lens (AC) is photographed, how tall is the image (DE) if the film on the opposite side (CD) is 0.3 inch from the lens.



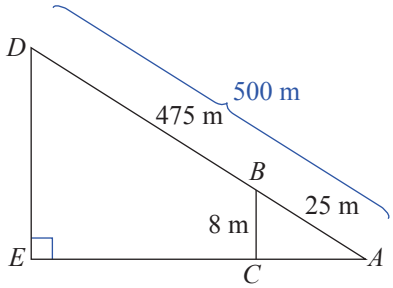
50. You and a friend are walking to class and want to figure out the height of the tree next to your building. Your friend is exactly 6 ft tall and casts 4 ft shadow. The tree casts a 12 ft shadow. How tall is the tree?

51. At what height off the table would you need to hit a ping pong ball for it to skim the net (0.5 feet tall) and hit on the edge of the opposite side of the table 9 feet away from you? Assume that the net is in the exact center of the table (with 4.5 feet on either side) and that your paddle is directly above the edge of the table.

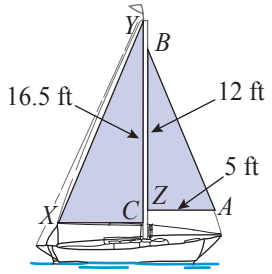


52. Your neighbors are hanging their holiday lights. The ladder they are currently using is 12 feet long and when leaned up against the house just reaches the top of their 8-foot tall porch. How long of a ladder will they need to reach the top of their chimney which is at a height of 32 feet? (Assume that both ladders are placed such that they make the same angle with the ground.)

53. The sloping surface of a hill (from base to peak) is 500 meters long. Paul starts at the base of the hill and walks uphill 25 meters which results in a gain of 8 meters in elevation. What is the height of the hill? (**Hint:** As shown on the accompanying drawing, there are two similar triangles ΔABC and ΔADE. Solve for DE.)



54. A sloop is a sailboat that has two triangular sails on a single mast. If the smaller sail is 12 feet along the mast (CB), and 5 feet along its bottom (AC), and the larger sail is 16.5 feet along the mast (ZY), how wide is the larger sail at the bottom (XZ) if $\triangle ABC$ and $\triangle XYZ$ are similar triangles? Round your answer to the nearest tenth.



55. If an airplane passes directly over your head at an altitude of 1 mile, how far (to the nearest hundredth of a mile) is the airplane from your position after it has flown 2 miles farther at the same altitude?

56. The GE Building in New York is 850 feet tall (70 stories). At a certain time of day, the building casts a shadow 100 feet long. Find the distance from the top of the building to the tip of the shadow (to the nearest tenth of a foot).

57. The base of a fire engine ladder is 30 feet from a building and reaches to a third floor window 50 feet above ground level. Find the length of the ladder to the nearest hundredth of a foot.

58. The Xerox Center building in Chicago is 500 feet tall. At a certain time of day, it casts a shadow that is 150 feet long. At that time of day, what is the distance (to the nearest tenth of a foot) from the tip of the shadow to the top of the Xerox building?

59. Before painting a picture on canvas, an artist must stretch the canvas on a rectangular wooden frame. To be sure that the corners of the canvas are true right angles, the artist can measure the diagonals of the stretched canvas. What should be the diagonal measure, to the nearest tenth of an inch, of a canvas whose sides are 24 inches and 30 inches in length?

60. While installing windows in a new home, a builder measures the diagonals of rectangular window casements to verify that their corners are true right angles. What should be the diagonal measure, to the nearest tenth of an inch, of a window casement with dimensions 36 inches and 54 inches.

61. A hiker hikes 9 kilometers north and then turns left and hikes 11 kilometers west. If she takes the shortest path, how long will she have to walk to get back? Assume the terrain is flat with no obstructions. Round the answer to the closest tenth.

Writing & Thinking

62. Kelly needs to determine whether two triangles are similar. She was given the following information.

For $\triangle ABC$ and $\triangle DEF$, $AB = 3.6$, $AC = 2.4$, $BC = 2$ and $DE = 9$, $DF = 6$, $EF = 5$.

What should be her first step?

63. If three whole numbers satisfy the Pythagorean Theorem, these three numbers are called a Pythagorean triple. For example, 3, 4, and 5 are a Pythagorean triple because $3^2 + 4^2 = 5^2$ (or $9 + 16 = 25$). Another Pythagorean triple is 5, 12, and 13 because $5^2 + 12^2 = 13^2$ (or $25 + 144 = 169$). Complete the following table by finding a, b, and c, and telling which sets of these three numbers (if any) constitute a Pythagorean triple. The first one is done for you.

m	n	$a = 2nm$	$b = m^2 + n^2$	$c = m^2 + n^2$	Pythagorean Triple?
5	1	10	24	26	yes: $10^2 + 24^2 = 26^2$
7	1				
3	2				
7	2				
5	3				
11	3				
13	7				

Extension: Choose some of your own numbers for m and n. Are your results Pythagorean triples? (**Note:** m must be larger than n so $m^2 - n^2$ will be positive.)