

Quadratic equations should be solved by factoring whenever possible.

Factoring is generally the easiest method. This fact was illustrated in Examples 11 and 12. However, the quadratic formula will always give the solutions whether they are rational, irrational, or nonreal (as shown in Example 13). Thus, the quadratic formula is a very useful tool. **In future courses in mathematics, every text and instructor will assume that you know or can easily remember the quadratic formula.**

13. Use the quadratic formula to solve the following quadratic equation.

$$x^2 + 3x + 5 = 0$$

Example 13 The Quadratic Formula and Nonreal Solutions

Use the quadratic formula to solve the following quadratic equation.

$$x^2 + x + 1 = 0$$

Solution

$$a = 1, b = 1, c = 1$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

There is no real solution. This example illustrates the fact that not every equation has real solutions. Such solutions are called **nonreal complex numbers** and are discussed in more depth in higher-level math courses.

Now work margin exercise 13.

Completion Example Answers

6. $20; \pm 2\sqrt{5}; -2 \pm 2\sqrt{5}$ 10. $a = 2, b = -5, c = -3; x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2 \cdot 2} = \frac{5 \pm 7}{4}$
 $x = 3$ and $x = -\frac{1}{2}$

Margin Exercise Answers

1. $x = \pm 2\sqrt{3}$ 2. $x = -5 \pm 2\sqrt{10}$ 3. $x = 1 \pm 5\sqrt{3}$ 4. $x = 2 \pm \sqrt{2}$ 5. no solution
 6. $x = 4 \pm 2\sqrt{3}$ 7. $x = -2 \pm \sqrt{5}$ 8. $x = \frac{-1 \pm \sqrt{13}}{4}$ 9. $x = \pm \frac{2\sqrt{30}}{3}$ 10. $x = \frac{-3 \pm \sqrt{5}}{4}$
 11. $x = \frac{3 \pm \sqrt{13}}{2}$ 12. $x = -\frac{5}{2}$ 13. no real solution

5.6 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

- A direct way of solving an equation of the form $x^2 = c$ is to use the _____ property.
- When using the square root property, we do not set the polynomial expression equal to _____.

3. When solving a quadratic equation, the quadratic formula will _____ work.
4. The expression $b^2 - 4ac$ is called the _____.
5. If $b^2 - 4ac > 0$, then there are _____ real solutions.
6. Quadratic equations should be solved by _____ whenever possible.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. Quadratic equations that are not easily solved using factoring might be solved by the square root method.
8. The quadratic formula will always work when solving quadratic equations.
9. When using the quadratic formula, if the discriminant is greater than zero, there are infinite solutions.
10. If the discriminant is less than zero, there is no real solution.

Practice

Solve the following quadratic equations by using the square root method. Write each radical in simplest form. See Examples 1 through 6.

- | | | |
|--------------------|---------------------------------|--------------------------------|
| 1. $x^2 = 121$ | 12. $5x^2 = 60$ | 22. $(x - 2)^2 = \frac{1}{16}$ |
| 2. $x^2 = 81$ | 13. $9x^2 = 4$ | 23. $(x - 6)^2 = 18$ |
| 3. $3x^2 = 108$ | 14. $4x^2 = 25$ | 24. $(x + 8)^2 = 75$ |
| 4. $5x^2 = 245$ | 15. $(x - 1)^2 = 4$ | 25. $2(x - 7)^2 = 24$ |
| 5. $x^2 = 35$ | 16. $(x + 3)^2 = 9$ | 26. $3(x + 11)^2 = 60$ |
| 6. $x^2 = 42$ | 17. $(x + 2)^2 = -25$ | 27. $(3x + 4)^2 = 27$ |
| 7. $x^2 - 62 = 0$ | 18. $(x - 5)^2 = 36$ | 28. $(2x + 1)^2 = 48$ |
| 8. $x^2 - 75 = 0$ | 19. $(x + 1)^2 = \frac{1}{4}$ | 29. $(5x - 2)^2 = 63$ |
| 9. $x^2 - 45 = 0$ | 20. $(x - 9)^2 = -\frac{9}{25}$ | 30. $(4x - 3)^2 = 125$ |
| 10. $x^2 - 98 = 0$ | 21. $(x - 3)^2 = \frac{4}{9}$ | 31. $3(x - 5)^2 + 5 = -25$ |
| 11. $3x^2 = 54$ | | 32. $2(x - 6)^2 - 11 = 25$ |

Rewrite each of the quadratic equations in the form $ax^2 + bx + c = 0$ with $a > 0$; then identify the constants a , b , and c .

33. $x^2 - 3x = 2$

34. $x^2 + 2 = 5x$

35. $x = 2x^2 + 6$

36. $5x^2 = 3x - 1$

37. $4x + 3 = 7x^2$

38. $x = 4 - 3x^2$

39. $4 = 3x^2 - 9x$

40. $6x + 4 = 3x^2$

41. $x^2 + 5x = 3 - x^2$

42. $x^2 + 4x - 1 = 2x + 3x^2$

Solve each of the quadratic equations by using the quadratic formula. See Examples 7 through 13.

43. $x^2 - 4x - 1 = 0$

44. $x^2 - 3x + 1 = 0$

45. $x^2 - 3x = 4$

46. $x^2 + 5x = 2$

47. $-2x^2 + x = -1$

48. $-3x^2 + x = -1$

49. $5x^2 + 3x - 2 = 0$

50. $-2x^2 + 5x - 1 = 0$

51. $9x^2 = 3x$

52. $4x^2 - 81 = 0$

53. $x^2 - 7 = 0$

54. $2x^2 + 5x - 3 = 0$

55. $x^2 + 4x = x - 2x^2$

56. $x^2 - 2x + 1 = 2 - 3x^2$

57. $3x^2 + 4x = 0$

58. $4x^2 - 10 = 0$

59. $\frac{2}{5}x^2 + x - 1 = 0$

60. $2x^2 + 3x - \frac{3}{4} = 0$

61. $-\frac{x^2}{2} - 2x + \frac{1}{3} = 0$

62. $\frac{x^2}{3} - x - \frac{1}{5} = 0$

63. $(2x + 1)(x + 3) = 2x + 6$

64. $(x + 5)(x - 1) = -3$

65. $\frac{5x + 2}{3x} = x - 1$

66. $-6x^2 = -3x - 1$

67. $4x^2 = 7x - 3$

68. $9x^2 - 6x - 1 = 0$

Solve each of the quadratic equations by using any method.

69. $2x^2 + 7x + 3 = 0$

70. $5x^2 - x - 4 = 0$

71. $3x^2 - 7x + 1 = 0$

72. $2x^2 - 2x - 1 = 0$

73. $10x^2 = x + 24$

74. $9x^2 + 12x = -2$

75. $3x^2 - 11x = 4$

76. $5x^2 = 7x + 5$

77. $-4x^2 + 11x - 5 = 0$

78. $-4x^2 + 12x - 9 = 0$

79. $10x^2 + 35x + 30 = 0$

80. $6x^2 + 2x = 20$

81. $25x^2 + 4 = 20$

82. $3x^2 - 4x + \frac{1}{3} = 0$

83. $\frac{3}{4}x^2 - 2x + \frac{1}{8} = 0$

84. $\frac{11}{2}x + 1 = 3x^2$

85. $\frac{3}{7}x^2 = \frac{1}{2}x + 1$

86. $\frac{35}{4x} = x - 1$