

5.5 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

- When a number is multiplied by itself, the product is said to be that number's _____.
- To reverse the process of squaring, find the _____ of the number.
- If a is a nonnegative real number, then \sqrt{a} is the _____ square root of a .
- In $\sqrt{102}$, the symbol $\sqrt{\quad}$ is called the _____ sign and 102 is the _____.
- When converted to decimal form, rational numbers either _____ or _____.
- When rationalizing the denominator, the objective is to find an _____ fraction that has a rational number for a denominator.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

- If a number is squared and the principal square root of the result is found, that square root is always equal to the original number.
- There is no real number that can be a square root of a negative number.
- The simplest form of a radical expression can be found by using prime factorization.
- The fraction $\frac{\sqrt{2}}{3}$ is in simplest form.

Practice

Simplify the following square roots. See Examples 1 and 2.

- | | | |
|-----------------|---------------------------|----------------------------|
| 1. $\sqrt{9}$ | 7. $\sqrt{169}$ | 13. $-\sqrt{100}$ |
| 2. $\sqrt{49}$ | 8. $\sqrt{361}$ | 14. $-\sqrt{144}$ |
| 3. $\sqrt{81}$ | 9. $\sqrt{\frac{1}{4}}$ | 15. $-\sqrt{0.0016}$ |
| 4. $\sqrt{36}$ | 10. $\sqrt{\frac{9}{16}}$ | 16. $-\sqrt{0.000004}$ |
| 5. $\sqrt{289}$ | 11. $\sqrt{0.04}$ | 17. $\sqrt{\frac{9}{25}}$ |
| 6. $\sqrt{121}$ | 12. $\sqrt{0.0081}$ | 18. $\sqrt{\frac{25}{81}}$ |

Estimates (rounded to the nearest ten-thousandth) of radicals are given. Show that these are reasonable estimates. See Example 3.

19. $\sqrt{74} \approx 8.6023$

21. $\sqrt{32} \approx 5.6569$

20. $\sqrt{18} \approx 4.2426$

22. $\sqrt{110} \approx 10.4881$

Use your knowledge of square roots to determine whether each number is rational, irrational, or not a real number.

23. $\sqrt{4}$

27. $\sqrt{\frac{2}{9}}$

24. $\sqrt{17}$

28. $-\sqrt{\frac{1}{4}}$

25. $\sqrt{169}$

29. $\sqrt{-36}$

26. $\sqrt{8}$

Simplify each of the following radical expressions. See Example 4.

30. $\sqrt{12}$

39. $-\sqrt{121}$

31. $-\sqrt{45}$

40. $\sqrt{\frac{1}{4}}$

32. $\sqrt{288}$

41. $\sqrt{\frac{32}{49}}$

33. $-\sqrt{63}$

42. $-\sqrt{\frac{11}{64}}$

34. $-\sqrt{72}$

43. $-\sqrt{\frac{125}{100}}$

35. $\sqrt{98}$

36. $-\sqrt{56}$

44. $\sqrt{\frac{28}{25}}$

37. $\sqrt{162}$

45. $\sqrt{\frac{147}{100}}$

38. $-\sqrt{125}$

Simplify the following radical expressions. Assume that all variables represent positive real numbers. See Examples 5 and 6.

46. $3\sqrt{2} + 5\sqrt{2}$

52. $3\sqrt{2} + 5\sqrt{3} - 2\sqrt{3} + \sqrt{2}$

47. $7\sqrt{3} - 2\sqrt{3}$

53. $\sqrt{5} + \sqrt{4} - 2\sqrt{5} + 6$

48. $4\sqrt{11} - 3\sqrt{11}$

54. $\sqrt{12} + \sqrt{27}$

49. $6\sqrt{5} + \sqrt{5}$

55. $\sqrt{32} - \sqrt{18}$

50. $8\sqrt{10} - 11\sqrt{10}$

56. $3\sqrt{5} - \sqrt{45}$

51. $\sqrt{7} + 6\sqrt{7} - 2\sqrt{7}$

57. $2\sqrt{7} + 5\sqrt{28}$

58. $\sqrt{50} - \sqrt{18} - 3\sqrt{12}$

59. $2\sqrt{48} - \sqrt{54} + \sqrt{27}$

60. $2\sqrt{20} - \sqrt{45} + \sqrt{36}$

61. $\sqrt{18} - 2\sqrt{12} + 5\sqrt{2}$

62. $\sqrt{8} - 2\sqrt{3} + \sqrt{27} - \sqrt{72}$

63. $\sqrt{80} + \sqrt{8} - \sqrt{45} + \sqrt{50}$

Multiply the following radical expressions and then simplify the results. Assume that all variables represent positive real numbers. See Examples 7 through 9.

64. $\sqrt{2}(3 - 4\sqrt{2})$

65. $2\sqrt{7}(\sqrt{7} + 3\sqrt{2})$

66. $3\sqrt{18} \cdot \sqrt{2}$

67. $2\sqrt{10} \cdot \sqrt{5}$

68. $-2\sqrt{6} \cdot \sqrt{8}$

69. $2\sqrt{15} \cdot 5\sqrt{6}$

70. $\sqrt{3}(\sqrt{2} + 2\sqrt{12})$

71. $\sqrt{2}(\sqrt{3} - \sqrt{6})$

72. $(3 + \sqrt{2})(5 - \sqrt{2})$

73. $(\sqrt{6} + 2)(\sqrt{6} - 2)$

74. $(2\sqrt{7} + 4)(\sqrt{7} - 3)$

75. $(5\sqrt{3} - 2)(2\sqrt{3} - 7)$

76. $(\sqrt{5} + 2\sqrt{2})^2$

77. $(2\sqrt{5} + 3\sqrt{2})^2$

78. $(\sqrt{2} + \sqrt{3})(\sqrt{5} - \sqrt{3})$

79. $(\sqrt{6} + \sqrt{5})(\sqrt{6} - \sqrt{2})$

80. $(\sqrt{11} + \sqrt{3})(\sqrt{11} - 2\sqrt{3})$

81. $(3\sqrt{7} + \sqrt{5})(3\sqrt{7} - \sqrt{5})$

Rationalize the denominator and simplify, if possible. Assume that all variables represent positive real numbers. See Examples 10 and 11.

82. $\frac{5}{\sqrt{2}}$

83. $\frac{7}{\sqrt{5}}$

84. $\frac{-3}{\sqrt{7}}$

85. $\frac{-10}{\sqrt{2}}$

86. $\frac{6}{\sqrt{3}}$

87. $\frac{8}{\sqrt{2}}$

88. $\frac{\sqrt{18}}{\sqrt{2}}$

89. $\frac{\sqrt{25}}{\sqrt{3}}$

90. $\frac{\sqrt{5}}{\sqrt{12}}$

91. $\frac{\sqrt{4}}{\sqrt{3}}$

92. $\sqrt{\frac{3}{8}}$

93. $\sqrt{\frac{9}{2}}$

94. $\sqrt{\frac{3}{5}}$

95. $\frac{-10}{3\sqrt{5}}$

96. $\frac{21}{5\sqrt{7}}$

Applications

Solve.

97. Isaac Newton fell asleep under an apple tree thinking about math. While he was sleeping, a squirrel knocked an apple off of a branch of the tree. The equation $t = \sqrt{\frac{2d}{9.8}}$ can be used to find the time t in seconds it takes for the apple to drop a certain distance d , where d is in meters. Round all answers to the nearest hundredth.
- If the apple was connected to a branch 2 m above Newton's head, how long would it take before the apple hit Newton's head?
 - If the squirrel knocked a second apple off a branch that was 5 m above Newton's head, how long would it take before the apple hit Newton's head?
 - Suppose the second apple missed Newton's head and landed on the ground instead. If Newton's head was 0.8 m above the ground, how long would it take for the apple to hit the ground?

Use the following two formulas associated with electricity to answer Exercises 61-64.

$$I = \sqrt{\frac{P}{R}} \quad \begin{array}{l} P = \text{power (in watts)} \\ I = \text{current (in amperes)} \\ E = \text{voltage (in volts)} \\ R = \text{resistance (in ohms, } \Omega) \end{array}$$

- What is the current in amperes of a light bulb that produces 150 watts of power and has a 25Ω resistance?
- If a light bulb has a resistance of 30Ω and produces 90 watts of power, what is its current in amperes?
- How many volts of electricity would Meghan need to produce 48Ω of resistance from a 300 watt lamp?
- A 5000Ω resistor is rated at 2.5 watts. What is the maximum voltage of electricity that should be connected across it?

- 102.** A nut company is determining how to package their new type of party mix. The marketing department is experimenting with different-sized cans for the party mix packaging. The designers use the equation $r = \sqrt{\frac{V}{h\pi}}$ to determine the radius of the can for a certain height h and volume V . The company decides they want the can to have a volume of $1200\pi \text{ cm}^3$. Keep your answers in simplified radical form.
- Find the radius of the can if the height is 12 cm.
 - Find the radius of the can if the height is 10 cm.
 - Find the radius of the can if the height is 8 cm.
- 103.** For a complete radio circuit, $d = \sqrt{2g} + \sqrt{2h}$, where d equals the visual horizon distance and g and h are the heights of the radio antennas at the respective stations. What is d when $g = 75$ ft and $h = 85$ ft?

Writing & Thinking

- 104.** Under what conditions is the expression \sqrt{a} not a real number?
- 105.** Discuss, in your own words, why the square root of a negative number is not a real number.
- 106.** In your own words, explain how to rationalize the denominator of a fraction containing the sum or difference of square roots in the denominator. Why does this work?