Common Error

A **common error** is to divide both sides of an equation by the variable x. This error can be illustrated by using the equation in Example 2.

Wrong Solution

$$3x^2 = 6x$$
$$\frac{3x^2}{x} = \frac{6x}{x}$$
$$3x = 6$$
$$x = 2$$

Do not divide by x, because you lose the solution $x = 0$.

Factoring is the method to use. By factoring, you will find all solutions as shown in the previous examples.

CAUTION

Completion Example Answers

8. $x^2 - 4x + 4 = 64$; $x^2 - 4x - 60 = 0$; $(x+6)(x-10) = 0$; $x + 6 = 0$ or $x - 10 = 0$; $x = -6$ or $x = 10$; The solutions are -6 and 10.

Margin Exercise Answers

1. $y = 7, \frac{5}{3}$ **2.** $v = 0, 2$ **3.** $x = 3$ (double root) **4.** $x = -1, 4$ **5.** $x = 4, 12$ **6.** $x = -\frac{7}{2}, \frac{4}{3}$
7. $x = -1, 3$ **8.** $x = -7, -1$ **9.** $x = -2, 0, 5$

5.4 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. When solving quadratic equations by factoring, it is necessary to have one side of the equation equal to ______.

2. The zero-factor property states that if the product of two or more factors equals zero, then at least one of the factors must be _______.

3. The factor theorem states that if $x = c$ is a root of a polynomial equation in the form $P(x) = 0$, then $x - c$ is a _______ of the polynomial $P(x)$.

4. In general, a quadratic equation has two solutions. If the two solutions are the same number, the equation is said to have a _________ solution or root.

5. Solutions can be checked by _______________ them one at a time for x in the equation.

6. Second-degree polynomials are called ____________ polynomials.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. When solving quadratic equations by factoring, it is important that all of the coefficients are integers.

8. The standard form for a quadratic equation is $ax^2 + bx = c$.

9. Not all quadratic equations can be solved by factoring.

10. All quadratic equations have two distinct solutions.

Practice

Solve each equation. See Example 1.

1. $(x-3)(x-2)=0$
2. $(x+5)(x-2)=0$
3. $(2x-9)(x+2)=0$
4. $(x+7)(3x-4)=0$
5. $0=(x+3)(x+3)$
6. $0=(x+10)(x-10)$
7. $(x+5)(x+5)=0$
8. $(x+5)(x-5)=0$
9. $2x(x-2)=0$
10. $3x(x+3)=0$
11. $(x+6)^2=0$
12. $5(x-9)^2=0$

Solve each equation by factoring.

13. $x^2-3x-4=0$
14. $x^2+7x+12=0$
15. $x^2-x-12=0$
16. $x^2-11x+18=0$
17. $0=x^2+3x$
18. $0=x^2-3x$
19. $x^2+8=6x$
20. $x^2=x+30$
21. $2x^2+2x-24=0$
22. $9x^2+63x+90=0$
23. $0=2x^2-5x-3$
24. $0=2x^2-x-3$
25. $3x^2-4x-4=0$
26. $3x^2-8x+5=0$
27. $2x^2-7x=4$
28. $4x^2+8x=-3$
29. $-2x=3x^2-8$
30. $6x^2+2=-7x$
31. $4x^2-12x+9=0$
32. $25x^2-60x+36=0$
33. $8x=5x^2$
34. $15x=3x^2$

35. $9x^2 - 36 = 0$

36. $4x^2 - 16 = 0$

37. $5x^2 = 10x - 5$

38. $2x^2 = 4x + 6$

39. $8x^2 + 32 = 32x$

40. $6x^2 = 18x + 24$

41. $\frac{x^2}{9} = 1$

42. $\frac{x^2}{2} = 8$

43. $\frac{x^2}{5} - x - 10 = 0$

44. $\frac{2}{3}x^2 + 2x - \frac{20}{3} = 0$

45. $\frac{x^2}{8} + x + \frac{3}{2} = 0$

46. $\frac{x^2}{6} - \frac{1}{2}x - 3 = 0$

47. $x^2 - x + \frac{1}{4} = 0$

48. $\frac{x^2}{3} - 2x + 3 = 0$

49. $x^3 + 8x = 6x^2$

50. $x^3 = x^2 + 30x$

51. $6x^3 + 7x^2 = -2x$

52. $3x^3 = 8x - 2x^2$

53. $0 = x^2 - 100$

54. $0 = x^2 - 121$

55. $3x^2 - 75 = 0$

56. $5x^2 - 45 = 0$

57. $x^2 + 8x + 16 = 0$

58. $x^2 + 14x + 49 = 0$

59. $3x^2 = 18x - 27$

60. $5x^2 = 10x - 5$

61. $(x-1)^2 = 4$

62. $(x-3)^2 = 1$

63. $(x+5)^2 = 9$

64. $(x+4)^2 = 16$

65. $(x+4)(x-1) = 6$

66. $(x-5)(x+3) = 9$

67. $27 = (x+2)(x-4)$

68. $-1 = (x+2)(x+4)$

69. $x(x+7) = 3(x+4)$

70. $x(x+9) = 6(x+3)$

71. $3x(x+1) = 2(x+1)$

72. $2x(x-1) = 3(x-1)$

73. $x(2x+1) = 6(x+2)$

74. $3x(x+3) = 2(2x-1)$

Applications

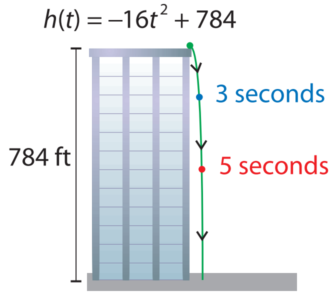
Solve.

75. A ball is dropped from the top of a building that is 784 feet high. The height of the ball above ground level is given by the polynomial function $h(t) = -16t^2 + 784$ where t is measured in seconds.

a. How high is the ball after 3 seconds? 5 seconds?

b. How far has the ball traveled in 3 seconds? 5 seconds?

c. When will the ball hit the ground? Explain your reasoning in terms of factors.



76. A tennis ball is dropped from a building. The position of the ball after t seconds is given by the polynomial function $s(t) = -4.9t^2 + 490$, where s is the height in meters of the ball.

a. Find $s(0)$. What does this value represent in the context of this problem?

b. How high is the tennis ball 2 seconds after it has been dropped?

c. How long before the tennis ball hits the ground?

77. A ball is thrown upward from an initial height of 96 feet with an initial velocity of 16 feet per second. After t seconds, the height of the ball can be described by the equation $h = -16t^2 + 16t + 96$.

a. What happens when $h = 0$?

b. Rewrite the equation with $h = 0$.

c. Solve the equation by factoring.

d. What does the answer to Part **c.** mean?

e. Do both solutions from Part **c.** make sense in the context of the problem? Explain why or why not.

78. Robin is putting the finishing touches on a quilt. The quilt is currently 80 inches long by 60 inches wide and she plans to add a border around the quilt. The width of the border on the sides will be twice the width of the border on the top and bottom of the quilt.

a. If x is the width of the border in inches that will be added to the top and bottom of the quilt, write an expression for the length and width of the quilt with the border added.

b. Write a simplified expression to find the area of the quilt with the border added.

c. Robin has a total of 5712 square inches of fabric to use for the back of the quilt. Use the expression from Part **b.** to write an equation to describe the total area of the back of the quilt.

d. Solve the quadratic equation from Part **c.** by factoring.

e. Do both of the solutions from Part **d.** make sense in the context of the problem?

f. What is the total length and width of the quilt?

79. We know that the area of a circle is proportional to the square of the radius. In fact, if the radius of a circle is r, then the area of the circle is $A = \pi r^2$. Let's determine how the area is changed when we double the radius.

a. Find the area of the circle for a radius of 1 in.

b. Find the area of the circle for a radius of 2 in.

c. Find the area of the circle for a radius of 4 in.

d. Find the area of the circle for a radius of 8 in.

e. Do you see the pattern? If you double the radius, by how many times does the area increase?

80. The St. Louis Arch is not quite in the shape of a parabola, but it can be closely modeled with the polynomial function $h(x) = -0.007x^2 + 0.003x + 625$, where x and $h(x)$ are both measured in feet and the center of the arch lies along the y-axis.

a. Find $h(0)$. What does this mean in this context?

b. Find $h(100)$. What does this mean in this context?

c. Find $h(300)$. Use this value to approximate the total distance between the two points at which the arch hits the ground.

d. Use your graphing calculator to sketch the graph and find the x-intercepts (to the nearest integer). What is the actual distance between the two points at which the arch hits the ground? (rounded to the nearest foot)

Writing & Thinking

81. When solving equations by factoring, one side of the equation must be 0. Explain why this is so.

82. In solving the equation $(x+5)(x-4) = 6$, why can't we just put one factor equal to 3 and the other equal to 2? Certainly $3 \cdot 2 = 6$.