Thus, for the original expression, we have the following.

$$18x^3 - 39x^2 + 18x = 3x\left(6x^2 - 13x + 6\right)$$
$$= 3x(2x - 3)(3x - 2)$$

Remember to include the original GCF in the final product

Now work margin exercise 7.

Tips to Keep in Mind while Factoring

1. When factoring polynomials, always look for a common factor first. Then, if there is one, remember to include this common factor as part of the answer.
2. **To factor completely** means to find factors of the polynomial such that none of the factors are themselves factorable.
3. Not all polynomials are factorable. (See $x^2 + 3x + 5$ in Example 4.) **Any polynomial that cannot be factored as the product of polynomials with integer coefficients is not factorable.**
4. Factoring can be checked by multiplying the factors. The product should be the original expression.

PROCEDURE

Margin Exercise Answers

1. $(x+3)(x+7)$ **2.** $-2(x+4)(x-5)$ **3.** $(4x-7)(2x+3)$ **4.** $3\left(x^2-2x+3\right)$
5. $(3x-2)(3x+1)$ **6.** $(x-7)(x+2)$ **7.** $3(x-2)(4x+5)$

5.2 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. When factoring a trinomial with leading coefficient 1, if two factors of the constant term do not exist, the trinomial is not _____.
2. The first step when factoring should be to factor out any common _____ factor.
3. To factor a trinomial with leading coefficient other than 1, the FOIL method is used with more of a _____ approach.
4. When factoring a trinomial, if the sigh of the constant term is _____, the signs of both factors will be the same.
5. The *ac*-method of factoring involves the method of factoring by _____.

6. To factor _____ means to find factors of the polynomial such that none of the factors themselves are factorable.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. In a trinomial such as $x^2 - 5x + 4$, one would need to find two factors of 4 whose sum is negative 5.

8. A trinomial is factorable if the middle term is the difference of the inner and outer products of two binomials.

9. The first step in the *ac*-method of factoring is to rewrite the middle term.

10. Factoring can be checked by multiplying the factors and verifying that the product matches the original polynomial.

Practice

Completely factor each polynomial. If a polynomial cannot be factored, write not factorable.

1. $x^2 + 9x + 18$

2. $y^2 - 7y - 30$

3. $x^2 - 6x - 27$

4. $y^2 - 5y - 14$

5. $x^2 - 27x + 50$

6. $x^2 - 15x + 36$

7. $2x^2 + 15x - 8$

8. $2x^2 + 9x - 35$

9. $6x^2 + 13x + 6$

10. $8y^2 + 10y - 25$

11. $2x^2 - 7x + 4$

12. $6x^2 - 35x - 5$

13. $2x^2 - 7x - 4$

14. $35y^2 + 9y - 18$

15. $18x^2 - 7x - 1$

16. $25x^2 + 5x - 6$

17. $12x^3 + 38x^2 + 20x$

18. $8x^3 - 6x^2 - 2x$

19. $2x^4 + 11x^2y - 15y^2$

20. $2x^4 + 11x^2y - 21y^2$

21. $2x^6 + 9x^3y^2 + 4y^4$

22. $5x^4 + 17x^2y^2 + 6y^4$

23. $-18x^2 + 72x - 8$

24. $-45y^2 + 30y + 120$

25. $-5y^2 + 40y - 60$

26. $-12x^2 + 22x + 4$

27. $21x^4 - 4x^3 - 32x^2$

28. $2x^4y^3 - 5x^3y^3 - 18x^2y^3$

29. $5x^4 + 17x^2y^2 + 6y^4$

30. $2x^6 + 9x^3y^2 + 4y^4$

31. $x^2(x-5) + 4x(x-5) - 21(x-5)$

32. $x^2(x+4) - 6x(x+4) - 15(x+4)$

33. $2x^2(2x+1)-9x(2x+1)-18(2x+1)$

34. $3x^2(4x-1)+19x(4x-1)+28(4x-1)$

35. $(2x+y)^2-9(2x+y)+20$

36. $(x-2y)^2+10(x-2y)+16$

37. $(x+3y)^2-14(x+3y)-32$

38. $(x+5y)^2+8(x+5y)+12$

39. $(6x-y)^2+8(6x-y)+7$

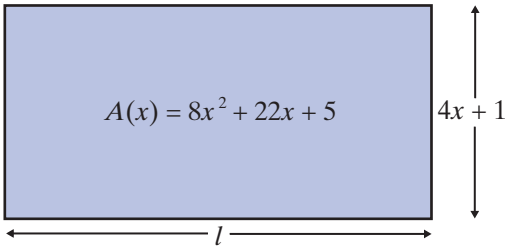
40. $5(3x-y)^2+15(3x-y)-20$

41. $4(2a+b)^2+4(2a+b)-24$

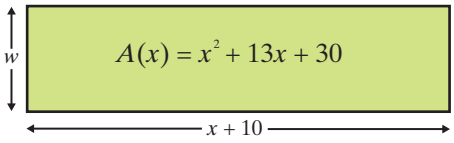
42. $-6(a-b)^2+6(a-b)+180$

Applications

43. The area (in square inches) of the rectangle shown is given by the polynomial function $A(x)=8x^2+22x+5$. If the width of the rectangle is $4x+1$ inches, what is the representation of the length?



44. The area (in square meters) of the rectangle shown is given by the polynomial function $A(x)=x^2+13x+30$. If the length of the rectangle is $(x+10)$ meters, what is the representation of the width?

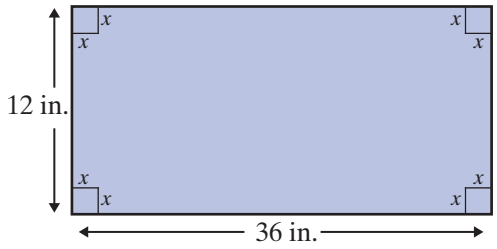


45. The volume of an open box is found by cutting equal squares (x units on a side) from a sheet of cardboard that is 12 inches by 36 inches. The function representing the volume is $V(x)=4x^3-96x^2+432x$, where $0<x<6$.

a. Factor this function in such a way that the factors represent the lengths of the sides of the box.

b. What is the value of $V(2)$?

c. What is the value of $V(4)$?



Writing & Thinking

46. The following statement is true:

$$4x^2 + 24x + 20 = (4x + 20)(x + 1) = (x + 5)(4x + 4) = (2x + 10)(2x + 2).$$

Explain how the trinomial can be factored in three ways. Is there some kind of error?

47. The following statement is true: $5x^2 + 5x - 60 = (5x + 20)(x - 3)$.
Explain why this is not the completely factored form of the trinomial.

48. Explain, in your own words, what is meant by factoring a polynomial.