5.1 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this section.

1. The result of multiplication is called the _____ and the numbers or expressions being multiplied are called _____ of the product.
2. The reverse of multiplication with polynomials is called _____.
3. GCF stands for _____ _____ _____. The GCF of a set of numbers is the _____ positive integer that is a factor of all numbers in the set.
4. Factoring polynomials with four or more terms can sometimes be accomplished by _____ terms and using the distributive property.
5. If the leading coefficient in a polynomial is a negative number, you may choose to factor out the _____ of the GCF.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

6. When finding the GCF of a polynomial, you need to consider only the coefficients.
7. An expression is factored completely if none of its factors can be factored.
8. One way to find the GCF of a set of numbers is to use the prime factorization of each number.
9. Binomials cannot be factored out of algebraic expressions.

Practice

Find the GCF for each set of terms. See Example 1.

1. $\{10, 15, 20\}$
2. $\{25, 30, 75\}$
3. $\{16, 40, 56\}$
4. $\{30, 42, 54\}$
5. $\{9, 14, 22\}$
6. $\{44, 66, 88\}$
7. $\{30x^3, 40x^5\}$
8. $\{15y^4, 25y\}$
9. $\{8a^3, 16a^4, 20a^2\}$
10. $\{36xy, 48xy, 60xy\}$
11. $\{26ab^2, 39a^2b, 52a^2b^2\}$
12. $\{28c^2d^3, 14c^3d^2, 42cd^2\}$
13. $\{45x^2y^2z^2, 75xy^2z^3\}$
14. $\{21a^5b^4c^3, 28a^3b^4c^3, 35a^3b^4c^2\}$

Simplify each expression. See Example 2.

15. $\frac{x^7}{x^3}$

16. $\frac{x^8}{x^3}$

17. $\frac{-8y^3}{2y^2}$

18. $\frac{12x^2}{2x}$

19. $\frac{9x^5}{3x^2}$

20. $\frac{-10x^5}{2x}$

21. $\frac{4x^3y^2}{2xy}$

22. $\frac{21x^4y^3}{-3xy^2}$

23. $\frac{8y^3-16y^2+24y}{8y}$

24. $\frac{18x^4+24x^3+36x^2}{6x^2}$

25. $\frac{34x^5-51x^4+17x^3}{17x^3}$

26. $\frac{14y^4+28y^3+12y^2}{2y^2}$

27. $\frac{110x^4-121x^3+11x^2}{11x}$

28. $\frac{15x^7+30x^6-45x^3}{15x^3}$

29. $\frac{-56x^4+98x^3-35x^2}{14x^2}$

30. $\frac{108x^6-72x^5+63x^4}{18x^4}$

31. $\frac{16y^6-56y^5-120y^4+64y^3}{16y^3}$

32. $\frac{20y^5-14y^4+21y^3+42y^2}{4y^2}$

Complete the factoring of the polynomial as indicated.

33. $3m+27=3(\quad)$

34. $2x+18=2(\quad)$

35. $5x^2-30x=5x(\quad)$

36. $6y^3-24y^2=6y^2(\quad)$

37. $13ab^2+13ab=13ab(\quad)$

38. $8x^2y-4xy=4xy(\quad)$

39. $-15xy^2-20x^2y-5xy=-5xy(\quad)$

40. $-9m^3-3m^2-6m=-3m(\quad)$

Factor each polynomial by finding the GCF (or –1 · GCF). See Examples 3 through 6.

41. $11x-121$

42. $14x+21$

43. $16y^3+12y$

44. $-3x^2+6x$

45. $-6ax+9ay$

46. $4ax-8ay$

47. $10x^2y-25xy$

48. $16x^4y-14x^2y$

49. $-18y^2z^2+2yz$

50. $-14x^2y^3-14x^2y$

51. $8y^2-32y+8$

52. $5x^2-15x-5$

53. $2xy^2-3xy-x$

54. $ad^2+10ad+25a$

55. $8m^2x^3 - 12m^2y + 4m^2z$

56. $36t^2x^4 - 45t^2x^3 + 24t^2x^2$

57. $-56x^4z^3 - 98x^3z^4 - 35x^2z^5$

58. $34x^4y^6 - 51x^3y^5 + 17x^5y^4$

59. $15x^4y^2 + 24x^6y^6 - 32x^7y^3$

60. $-3x^2y^4 - 6x^3y^4 - 9x^2y^3$

Factor each expression by factoring out the common binomial factor. See Example 7.

61. $7y^2(y+3) + 2(y+3)$

62. $6a(a-7) - 5(a-7)$

63. $3x(x-4) + (x-4)$

64. $2x^2(x+5) + (x+5)$

65. $4x^3(x-2) - (x-2)$

66. $9a(x+1) - (x+1)$

67. $10y(2y+3) - 7(2y+3)$

68. $a(x+5) + b(x+5)$

69. $a(x-2) - b(x-2)$

70. $3a(x-10) + 5b(x-10)$

Factor each of the polynomials by grouping. If a polynomial cannot be factored, write "not factorable." See Examples 8 through 12.

71. $bx + b + cx + c$

72. $3x + 3y + ax + ay$

73. $x^3 + 3x^2 + 6x + 18$

74. $2z^3 - 14z^2 + 3z - 21$

75. $10a^2 - 5az + 2a + z$

76. $x^2 - 4x + 6xy - 24y$

77. $3x + 3y - bx - by$

78. $ax + 5ay + 3x + 15y$

79. $5xy + yz - 20x - 4z$

80. $x - 3xy + 2z - 6zy$

81. $z^2 + 3 + az^2 + 3a$

82. $x^2 - 5 + x^2y + 5y$

83. $6ax + 12x + a + 2$

84. $4xy + 3x - 4y - 3$

85. $xy + x + y + 1$

86. $xy + x - y - 1$

87. $10xy - 2y^2 + 7yz - 35xz$

88. $7xy - 3y + 2x^2 - 3x$

89. $3xy - 4uy - 6vx + 8uv$

90. $xy + 5vy + 6ux + 30uv$

91. $3ab + 4ac + 2b + 6c$

92. $24y - 3yz + 2xz - 16x$

93. $6ac - 9ad + 2bc - 3bd$

94. $2ac - 3bc + 6ad - 9bd$

Applications

Solve.

95. Bonnie volunteers to bring bags of candy to her child's class for the Halloween party this year. She buys one bag of candy A containing 150 pieces of candy, one bag of candy B containing 180 pieces of candy, and one bag of candy C containing 330 pieces of candy. She needs to use all the candy to create identical treat bags. How many treat bags can Bonnie make so that each one has the same number and variety of candy? How many of each type of candy will be in each bag?

96. The area of a rectangular photo can be represented by the polynomial $15x^2 + 5x$.

a. If $x = 2$ inches, find the area of the photo.

b. Factor the polynomial to find a variable expression for the length and width of the photo.

c. If $x = 2$ inches, use the answer from Part **b.** to find the length and the width of the photo.

d. Find the area of the photo by multiplying the length and width values from Part **c.**

e. Are the answers from Parts **a.** and **d.** the same? Explain why or why not.

97. A circus performer is shot vertically into the air with an initial velocity of 48 feet per second. The height of the performer above the ground in feet can be described by the polynomial $48x - 16x^2$ after x seconds.

a. Find the height of the circus performer after 2 seconds.

b. Factor the polynomial $48x - 16x^2$.

c. Use the factored form of the polynomial from Part **b.** to find the height of the circus performer after 2 seconds.

d. Are the answers from Parts **a.** and **c.** the same? Explain why or why not.

Writing & Thinking

98. Explain why the GCF of $-3x^2 + 3$ is 3 and not -3.