5. Evaluate the polynomial for the given values.

a. $P(x) = x^3 + 4x^2 - 7x - 9$, for $P(3)$.

b. $P(x,y) = 3xy^2 + 2xy - 5x + 6y + 13$, for $x = -4$ and $y = -1$

Example 5 Evaluating Polynomials

a. For the polynomial $P(x) = x^3 - 2x^2 + 3x + 5$, find $P(4)$.

b. Evaluate the polynomial $P(x, y) = 2x^2y - xy + 3x - 4y + 15$ for $x = -1$ and $y = -6$.

Solution

a. Substitute 4 for x throughout the polynomial.

$$\begin{aligned} P(4) &= 4^3 - 2(4)^2 + 3(4) + 5 = 64 - 32 + 12 + 5 \\ &= 32 + 12 + 5 \\ &= 49 \end{aligned}$$

b. Substitute -1 for x and -6 for y throughout the polynomial.

$$\begin{aligned} P(-1, -6) &= 2(-1)^2(-6) - (-1)(-6) + 3(-1) - 4(-6) + 15 \\ &= -12 - 6 - 3 + 24 + 15 \\ &= -21 + 39 \\ &= 18 \end{aligned}$$

Now work margin exercise 5.

Margin Exercise Answers

1. a. $x^4 + 2x^3 + 3x^2 + 5$ **b.** $4x^2 + 9x + 5$ **2.** $7x^3 - 8x^2 - 3x - 2$ **3.** $-2xy^2 + 7x - 5y$ **4. a.** $-3x^2 - 4x + 2$ **b.** $3x^4 - 9x^3 + 2x^2 + 12x - 2$ **5. a.** 33 **b.** 23

4.4 Exercises

Concept Check

Fill-in-the-Blank. Complete the sentences using information found in this chapter.

1. A monomial in x is an expression of the form kx^n, where n is a _____ number and k is the _____.

2. In general, expressions with _____ in the denominator are not polynomials.

3. A _____ is a monomial or the indicated sum and/or difference of monomials.

4. The coefficient of the term of the largest degree is the _____ coefficient.

5. The _____ of two or more polynomials can be found by combining like terms

6. In function notation, indicates that P is the ____ of the polynomial and x is the _____ used in the polynomial.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. A nonzero constant is a monomial with no degree.

8. A monomial is a polynomial with one term.

9. When subtracting one polynomial from another polynomial, only the first term of the polynomial is subtracted.

Practice

State whether the expression is or is not a polynomial. If the expression is a polynomial, state its degree, its classification as a monomial, binomial, or trinomial, and its leading coefficient.

1. $x^3 - x^2$

2. 9

3. $-3x^{\frac{1}{2}} + x$

4. $x^4 + 8x^3 - y^2$

5. $\frac{1}{2}y^2 + \frac{5}{4}y^3 - \frac{7}{4}y$

6. $x^2 + y^2 - \frac{1}{y}$

7. 0

8. $-\sqrt{2}$

9. $\left(x^5 - y^3\right)^{\frac{1}{2}}$

10. $\frac{3}{2}x^2 - \sqrt{3}x - 7$

11. $7x^2 - 6x + 9x^{\frac{2}{3}}$

12. $\frac{x^3 - 3y^2}{x}$

Find the indicated sums and differences. Simplify each answer. See Examples 1 and 3.

13. $\left(3x^2 - 5x + 1\right) + \left(x^2 + 2x - 7\right)$

14. $\left(5x^2 + 8x - 3\right) + \left(-2x^2 + 6x - 4\right)$

15. $\left(x^2 - 9x + 2\right) + \left(-x^2 + 2x - 8\right)$

16. $\left(7x^2 - 4x + 6\right) + \left(4x^2 - 2x + 5\right)$

17. $\left(x^2 + y^2\right) + \left(2x^2 - 5y^2\right)$

18. $\left(x^2 - 3xy + y^2\right) + \left(2x^2 - 5xy - y^2\right)$

19. $\left(2x^2 + 3x + 8\right) - \left(x^2 + 4x - 2\right)$

20. $\left(6x^3 - 5x + 1\right) - \left(2x^3 + 3x - 4\right)$

21. $\left(2x^4 + 3x\right) - \left(5x^3 + 4x + 3\right)$

22. $\left(2x^3 - 3x^2 + 6\right) - \left(x^4 + x + 1\right)$

23. $\left(5x^2 + 6x - 1\right) + \left(x^4 - 3x^2 + 2x\right)$

24. $\left(7x^2 - 2xy + 3y^2\right) + \left(-3x^2 - 2xy + 5y^2\right)$

25. $\left(4x^3 - 7x^2 + 3x + 2\right) - \left(-2x^3 - 5x - 1\right)$

26. $\left(4x^2 - 8xy - 2y^2\right) + \left(-9x^2 + 5xy - 6y^2\right)$

27. $\left(3x^2 - 2y^2\right) + \left(7xy + 4y^2\right) - \left(-6x^2 - 6xy + 8y^2\right)$

28. $\left(9xy+8y^2\right)-\left(6x^2-8xy\right)+\left(5x^2-3xy+7y^2\right)$

29. $\left(5x^3-14x^2\right)-\left(5x^2+2x+1\right)-\left(-7x^3+2x^2-13\right)$

30. $\left(7x^3+4x^2-x\right)+\left(3x^3-4x+5\right)-\left(8x^3+x^2-x+3\right)$

31. $x^3-\left[3x^2-1-\left(x^3+4x^2+1\right)\right]+\left(3x^3-3x^2-2\right)$

32. $3x-4xy+\left[6y+\left(4x+3xy+2y\right)\right]-\left[-6x-\left(xy-4y\right)\right]$

33. $x^2-2xy+\left[y^2-\left(3xy+2y^2\right)-\left(3x^2-xy-2y^2\right)\right]$

34. $\left[\left(4x^2-3x\right)-\left(2x^2+5x\right)\right]+\left[\left(x^2-6x\right)+\left(-3x^2+x\right)\right]$

35. $\left[\left(2x+xy-y\right)+\left(x-2xy+4y\right)\right]-\left[\left(-3x+5xy+y\right)-\left(2x+3xy-2y\right)\right]$

Find each sum. See Example 2.

36.
$$\begin{array}{r} 2x^2-5x-6 \\ \underline{-3x^2+2x-1} \end{array}$$

37.
$$\begin{array}{r} x^3+2x^2+x-2 \\ \underline{x^3-2x^2-3x-1} \end{array}$$

38.
$$\begin{array}{r} 5x^3-4x^2\qquad -9 \\ \underline{2x^3-3x^2-6x+5} \end{array}$$

39.
$$\begin{array}{r} 3x^4+3x^3+x^2+x+2 \\ \underline{7x^4-x^3-5x^2+x-1} \end{array}$$

40.
$$\begin{array}{r} 14x^3+13x^2+10x-13 \\ \underline{20x^3\qquad\qquad -18x+25} \end{array}$$

Find each difference. See Example 4.

41.
$$\begin{array}{r} 9x^2-2x+3 \\ \underline{-\left(4x^2+5x-2\right)} \end{array}$$

42.
$$\begin{array}{r} -3x^2+7x-6 \\ \underline{-\left(2x^2-x+6\right)} \end{array}$$

43.
$$\begin{array}{r} 5x^3\qquad -10x+15 \\ \underline{-\left(x^3-4x^2-3x-9\right)} \end{array}$$

44.
$$\begin{array}{r} x^3-8x^2+12x+5 \\ \underline{-\left(-3x^3+8x^2+2x+5\right)} \end{array}$$

45.
$$\begin{array}{r} 2x^4-5x^3-6x^2+7x+7 \\ \underline{-\left(x^4\qquad +2x^2+4x+10\right)} \end{array}$$

Evaluate each polynomial in for the specified value(s) of the variable(s). See Example 5.

46. Given $P(x)=2x^2-x+3$; find $P(1)$.

47. Given $P(x)=3x^2-2x+5$; find $P(2)$.

48. Given $P(x)=3-x^2$; find $P(-2)$.

49. Given $P(x)=x^3-2x^2+x-1$; find $P(2)$.

50. Given $P(x)=x^3+x^2-4$; find $P(-3)$.

51. Given $P(x)=4x^3-2x^2-1$; find $P(-4)$.

52. Given $P(x, y) = 2x^2 - 3xy + y^2$; find $P(2, -2)$.

53. Given $P(x, y) = 4x - 2xy + 5y$; find $P(1, 1)$.

54. Given $P(x, y, z) = 3x + 4xy - 2yz + z$; find $P(1, 0, 2)$.

55. Given $P(x, y, z) = 2xyz - 3x + yz - xz$; find $P(2, -1, 2)$.

56. Use a graphing calculator to graph the following linear functions.

a. $P(x) = -2x + 5$

b. $P(x) = \frac{1}{4}x$

57. Use a graphing calculator to graph the following quadratic functions.

a. $P(x) = -x^2$

b. $P(x) = x^2 - 4x + 4$

58. Use a graphing calculator to graph the following cubic functions.

c. $P(x) = x^3$

d. $P(x) = x^3 - 4x$

Writing & Thinking

59. Write the definition of a polynomial.

60. Explain, in your own words, how to subtract one polynomial from another.

61. Describe what is meant by the degree of a polynomial in x.

62. Give two examples that show how the sum of two binomials might not be a binomial.