Remember that the choice of steps in dealing with exponents is yours. As long as you correctly apply the properties of exponents, the answer will be the same regardless of the order of the steps.

Summary of Properties and Rules for Exponents

If a and b are nonzero real numbers and m and n are integers:

1. The exponent 1: $a^1 = a$
2. The exponent 0: $a^0 = 1$
3. Product rule: $a^m \cdot a^n = a^{m+n}$
4. Quotient rule: $\dfrac{a^m}{a^n} = a^{m-n}$
5. Negative exponents: $a^{-n} = \dfrac{1}{a^n}$

The Power Rules

6. Power rule: $(a^m)^n = a^{mn}$
7. Power of a product: $(ab)^n = a^n b^n$
8. Power of a quotient: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

DEFINITION

Margin Exercise Answers

1. a. 32 **b.** 125 **c.** $-10x^7$ **d.** $-35a^2b^7$ **2. a.** 1 **b.** 1 **c.** 1 **d.** 1 **3. a.** x^2 **b.** 1 **c.** $3x^8$ **d.** $7x^6y^4$ **4.** a. $\dfrac{1}{x^7}$ **b.** $\dfrac{x^3}{y^3}$ **c.** $\dfrac{1}{x^4}$ **d.** $\dfrac{4}{x^3}$ **5. a.** $\dfrac{1}{x^{15}}$ **b.** $\dfrac{-27x^3}{y^9}$ **c.** $\dfrac{16b^2}{a^2}$ **6.** $\dfrac{y^9}{8x^{18}}$ **7.** $\dfrac{64}{225x^{18}y^2}$

4.1 Exercises

Concept Check

Fill-in-the-Blank. Complete each sentence using information found in this section.

1. Exponents can be used to indicate repeated ____ by the same number.
2. In the expression x^5, x is the ____ and 5 is the ____.
3. To use the ____ rule, multiply powers with the same base, keep the base, and add the exponents.
4. If a is a nonzero real number, then a^0 = ____.

5. An expression is considered simplified if each base appears ____ and each base has only ____ exponents.

6. To raise a power to a power, ____ the exponents.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If a constant does not have an exponent written, it is assumed that the exponent is 0.

8. If a is a nonzero real number an n is an integer, then $a^{-n} = -a^n$.

9. Since the product rule is stated for integer exponents, the rule is also valid for 0 and negative exponents.

10. When using the quotient rule, you should subtract the smaller exponent from the larger exponent.

Practice

Use the properties of exponents to simplify the given expressions. Answers should contain only positive exponents.

1. $\left(7^2\right)\left(7^0\right)$
2. 7^{-2}
3. $3 \cdot 2^2$
4. -5^{-2}
5. $(-8)^{-2}$
6. $x^3 \cdot x^5$
7. $x^2 \cdot x^{-1}$
8. $x^{-2} \cdot x^3 \cdot x^5$
9. $y^{-3} \cdot y^{-2} \cdot y^0$
10. $\dfrac{x^{12}}{x^4}$
11. $\dfrac{x^2}{x^{-1}}$
12. $\dfrac{y^2}{y^{-5}}$
13. $\dfrac{x^3 x^5}{x^4}$
14. $\dfrac{x^0 x^3}{x^6}$
15. $\dfrac{x \cdot x^3}{x^5}$
16. $\dfrac{x^{-1} x^3}{x^{-4}}$
17. $\dfrac{x \cdot x^{-2}}{x^2 x^{-3}}$
18. $\dfrac{x^{16}}{x^{-2} x^{-8}}$
19. $\left(x^4\right)^2$
20. $\left(x^2\right)^{-2}$
21. $\left(x^0\right)^{-1}$
22. $\left(-x^3\right)^0$
23. $\left(y^0 y^{-1}\right)^5$
24. $\left(x^3 x^{-3}\right)^0$
25. $\dfrac{y^2 y^4}{y}$
26. $\dfrac{y \cdot y^4}{y}$
27. $\dfrac{x^5 x^2}{\left(x^2\right)^2}$
28. $\dfrac{x^{10} x^{-3}}{x^3 x^{-1}}$
29. $\dfrac{x^8 x^{-2}}{\left(x^2\right)^3}$
30. $\dfrac{\left(x^{-2}\right)^3}{x \cdot x^{-3}}$
31. $\dfrac{\left(y^2\right)^4}{y^{-2} y^{-1}}$
32. $\left(\dfrac{y^2 y^{-1}}{y^5 y^2}\right)^{-2}$
33. $\left(\dfrac{x^2 x^0}{x^4 x^{-1}}\right)^{-3}$
34. $\left(\dfrac{x^{-3} x^0}{x^2 x}\right)^3$
35. $\left(\dfrac{x^5 x^{-2}}{x \cdot x^{-3}}\right)^2$
36. $x^k \cdot x$

37. $x^k \cdot x^3$

38. $x^k \cdot x^{2k}$

39. $x^{3k} \cdot x^4$

40. $\dfrac{x^k}{x^2}$

41. $\dfrac{x^{2k}}{x^k}$

42. $\dfrac{x^{k+1}}{x^3}$

43. $\left(x^k\right)^2$

44. $\left(x^5\right)^k$

45. $x\left(x^2\right)^k$

46. $\dfrac{x^2x^k}{\left(x^2\right)^k}$

47. $\dfrac{x^{k+1}x^{-2}}{x^4}$

48. $\dfrac{x^{k+3}x}{x^{-2}}$

49. $\left(\dfrac{-3x^{-2}}{y^3}\right)^{-1}$

50. $\left(\dfrac{2ab^4}{3b^2}\right)^{-3}$

51. $\left(\dfrac{x^2y^{-3}}{3x^{-1}y}\right)^{-1}$

52. $\left(x^k y^m\right)^2$

53. $\left(x^{4n}y^3\right)\left(x^n y^{-k}\right)$

54. $\left(x^{k+1}y^{3k}\right)\left(x^2 y^{-k}\right)$

55. $\left(\dfrac{a^2b}{ab^{-2}}\right)\left(\dfrac{a^{-3}b}{b^{-3}}\right)$

56. $\left(\dfrac{x^2y^{-3}}{y^{-1}}\right)^2\left(\dfrac{xy^2}{2y}\right)^{-1}$

57. $\left(\dfrac{x^4y}{2}\right)^2\left(\dfrac{y^3}{x^2}\right)^{-1}$

58. $\left(\dfrac{3x}{2x^2y^2}\right)^{-1}\left(\dfrac{y^3}{2x}\right)^2$

59. $\left(\dfrac{5x^3y}{x^{-2}y^3}\right)^{-1}\left(\dfrac{4x^{-2}y^{-1}}{15xy^4}\right)^{-1}$

60. $\dfrac{\left(7x^3y^4\right)^0}{\left(2x^2y\right)\left(xy^{-3}\right)^{-1}}$

61. $\dfrac{\left(4^{-2}x^{-3}y\right)^{-1}}{\left(x^{-2}y^2\right)^3\left(5xy^{-2}\right)^{-1}}$

62. $\dfrac{\left(6x^2y\right)\left(x^{-1}y^3\right)^2}{\left(x^{-1}y\right)^2\left(3x^2y\right)^3}$

63. $\dfrac{\left(x^{-3}y^{-5}\right)^{-2}\left(x^2y^{-3}\right)^3}{\left(x^3y^{-4}\right)^2\left(x^{-1}y^{-2}\right)^{-2}}$

64. $\dfrac{(4xy)^2\left(x^{-2}y^2\right)^{-1}}{\left(3x^3y\right)^{-2}\left(2x^2y^{-2}\right)^3}$

Writing & Thinking

65. Without looking at the text, show that $\left(\dfrac{x}{y}\right)^{-n} = \left(\dfrac{y}{x}\right)^n$ by using the Power Rules and the Rule for Negative Exponents.