To find Sally's z-score, substitute the data value 1280, the mean 1000, and the standard deviation 140 into the formula and simplify.

$$\text{Sally: } z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{1280 - 1000}{140} = 2$$

To find Janet's z-score, substitute the data value 1440, the mean 1200, and the standard deviation 160 into the formula and simplify.

$$\text{Janet: } z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{1440 - 1200}{160} = 1.5$$

Now compare the two z-scores. Sally has a z-score of 2, which means her score is 2 standard deviations above the mean. Janet has a z-score of 1.5, which means her score is 1.5 standard deviations above the mean. While both scores are above the mean, Sally's z-score is farther from the mean than Janet's, which means that Sally performed better.

Now work margin exercise 5.

Margin Exercise Answers

1. a. 100; **b.** 50 **2. a.** Approximately 95%; **b.** Approximately 68% **3. a.** Approximately 68%; **b.** Approximately 95%; **c.** Approximately 84%; **d.** Approximately 16% **4. a.** 2.4; **b.** −1.5; **c.** 2.65 **5.** Brian has a z-score of 1.5 while Michael has a z-score of 1.25, meaning Brian performed better.

10.5 Exercises

Concept Check

Fill-in-the-blank. Complete the sentences using information found in this section.

1. A _____ _____ is one in which the distribution is bell-shaped with most of the data points clustered in the middle and tapered on the ends and is symmetrical.
2. The normal distribution is completely defined by its mean and _____ _____.
3. In a normal distribution, the _____, _____, and _____ are all the same.
4. When the distribution of data is bell-shaped it is possible to estimate the percentage of data values that fall within a few standard deviations using the _____ _____.
5. The _____ tells us how many standard deviations a data value is from the mean.
6. If a data value is below the mean, then the z-score is _____.

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

7. If a z-score is positive, then the data value is above the mean.

8. According to the Empirical Rule, approximately 95% of the data values fall within one standard deviation above and below the mean.

9. The total area under the normal curve is 1.

10. The normal distribution is bimodal.

Practice

1. Given $\mu = 60$ and $\sigma = 3$. Find the following.

a. The data value that is two standard deviations above the mean.

b. The data value that is three standard deviations above the mean.

c. The data value that is one standard deviation below the mean.

d. The data value that is two standard deviations below the mean.

2. Given $\mu = 45$ and $\sigma = 2.5$. Find the following.

a. The data value that is one standard deviation above the mean.

b. The data value that is two standard deviations above the mean.

c. The data value that is two standard deviations below the mean.

d. The data value that is three standard deviations below the mean.

3. Given $\mu = 30$ and $\sigma = 2$. Find the following.

a. The data value that is two standard deviations above the mean.

b. The data value that is three standard deviations above the mean.

c. The data value that is one standard deviation below the mean.

d. The data value that is three standard deviations below the mean.

Find the z-score for each given value.

4. $\mu = 50$ and $\sigma = 2$

a. $x = 45$

b. $x = 54.5$

c. $x = 51$

5. $\mu = 1200$ and $\sigma = 75$

a. $x = 1357.5$

b. $x = 1102.5$

c. $x = 1290.75$

6. $\mu = 80$ and $\sigma = 4.5$

a. $x = 69.875$

b. $x = 89.45$

c. $x = 68.3$

7. $\mu = 10$ and $\sigma = 0.5$

a. $x = 10.82$

b. $x = 9.15$

c. $x = 11.2$

Use the Empirical Rule to answer the following questions.

8. The lifetime of projector light bulbs are approximately normally distributed with a mean of 1000 hours and a standard deviation of 60.

a. What percentage of the light bulbs last between 940 and 1060 hours?

b. Identify the range of hours that 95% of the projector light bulbs last.

9. Exam scores were approximately normally distributed with a mean of 70 and a standard deviation of 4.

a. What percentage of the exam scores were between 62 and 78 hours?

b. Identify the range of exam scores that 68% of students are likely to earn.

10. The lifetime of car batteries are approximately normally distributed with a mean of 40,000 miles and a standard deviation of 2,000 miles.

a. What percentage of the car batteries last between 38,000 and 42,000 miles?

b. Identify the range of miles that 95% of the car batteries last.

11. The heights of female basketball players are approximately normally distributed with a mean of 72 inches and a standard deviation of 2 inches.

a. What percentage of the female basketball players have heights between 68 inches and 76 inches?

b. Identify the range of heights that includes 99.7% of the female basketball players.

12. The lifetime of car tires are approximately normally distributed with a mean of 14,000 miles and a standard deviation of 600 miles.

a. What percentage of the car tires last between 12,200 and 15,800 miles?

b. Identify the range of miles that 68% of the car tires last.

Use the empirical rule to answer the following questions.

A placement exam is given to all entering students at a local university. The scores are approximately normally distributed with a mean of 50 and a standard deviation of 4.

13. What percent of students scored between 46 and 54?

14. What percent of students scored between 42 and 58?

15. What percent of students scored between 38 and 62?

16. What percent of students scored less than 54?

17. What percent of students scored less than 58?

18. What percent of students scored less than 46?

19. What percent of students scored less than 42?

20. What percent of students scored more than 38?

21. What percent of students scored more than 46?

22. What percent of students scored more than 54?

23. What percent of students scored more than 58?

24. What percent of students scored more than 62?

The battery charge length of the most recent cell phones is approximately normally distributed with a mean of 12 hours and a standard deviation of 1 hour.

25. What percent of batteries lasted between 11 and 13 hours?

26. What percent of batteries lasted between 10 and 14 hours?

27. What percent of batteries lasted between 9 and 15 hours?

28. What percent of batteries lasted less than 11 hours?

29. What percent of batteries lasted less than 14 hours?

30. What percent of batteries lasted less than 9 hours?

31. What percent of batteries lasted more than 11 hours?

32. What percent of batteries lasted more than 14 hours?

33. What percent of batteries lasted more than 15 hours?

34. What percent of batteries lasted more than 13 hours?

Applications

Solve.

35. Jordan and Alicia each took a standardized test for their respective areas for graduate school. Although they took different tests they want to compare their performances. Jordan scored 800, and his test had a mean of 600 with a standard deviation of 80. Alicia scored 1200, and her test had a mean of 1000 with a standard deviation of 100. Both tests' scores are approximately normally distributed. Who performed better?

36. Willow and Lydia are both taking biology but with different teachers. Although they took different tests they want to compare their performances. Willow scored 86, and her test had a mean of 80 with a standard deviation of 4. Lydia scored 82, and her test had a mean of 78 with a standard deviation of 2. Both tests' scores are approximately normally distributed. Who performed better?

37. Jameson and Marie each took a standardized test for their respective areas for graduate school. Although they took different tests they want to compare their performances. Jameson scored 176, and his test had a mean of 151 with a standard deviation of 10. Marie scored 334, and her test had a mean of 300 with a standard deviation of 17. Both tests' scores are approximately normally distributed. Who performed better?

38. Brian and Isabella are both taking psychology at their respective universities. Although they took different final exams they want to compare their performances. Brian scored 85, and his test had a mean of 80 with a standard deviation of 4. Isabella scored 76, and her test had a mean of 68 with a standard deviation of 4. Both exams' scores are approximately normally distributed. Who performed better?

Writing & Thinking

39. Explain the characteristics of a normal distribution.

40. In your own words, explain the empirical rule.