Skill Check 2

If the odds on a bet are 4 : 1 against, what is the probability of winning?

In fact, rather than being an expression for the probability of winning, the term "odds against" represents the ratio of the chance of losing to the chance of winning. In our example, the phrase "odds against of 3 : 1" means that 3 out of 4 times you're likely to lose. Since odds are ratios, you can also express odds as fractions.

$$\text{Odds against} = \frac{P(\text{losing})}{P(\text{winning})}$$

In this particular case, the fraction is

$$\text{Odds against winning} = \frac{\left(\frac{3}{4}\right)}{\left(\frac{1}{4}\right)} = \frac{3}{1} = 3.$$

Skill Check Answers

1.

$x_i P(x_i)$
1.35
1.80
2.25

$E(X) = 5.4$

2. $\frac{1}{5}$ or 20%

We should note that in the betting world, "odds against" are most often quoted because it is the most convenient way to understand the payout if the bet is a successful one for the player. So odds of 20 : 1 mean that for every \$1.00 you bet, the bookie will pay you \$20.00 if you win. The bookie is out to make money; in reality, the odds represent how much the bookie is willing to pay out rather than the true ratio.

Finally, note that odds may also be expressed as "odds for" (also called "odds in favor"), in which case the ratio is

$$\text{Odds for} = \frac{P(\text{winning})}{P(\text{losing})}.$$

8.7 EXERCISES

PRACTICE

Calculate the expected value of each scenario. Round your answer to the nearest hundredth when necessary.

1.

x_i	$P(x_i)$
1	0.21
2	0.58
3	0.06
4	0.15
5	0

2.

x_i	$P(x_i)$
–\$1.50	0.3
\$0.00	0.5
\$2.75	0.1
\$5.00	0.1

3.

x_i	$P(x_i)$
25	$\frac{1}{3}$
15	$\frac{2}{5}$
10	$\frac{1}{15}$
5	$\frac{1}{5}$

4. Let x_i represent the number of even numbers showing when a pair of standard six-sided dice are rolled.

x_i	$P(x_i)$
0	$\frac{9}{36}=\frac{1}{4}=0.25$
1	$\frac{18}{36}=\frac{1}{2}=0.5$
2	$\frac{9}{36}=\frac{1}{4}=0.25$

APPLICATIONS

5. Suppose Piper eats out twice a week 15% of the time, she eats out once a week 35% of the time, and she doesn't eat out anytime during the week 50% of the time. What is the expected value for the number of times Piper eats out during a week?

6. Suppose that you and a friend are playing cards and decide to make a bet. If you draw two aces in succession from a standard deck of cards without replacing the first card, you win $50.00. Otherwise, you pay your friend $10.00.
- **a.** What is the expected value of your bet?
- **b.** If the same bet was made 25 times, how much would you expect to win or lose?

7. A European roulette wheel has only one green slot instead of two. Using Example 2 from this section as a guide, calculate the expected winnings on a European roulette wheel if a player bets $1.00 on red to play the game.

8. Jim likes to day-trade on the Internet. On a good day, he averages a $1100 gain. On a bad day, he averages a $900 loss. Suppose that he has good days 25% of the time, bad days 35% of the time, and the rest of the time he breaks even.
- **a.** What is the expected value for one day of Jim's day-trading hobby?
- **b.** If Jim day-trades every weekday for three weeks, how much money should he expect to win or lose?

9. A university in town is raffling off $20,000 for student scholarships. You can buy one ticket for $10, three tickets for $25, or five tickets for $40. Assume that the university sells 10,000 tickets.
- **a.** Find the expected value for each of the three ticket options: purchasing just one ticket, purchasing three tickets, or purchasing five tickets.
- **b.** Should you buy one, three, or five tickets in order to maximize the money you expect to have at the end of the raffle?

10. You need to borrow money from your sister. She's feeling quirky on the day you ask and says she wants you to flip a coin. Heads, you get $15, tails you get $5. Thinking this is weird, you ask your mother for money instead. She says she'll let you roll a die and she'll give you $2 times the number that appears on the die. Before agreeing to either of these unique offers from the "mathy" folk in your family, you decide to see which is the better offer by calculating the expected value for each method (realizing that you too fit the bill of a "mathy" member of your family). Which offer should you take? Explain your reasoning.

11. Assume that stock in Degree Compass, a predictive analytics company in higher education, returns the percentages shown in the table.

Degree Compass Stock Returns	
Annual Return Rate	**Probability**
15%	0.17
30%	0.51
45%	0.32

Calculate the expected value of the return rate for stock in Degree Compass.

12. During the NCAA basketball tournament season, affectionately called *March Madness*, part of one team's strategy is to always foul their opponent's tall forward. Because he is so tall, he makes 57% of shots he takes close to the basket. However, when he is fouled, his free throw shooting percentage is only 51.5%. The shots he makes close to the basket are worth two points and each of the two free throw shots after being fouled are worth one point.

a. Calculate the expected value of the number of points the forward makes when he takes a shot close to the basket.

b. Calculate the expected value of the number of points the forward makes when he shoots two foul shots.

c. Based on these expected values, is fouling the tall forward a good strategy? Explain your answer.

13. On your next multiple-choice test, each question has four incorrect answers and one correct answer to choose from. Your professor tells you that each correct answer you make, you receive 1 point, but you lose $\frac{1}{4}$ point for each incorrect answer.

a. What is your expected gain or loss on a question if you have no idea of the correct answer and end up simply guessing?

b. What is your expected gain or loss if you guess on all 25 questions?

14. The stock prices of Web Movies on the 1st of the month for the first 6 months of 2014 were \$177.41, \$207.90, \$214.63, \$239.09, \$242.19, and \$259.99 respectively.

a. Calculate the average change in the stock prices of Web Movies in a month.

b. Use linearity of expectation to estimate the stock price on January 1st, 2015.

15. If the odds on a bet are $6:1$ against, what is the probability of winning?

16. Suppose the probability of a football team winning a playoff game is 0.25. What are the odds of winning?

17. Suppose the odds of a teenage male having an accident are $2:3$. What is the probability of a teenage male having an accident?

18. An insurance company claims the probability of surviving a certain type of cancer is 95%. What are the odds of surviving?

19. The UVest investment company publishes that the odds of increasing your wealth with their company is $5:2$. What is the probability of UVest increasing your investment?

20. Odds against being struck by lightning in one year are 1,000,000 to 1.[1]

a. If you live to be 80, what are the odds against being struck by lightning over your lifetime? Assume each year has the same probability.

b. The National Weather Service gives the odds against being struck by lightning over an 80-year lifetime as 10,000 to 1. Why do you think this is different from the answer you got in part **a.**?

21. Overall odds in favor of winning in a state lottery game are 4.63 : 1.

a. Find the probability of winning in the lottery game.

b. The prize for this lottery game is $100. If the cost to play the game is $2.00, what is the expected value for playing this game?

22. Suppose the odds on a bet are 10 : 1 against. Your friend tells you he thinks the odds are too generous. Odds are considered less generous if the probability of losing is greater. Write down some less generous odds.

1 NWS Lightning Safety, http://www.lightningsafety.noaa.gov