Example 8: Applying the Inclusion-Exclusion Principle

A standard deck of playing cards has 52 cards (26 of which are red and 26 of which are black) divided into 4 suits (clubs, spades, diamonds, and hearts), where there are 13 of each suit (ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king). Of these cards, 12 are considered face cards (4 kings, 4 queens, and 4 jacks). Find the number of cards in a standard deck that are either clubs or face cards.

Solution

We start the solution by writing what we are looking for using set notation.

$$|\text{clubs} \cup \text{face cards}| = |\text{clubs}| + |\text{face cards}| - |\text{clubs} \cap \text{face cards}|$$

If the set A consists of clubs and the set B consists of face cards, then this is equivalent to

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

There are 13 clubs in the deck and there are 12 face cards. However, there are 3 face cards that are also clubs (king of clubs, queen of clubs, and jack of clubs). Therefore,

$$|A \cup B| = 13 + 12 - 3 = 22.$$

So, the number of cards that are either clubs or face cards is 22.

Skill Check 3

Find the number of playing cards that are either even (2, 4, 6, 8, 10) or are diamonds.

Skill Check Answers

1. {n, e, t, o}
2. $A \cap B = \{\text{o}\}$, so $(A \cap B)' = \{$a, b, c, d, e, f, g, h, i, j, k, l, m, n, p, q, r, s, t, u, v, w, x, y, z$\}$.
   $A' = \{$a, b, c, e, f, g, i, j, k, l, m, p, q, r, s, t, v, w, x, y, z$\}$ and
   $B' = \{$a, b, d, e, f, g, h, i, j, l, m, n, p, q, s, t, u, v, w, x, y, z$\}$, so
   $A' \cup B' = \{$a, b, c, d, e, f, g, h, i, j, k, l, m, n, p, q, r, s, t, u, v, w, x, y, z$\}$.
   Thus, $(A \cap B)' = A' \cup B'$.
3. 28

8.2 EXERCISES

PRACTICE

Use the given sets to solve each problem.

$U = \{1, 2, 3, \ldots, 20\}$ $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 4, 6, 8, 10, 12\}$ $C = \{5, 7, 9, 11, 13, 15\}$

1. Find $A \cup B$.
2. Find $A \cup C$.

3. Find $B \cap C$.

4. Find $A \cap C$.

5. Verify $(A \cup B)' = A' \cap B'$.

6. Verify $(A \cap B)' = A' \cup B'$.

Use the given sets to solve each problem.

$U = \{a, b, c, d, \ldots, z\}$ $A = \{n, u, m, b, e, r, s\}$ $B = \{r, u, l, e\}$

7. Find $A \cup B$.

8. Find $A \cap B$.

9. Find $|A \cap B|$.

10. Verify $(A \cup B)' = A' \cap B'$.

11. Verify $(A \cap B)' = A' \cup B'$.

Use the given sets to solve each problem.

$U = \{A, B, C, D, \ldots, Z\}$ $A = \{I, C, E\}$ $B = \{C, U, B, E\}$

12. Find $A \cup B$.

13. Find $A \cap B$.

14. Find $|A \cap B|$.

15. Verify $(A \cup B)' = A' \cap B'$.

16. Verify $(A \cap B)' = A' \cup B'$.

Use the given sets to solve each problem.

$U = \{A, B, C, D, \ldots, Z\}$ $A = \{F, A, C, T, O, R\}$ $B = \{P, R, O, D, U, C, T\}$

17. Find $A \cup B$.

18. Find $A \cap B$.

19. Find $|A \cap B|$.

20. Verify $(A \cup B)' = A' \cap B'$.

21. Verify $(A \cap B)' = A' \cup B'$.

Use the given sets to solve each problem.

$U = \{a, b, c, d, \ldots, z\}$ $M = \{b, r, i, d, g, e\}$

$N = \{g, a, t, o, r\}$ $K = \{b, a, l, i, s, t, e, r\}$

22. Find $N \cap (M \cup K)$.

23. Find $N \cup (M \cap K)$.

24. Find $K \cup (N \cap M)$.

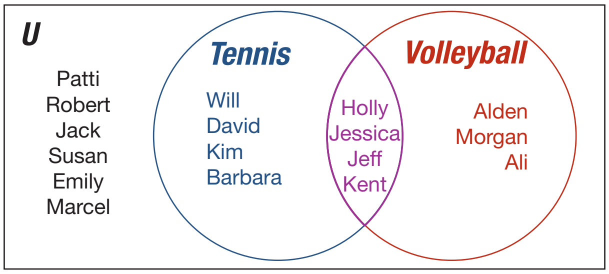
25. Verify $(M \cup K)' = M' \cap K'$.

26. Verify $(M \cap N)' = M' \cup N'$.

APPLICATIONS

27. A grocery store found that 275 of its customers use push carts to shop, 185 used a carry basket to shop, and that 145 used both a push cart and a carry basket. How many customers use only a push cart or a carry basket? Draw the Venn diagram.

Use the Venn diagram to solve each problem.



28. Which students played only tennis?

29. Determine which students played tennis or volleyball.

30. Determine which students played tennis and volleyball.

31. Find the number of students that play tennis or volleyball.

Solve each problem.

32. Determine the number of playing cards in a standard deck that are red cards or face cards.

33. Determine the number of playing cards in a standard deck that are odd numbered cards or black cards.

WRITING & THINKING

Show that each pair of sets is equal by drawing a Venn diagram of each set.

34. $A \cap B$ and $B \cap A$

35. $A \cup B$ and $B \cup A$

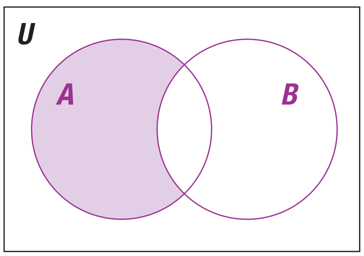
36. $(A \cap B) \cap C$ and $A \cap (B \cap C)$

37. $(A \cup B) \cup C$ and $A \cup (B \cup C)$

38. $A \cup \varnothing$ and A

Use set notation to represent each shaded region.

39.



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