This time, column 2 has a negative entry in the bottom row. Thus, we choose column 2 to be the pivot column. However, this example is different than previous ones because now there are no positive entries in the second column. Thus, the method we used in earlier examples to identify a pivot row won't work here. What does this mean?

Note that the bottom row corresponds to the equation $x_1 - 5x_2 + 2s_1 + z = 4$, or equivalently $z = 4 - x_1 + 5x_2 - 2s_1$. Since increasing x_1 and s_1 will decrease z (and because x_1 and s_1 are nonbasic variables), z will be optimized if $x_1 = 0$ and $s_1 = 0$. The second row corresponds to the equation $-x_1 - 2s_1 + s_2 = 0$, and if we substitute $x_1 = 0$ and $s_1 = 0$ into this equation, we obtain $s_2 = 0$. The first row corresponds to the equation $x_1 - 2x_2 + x_3 + s_1 = 2$, and if we substitute $x_1 = 0$ and $s_1 = 0$ into this equation, we obtain $-2x_2 + x_3 = 2$; or equivalently $x_3 = 2 + x_2$. Going back to $z = 4 - x_1 + 5x_2 - 2s_1$, we see that increasing x_2 will increase the value of z. However, looking at $x_3 = 2 + x_2$, we see that increasing x_2 will increase the value of x_3.

Therefore, if we increase the size of x_2 without bound, we will obtain a value of x_3 that gets infinitely large, and thus $f(x_1, x_2, x_3) = x_1 + x_2 + 2x_3$ gets infinitely large.

So, we conclude that $f(x_1, x_2, x_3)$ has no maximum value because it is unbounded. In addition, we observe that when using the simplex method to solve maximization problems with no solution, there is some point in the process in which a pivot column in a simplex tableau will have no positive entries.

7.3 EXERCISES

PRACTICE

For each given simplex tableau:

a. identify the basic and nonbasic variables,
b. find a basic feasible solution corresponding to this tableau,
c. find the pivot element, and
d. perform one pivot operation.

1.

x_1	x_2	s_1	s_2	z	
3	1	1	0	0	6
1	2	0	1	0	4
−7	−5	0	0	1	0

2.

x_1	x_2	s_1	s_2	z	
3	0	1	4	0	10
6	1	0	2	0	15
9	0	0	−8	1	30

3.

x_1	x_2	x_3	s_1	s_2	z	
1	1	1	1	0	0	5
3	0	3	−1	1	0	3
−3	0	−11	7	0	1	35

4.

x_1	x_2	x_3	s_1	s_2	s_3	z	
9	0	3	1	−1	0	0	54
1	1	1	0	1	0	0	18
3	0	7	0	−1	1	0	58
−14	0	−6	0	6	0	1	108

For each given maximization problem, find an optimal solution if one exists. If there are multiple solutions, find at least two. If there are no solutions, explain why. See Examples 1, 4, and 5.

5. Maximize $f(x_1, x_2) = 5x_1 + 6x_2$ subject to the following constraints.

$$\begin{cases} x_1 + 2x_2 \le 10 \\ 3x_1 + x_2 \le 15 \end{cases}$$

6. Maximize $f(x_1, x_2) = 3x_1 + x_2$ subject to the following constraints.

$$\begin{cases} x_1 + 4x_2 \le 5 \\ 2x_1 + x_2 \le 3 \\ 3x_1 + 2x_2 \le 5 \end{cases}$$

7. Maximize $f(x_1, x_2) = 4x_1 + 8x_2$ subject to the following constraints.

$$\begin{cases} 3x_1 + x_2 \le 6 \\ x_1 + 2x_2 \le 4 \end{cases}$$

8. Maximize $f(x_1, x_2, x_3) = 5x_1 + 7x_2 + 2x_3$ subject to the following constraints.

$$\begin{cases} x_1 + 2x_2 - x_3 \le 5 \\ 2x_1 + x_2 - 2x_3 \le 4 \end{cases}$$

9. Maximize $f(x_1, x_2, x_3) = 5x_1 + 6x_2 + 3x_3$ subject to the following constraints.

$$\begin{cases} 3x_1 + x_2 + x_3 \le 18 \\ x_1 + 4x_2 + x_3 \le 18 \\ x_1 + x_2 + 2x_3 \le 19 \end{cases}$$

10. Maximize $f(x_1, x_2) = 7x_1 - 3x_2$ subject to the following constraints.

$$\begin{cases} x_1 + x_2 \le 15 \\ 4x_1 + 2x_2 \le 21 \end{cases}$$

11. Maximize $f(x_1, x_2, x_3) = 6x_1 + 5x_2 + 4x_3$ subject to the following constraints.

$$\begin{cases} 2x_1 + x_2 \le 46 \\ x_1 + 3x_3 \le 54 \\ x_1 + x_2 + x_3 \le 60 \end{cases}$$

12. Maximize $f(x_1, x_2, x_3) = 7x_1 + 4x_2 + x_3$ subject to the following constraints.

$$\begin{cases} 2x_1 + x_2 + x_3 \le 40 \\ x_1 + x_2 + 2x_3 \le 30 \end{cases}$$

APPLICATIONS

Use the simplex method to solve each given application. See Examples 2 and 3.

13. A tool company manufactures drills and table saws. Suppose it costs the company \$30 to produce each drill, and the company makes a profit of \$6 for each drill sold. Moreover, it costs the company \$80 to produce each table saw, and the company makes a profit of \$11 for each table saw sold. In addition, suppose the company does not expect the total daily demand for the tools to exceed 100, and the company will halt production for the day once they spend a total of \$4500 in inventory. How many drills and table saws should the company produce and sell to optimize daily profit?

14. A company sells two kinds of fruit juices: Tropical Blend and Florida Sunshine. Suppose the Tropical Blend juice is 40% orange juice and 60% pineapple juice, and the Florida Sunshine juice is 60% orange juice and 40% pineapple juice. The company has a daily supply of 100 gallons of orange juice and 120 gallons of pineapple juice. In addition, suppose the company makes a profit of $0.50 per gallon of Tropical Blend sold and $0.40 per gallon of Florida Sunshine sold. How many gallons of Tropical Blend and Florida Sunshine juice should be sold in order for the company to make an optimal daily profit?

15. A company produces and sells two models of roller skates. The following table summarizes how long it takes (in minutes) to assemble and package one pair of each model.

	A	**B**
Assemble	45	60
Package	10	5

Suppose the company makes a $3.50 profit for each pair of model A roller skates sold and a $4 profit for each pair of model B roller skates sold. In addition, suppose the time available for assembling and packaging is 1200 hours and 200 hours, respectively. How many pairs of each model of roller skate should the company sell to maximize profit?

16. A candy store sells 2 kinds of trail mix. The standard trail mix consists of 50% peanuts and 50% M&M's, and the Chocolate Lover's trail mix consists of 37.5% peanuts and 62.5% M&M's. Suppose the candy store has a daily inventory of 100 pounds of peanuts and 150 pounds of M&M's. In addition, suppose the store makes $0.25 in profit for each pound of standard trail mix sold and $0.20 in profit for every pound of Chocolate Lover's sold. How many pounds of each kind of trail mix should the store sell to maximize daily profit?

17. A contractor builds two types of homes. Type A requires one lot, $200,000 in capital, and 165 worker-days of labor. Type B requires one lot, $250,000 of capital, and 150 worker-days of labor. The contractor owns 170 lots, has $40,000,000 available in capital, and has 27,600 worker-days of labor. The profit for selling each Type A home is $45,000, and the profit for each Type B home is $56,000. How many of each type of home should the contractor build to maximize profit?

18. A shoe manufacturer makes running shoes and basketball shoes. Each pair of running shoes requires 2 units of leather, 5 units of cotton, and 1 unit of rubber. Each pair of basketball shoes requires 4 units of leather, 4 units of cotton, and 2 units of rubber. Shipments are such that leather is limited to 120 units per day, cotton is limited to 150 units per day, and rubber is limited to 80 units per day. If the profits of the shoe manufacturer are $15 per pair of running shoes and $25 per pair of basketball shoes, how many pairs of each kind of shoe should be produced to maximize profit? What is the maximum profit?

19. A landlord has a brand new, empty apartment complex with 110 units available for rent; 25 of them have one bedroom, 50 have two bedrooms, and the other 35 have three bedrooms. He has set the rent at $550 per month for a one-bedroom unit, $850 per month for two bedrooms, and $1150 per month for three bedrooms. Assume that he must rent to one person per bedroom, and the laws restrict him to at most 200 occupants in the complex. How many of each type of apartment should the landlord rent to maximize the revenue? What is the maximum revenue?

20. A manufacturer produces 3 models of wooden rocking chairs. The time required for assembling, finishing, and packaging for each model are as follows.

	Model A	**Model B**	**Model C**
Assembling	2 hr 30 min	2 hr 30 min	3 hr 30 min
Finishing	1 hr 30 min	1 hr 20 min	1 hr 30 min
Packaging	30 min	45 min	1 hr

The total time for assembling, finishing, and packaging is 4500 hours, 2200 hours, and 1200 hours, respectively. The profit for each model is $55 (Model A), $60 (Model B), and $77 (Model C). How many of each type should be produced to maximize profit? What is the maximum profit?

21. A furniture store manufactures bookshelves, bed frames, and recliners. Each bookshelf uses 9 units of wood, each bed frame uses 23 units of wood, and each recliner uses 5 units of wood. Each bookshelf uses 1 unit of steel, each bed frame uses 4.5 units of steel, and each recliner uses 7.5 units of steel. It takes 35 minutes to assemble a bookshelf, 80 minutes to assemble a bed frame, and 75 minutes to assemble a recliner. Suppose the store has 600 units of wood and 200 units of steel in its inventory. Also, suppose the total time available for assembly is 3025 minutes. Finally, the price the store sells each item is $200 (bookshelf), $550 (bed frame), and $600 (recliner). How many of each kind of furniture should the store produce to maximize revenue? What is the maximum revenue?

22. A grower has 60 acres of land available for planting 3 crops. It costs $150 to produce an acre of carrots and the profit is $50 per acre. It costs $120 to produce an acre of potatoes and the profit is $42 per acre. Finally, it costs $180 to produce an acre of beets and the profit is $55 per acre. Assume the grower's cost cannot exceed $8700, and the grower must plant at least 10 acres of beets. How many acres of each crop should the grower plant in order to maximize profit? What is the maximum profit?

23. A shoe company is advertising their latest running shoe. They have a budget of $8000 per month for advertising. Newspaper ads cost $120 each and can occur a maximum of 20 times per month. Radio ads cost $400 each and can occur a maximum of 24 times a month. Suppose each newspaper ad reaches 8000 women under the age of 24 and each radio ad reaches 10,000 women of this age group. If the company wants to maximize its ad exposure to women under 24, how many of each ad should it purchase? What is the maximum number of exposures?

24. A candidate running for president wants to purchase radio and television ads to maximize his exposure. Suppose he has a $1 million budget for ads in his campaign, and election day is 6 months away. Each radio ad costs him $800 and can occur a maximum of 100 times over the next 6 months. Each television ad costs $4000, and can occur a maximum of 300 times over the next 6 months. Suppose each radio ad reaches 20 thousand voters and each television ad reaches 80 thousand voters. How many of each kind of ad should this presidential candidate purchase in order to maximize how many voters he reaches?

25. A researcher is studying how the rabbit population grows in two controlled environments. He wants each male rabbit to be in environment A for 30 minutes and each female rabbit to be in environment A for 20 minutes. He wants each male rabbit to be in environment B for 18 minutes and each female rabbit to be in environment B for 30 minutes. Suppose the researcher has the following time limits: 1040 minutes available for environment A and 750 minutes for environment B. How many male and female rabbits should the researcher use to maximize the total number of rabbits he uses in his study?

26. Biologists at a university have a research pond on their grounds that is stocked with bluegill and bass. Suppose each bluegill consumes 2 units of food per day while each bass consumes 8 units of food per day. In addition, suppose each bluegill produces 1 unit of waste per day while each bass produces 5 units of waste per day. Suppose the total daily supply of food for this pond is no more than 220 units. Moreover, in order to sustain life in the pond, the biologists want no more than a total of 120 units of waste excreted in the pond per day, and the bluegill population should be no more than 50. How many bluegill and bass should the biologists stock in this pond to maximize the number of fish they can study?

27. A botanist is studying how three different types of plant (onion, turnip, and radish) respond to three different controlled environments (A, B, and C). The following table describes how many days each plant is to be tested in each environment.

	Onion	**Turnip**	**Radish**
Environment A	4	1	3
Environment B	2	3	2
Environment C	1	2	1.5

There is limited time available in these environments: the researcher has 27 days for Environment A, 23 days for Environment B, and 15 days for Environment C. How many of each kind of plant should the researcher test in order to maximize the number of plants she is able to test?

28. A natural habitat in a zoo contains several feeding areas for lions, tigers, and bears. There are 3 foods (A, B, and C) available at each of these feeding areas. The following table describes how many pounds of each type of food is required to feed one animal.

	Food A	Food B	Food C
Lion	4	1	3
Tiger	5	3	2
Bear	3	6	2

Suppose the zoo has the following amounts of food available: 700 pounds (A), 490 pounds (B), and 455 pounds (C). How many of each type of animal can the zoo support so that the number of lions, tigers, and bears is a maximum?

WRITING & THINKING

29. Look once again at Example 4 in this section. Is the constraint $2x_1 + 5x_2 \le 80$ necessary? Explain your answer.

30. Consider a store that manufactures and sells two brands of clocks, say brand A and brand B. Suppose x_1 and x_2 denote the number of brand A and brand B clocks sold, respectively. Suppose the profit to be maximized (in dollars) is $f(x_1, x_2) = 5x_1 + 6x_2$. Moreover, suppose we have the following constraints.

$$\begin{cases} x_1 + 2x_2 \le 15 \\ 3x_1 + 2x_2 \le 26 \end{cases}$$

a. Use the simplex method to find the exact values of x_1 and x_2 that maximize f with the constraints. Do these values make sense in the context of this problem? Explain.

b. If your answer to part a. is no, can you find a solution that is optimal, feasible, and makes sense in the context of this problem? Explain.

31. Recall that it is important to always use the pivot row when doing row operations in any given iteration of the pivoting process. This exercise explores a possible pitfall of failing to do this. Look back at Example 3 of this section. At one stage in this problem, we arrived at the following simplex tableau.

$$\left[\begin{array}{ccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & z & \\ \hline \frac{6}{7} & \frac{8}{7} & 1 & \frac{4}{7} & 0 & 0 & 0 & \frac{4800}{7} \\ \frac{6}{7} & \frac{1}{7} & 0 & -\frac{10}{7} & 1 & 0 & 0 & \frac{2700}{7} \\ \frac{18}{7} & -\frac{11}{7} & 0 & -\frac{16}{7} & 0 & 1 & 0 & \frac{6840}{7} \\ \hline -\frac{9}{70} & \frac{22}{35} & 0 & \frac{32}{35} & 0 & 0 & 1 & \frac{7680}{7} \end{array}\right]$$

a. Explain why $\frac{18}{7}$ is the pivot element.

b. Since $\frac{6}{7}$ appears in rows 1 and 2 of the first column, the row operation $-R_2 + R_1$ seems like an intuitive first step in introducing a 0 in the top left entry. Also note that it doesn't use the pivot row, R_3. Try the sequence of row operations: $-R_2 + R_1$, $-\frac{1}{3}R_3 + R_1$, $\frac{1}{20}R_3 + R_4$, then $\frac{7}{18}R_3$. Do you obtain the optimal feasible solution of $(x_1, x_2, x_3, s_1, s_2, s_3, z) = (380, 0, 360, 0, 60, 0, 1146)$ with this new tableau?

c. How many basic variables are in the tableau you obtained in part b.? Is the number of basic variables different than 3, which is the number of constraints in this problem?

d. Can you do any additional row operations on the tableau you obtained in part b. to introduce another basic variable and arrive at the optimal feasible solution? Explain your answer.