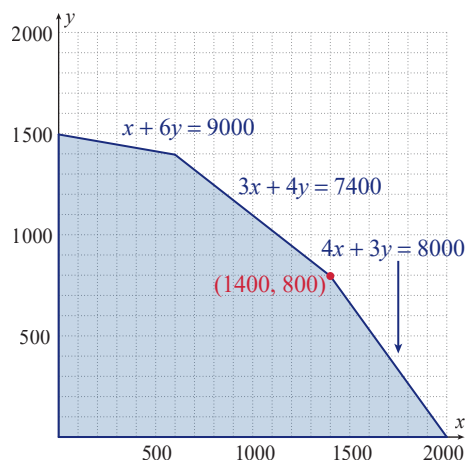


Next we sketch the feasible region.



The profit function in this situation is $f(x, y) = 7x + 8y$, and our goal is to maximize this function. The vertices of the feasible region are $(0, 1500)$, $(600, 1400)$, $(1400, 800)$, $(2000, 0)$, and $(0, 0)$. Of course, manufacturing 0 units of Model *A* and Model *B* is not going to be profitable, so we really only need to evaluate f at four vertices:

$$f(0, 1500) = 12,000$$

$$f(600, 1400) = 15,400$$

$$f(1400, 800) = 16,200$$

$$f(2000, 0) = 14,000$$

From these calculations, we conclude that the maximum profit is generated from making 1400 units of Model *A* and 800 units of Model *B*. The maximum profit would be \$16,200.

7.2 EXERCISES

💡 PRACTICE

Find the minimum and maximum values of the given functions, subject to the given constraints. See Examples 2 and 3.

1. Objective Function:

$$f(x, y) = 2x + 3y$$

Constraints:

$$\begin{cases} x \geq 0, y \geq 0 \\ x + y \leq 7 \end{cases}$$

2. Objective Function:

$$f(x, y) = 4x + y$$

Constraints:

$$\begin{cases} x \geq 0, y \geq 0 \\ x + y \leq 3 \end{cases}$$

3. Objective Function:

$$f(x, y) = 2x + 5y$$

Constraints:

$$\begin{cases} x \geq 0, y \geq 0 \\ x + y \leq 7 \end{cases}$$

4. Objective Function:

$$f(x, y) = 7x + 4y$$

Constraints:

$$\begin{cases} x \geq 0, y \geq 0 \\ 3x + y \leq 3 \end{cases}$$

5. Objective Function:

$$f(x, y) = 5x + 6y$$

Constraints:

$$\begin{cases} 0 \leq x \leq 7 \\ 0 \leq y \leq 10 \\ 8x + 5y \geq 40 \end{cases}$$

6. Objective Function:

$$f(x, y) = 9x + 7y$$

Constraints:

$$\begin{cases} 0 \leq x \leq 20 \\ 0 \leq y \leq 10 \\ 6x + 12y \geq 140 \end{cases}$$

7. Objective Function:

$$f(x, y) = 6x + 4y$$

Constraints:

$$\begin{cases} 0 \leq x \leq 4 \\ 0 \leq y \leq 5 \\ 4x + 3y \geq 10 \end{cases}$$

8. Objective Function:

$$f(x, y) = 3x + 7y$$

Constraints:

$$\begin{cases} 0 \leq x \leq 8 \\ 0 \leq y \leq 6 \\ 7x + 10y \geq 50 \end{cases}$$

9. Objective Function:

$$f(x, y) = 6x + 8y$$

Constraints:

$$\begin{cases} x \geq 0, y \geq 0 \\ 4x + y \leq 16 \\ x + 3y \leq 15 \end{cases}$$

10. Objective Function:

$$f(x, y) = x + 2y$$

Constraints:

$$\begin{cases} x \geq 0, y \geq 0 \\ 3x + y \leq 45 \\ x + 3y \leq 24 \end{cases}$$

11. Objective Function:

$$f(x, y) = 6x + y$$

Constraints:

$$\begin{cases} x \geq 0, y \geq 0 \\ 3x + 4y \geq 24 \\ 3x + 4y \leq 48 \end{cases}$$

12. Objective Function:

$$f(x, y) = 15x + 30y$$

Constraints:

$$\begin{cases} x \geq 0, y \geq 0 \\ 5x + 7y \geq 70 \\ 5x + 7y \leq 140 \end{cases}$$

13. Objective Function:

$$f(x, y) = 3x + 10y$$

Constraints:

$$\begin{cases} x \geq 0 \\ 2x + 4y \geq 8 \\ 5x - y \leq 10 \\ x + 3y \leq 40 \end{cases}$$

14. Objective Function:

$$f(x, y) = 20x + 30y$$

Constraints:

$$\begin{cases} x \geq 0 \\ 12x + 6y \geq 120 \\ 9x - 6y \leq 144 \\ x + 4y \leq 12 \end{cases}$$

APPLICATIONS

15. A plane carrying relief food and water can carry a maximum of 50,000 pounds and is limited in space to carrying no more than 6000 cubic feet. Each container of water weighs 60 pounds and takes up 1 cubic foot, and each container of food weighs 50 pounds and takes up 10 cubic feet. What is the region of constraint for the numbers of containers of food and water that the plane can carry?

16. A furniture company makes two kinds of sofas, the Standard model and the Deluxe model. The Standard model requires 40 hours of labor to build, and the Deluxe model requires 60 hours of labor to build. The finish of the Deluxe model uses both teak and fabric, while the Standard uses only fabric, with the result that each Deluxe sofa requires 5 square yards of fabric and each Standard sofa requires 8 square yards of fabric. Given that the company can use 200 hours of labor and 25 square yards of fabric per week building sofas, what is the region of constraint for the numbers of Deluxe and Standard sofas the company can make per week?
17. Sarah is looking through a clothing catalog, and she is willing to spend up to \$80 on clothes and \$10 for shipping. Shirts cost \$12 each plus \$2 shipping, and a pair of pants costs \$32 plus \$3 shipping. What is the region of constraint for the numbers of shirts and pairs of pants Sarah can buy?
18. Suppose you inherit \$75,000 from a previously unknown (and highly eccentric) uncle and that the inheritance comes with certain stipulations regarding investments. First, the dollar amount invested in bonds must not exceed the dollar amount invested in stocks. Second, a minimum of \$10,000 must be invested in stocks, and a minimum of \$5000 must be invested in bonds. Finally, a maximum of \$40,000 can be invested in stocks. What is the region of constraint for the dollar amounts that can be invested in the two categories of stocks and bonds?
19. A manufacturer produces two models of computers. The times (in hours) required for assembling, testing, and packaging each model are listed in the following table.

Process	Model X	Model Y
Assemble	2.5	3
Test	2	1
Package	0.75	1.25

The total times available for assembling, testing, and packaging are 4000 hours, 2500 hours, and 1500 hours, respectively. The profits per unit are \$50 for Model X and \$52 for Model Y. How many of each type should be produced to maximize profit? What is the maximum profit?

20. A manufacturer produces two types of fans. The times (in minutes) required for assembling, packaging, and shipping each type are listed in the following table.

Process	Type X	Type Y
Assemble	20	25
Package	40	10
Ship	10	7.5

The total times available for assembling, packaging, and shipping are 4000 minutes, 4800 minutes, and 1500 minutes, respectively. The profits per unit are \$4.50 for Type X and \$3.75 for Type Y. How many of each type should be produced to maximize profit? What is the maximum profit?

21. Ashley is making a set of patchwork curtains for her apartment. She needs a minimum of 16 yards of the solid material, at least 5 yards of the striped material, and at least 20 yards of the flowered material. She can choose between two sets of precut bundles. The olive-based bundle costs \$10 per bundle and contains 8 yards of the solid material, 1 yard of the striped material, and 2 yards of the flowered material. The cranberry-based bundle costs \$20 per bundle and includes 2 yards of the solid material, 1 yard of the striped material, and 7 yards of the flowered material. How many of each bundle should Ashley buy to minimize her cost and yet buy enough material to complete the curtains? What is her minimum cost?
22. A volunteer has been asked to drop off some supplies at a facility housing victims of a hurricane evacuation. The volunteer would like to bring at least 60 bottles of water, 45 first aid kits, and 30 security blankets on his visit. The relief organization has a standing agreement with two companies that provide victim packages. Company A can provide packages of 5 water bottles, 3 first aid kits, and 4 security blankets at a cost of \$1.50. Company B can provide packages of 2 water bottles, 2 first aid kits, and 1 security blanket at a cost of \$1.00. How many of each package should the volunteer pick up to minimize the cost? What total amount does the relief organization pay?
23. On your birthday your grandmother gave you \$25,000, but told you she would like you to invest the money for 10 years before you use any of it. Since you wish to respect your grandmother's wishes, you seek out the advice of a financial adviser. She suggests you invest at least \$15,000 in municipal bonds yielding 6% and no more than \$5000 in Treasury bills yielding 9%. How much should be placed in each investment so that income is maximized?
24. A boutique cell phone manufacturer produces two models: a retro model flip phone and a smart phone. The manufacturer's quota per day is to produce at least 100 flip phones and 80 smart phones. No more than 200 flip phones and 170 smart phones can be produced per day due to limitations on production. A total of at least 200 phones must be shipped every day.
- If the production costs are \$5 for a flip phone and \$7 for a smart phone, how many of each model should be produced on a daily basis to minimize cost and what would that cost be?
 - If each flip phone results in a \$2 loss but each smart phone results in a \$5 gain, how many of each model should be manufactured daily to maximize profit? What is the maximum profit if this number of phones is produced?