

If we defined matrix  $A$  to be  $\begin{bmatrix} 7 & 4 \\ 1 & 2 \end{bmatrix}$ , we would find the following to be its inverse.

NORMAL FLOAT AUTO REAL Radian MP

$[A]^{-1}$

$$\begin{bmatrix} 0.2 & -0.4 \\ -0.1 & 0.7 \end{bmatrix}$$

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$$\begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \\ -\frac{1}{10} & \frac{7}{10} \end{bmatrix}$$

## 6.5 EXERCISES

### PRACTICE

Write each of the following systems of equations as a single matrix equation. See Example 1.

1.  $\begin{cases} 14x - 5y = 7 \\ x + 9y = 2 \end{cases}$
2.  $\begin{cases} x - 5 = 9y \\ 3y - 2x = 8 \end{cases}$
3.  $\begin{cases} -6 - 2y = x \\ 9x + 14 = 3y \end{cases}$
4.  $\begin{cases} x - y = 5 \\ 2 - z = x \\ z - 3y = 4 \end{cases}$
5.  $\begin{cases} 3x_1 - 7x_2 + x_3 = -4 \\ x_1 - x_2 = 2 \\ 8x_2 + 5x_3 = -3 \end{cases}$
6.  $\begin{cases} x_3 = x_2 \\ x_2 = x_1 \\ x_1 = x_3 \end{cases}$
7.  $\begin{cases} \frac{3x - 8y}{5} = 2 \\ y - 2 = 0 \end{cases}$
8.  $\begin{cases} x - 7y = 5 \\ \frac{6 + x}{2} = 3y - 2 \end{cases}$
9.  $\begin{cases} 4x = 3y - 9 \\ 13 - 2x = -4y \end{cases}$
10.  $\begin{cases} -\frac{7}{3}y = \frac{5 - x}{6} \\ x - 5(y - 3) = -2 \end{cases}$
11.  $\begin{cases} 2x - y = -3z \\ y - x = 17 \\ 2 + z + 4x = 5y \end{cases}$
12.  $\begin{cases} 2x_1 - 3x_3 = 7 \\ x_2 - 10x_3 = 0 \\ 2x_1 - x_2 + x_3 = 1 \end{cases}$

Find the inverse of each of the following matrices, if possible. See Examples 2 and 3.

13.  $\begin{bmatrix} 0 & 4 \\ -5 & -1 \end{bmatrix}$
14.  $\begin{bmatrix} -2 & -2 \\ -1 & 2 \end{bmatrix}$
15.  $\begin{bmatrix} 3 & 4 \\ -4 & -5 \end{bmatrix}$
16.  $\begin{bmatrix} -1 & -1 \\ -\frac{1}{4} & -\frac{1}{2} \end{bmatrix}$
17.  $\begin{bmatrix} -\frac{1}{5} & 0 \\ \frac{1}{5} & \frac{1}{2} \end{bmatrix}$
18.  $\begin{bmatrix} -7 & 2 \\ 7 & -2 \end{bmatrix}$
19.  $\begin{bmatrix} -2 & -4 & -2 \\ 1 & -4 & 1 \\ 4 & -3 & 4 \end{bmatrix}$
20.  $\begin{bmatrix} -3 & 0 & -4 \\ 2 & 5 & 4 \\ 1 & -5 & -2 \end{bmatrix}$
21.  $\begin{bmatrix} -\frac{5}{11} & -\frac{8}{11} & 1 \\ \frac{13}{11} & \frac{12}{11} & -2 \\ \frac{11}{11} & \frac{11}{11} & -2 \\ -\frac{2}{11} & -\frac{1}{11} & 0 \end{bmatrix}$

$$22. -\frac{1}{31} \begin{bmatrix} 17 & -8 & -2 \\ 1 & 5 & 9 \\ -6 & 1 & 8 \end{bmatrix} \quad 23. \begin{bmatrix} -1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & 4 & -1 \end{bmatrix} \quad 24. \begin{bmatrix} -1 & 0 & -1 \\ -\frac{3}{2} & \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{4} \end{bmatrix}$$

$$25. \begin{bmatrix} -\frac{6}{5} & -\frac{2}{5} & -1 \\ \frac{3}{5} & \frac{1}{5} & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad 26. \begin{bmatrix} 2 & -2 & 1 \\ -2 & 2 & -3 \\ 1 & 0 & 2 \end{bmatrix} \quad 27. \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$28. \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad 29. \begin{bmatrix} \frac{2}{3} & \frac{8}{9} & \frac{1}{9} \\ -\frac{1}{3} & \frac{2}{9} & -\frac{2}{9} \\ -\frac{1}{3} & -\frac{7}{9} & -\frac{2}{9} \end{bmatrix} \quad 30. \begin{bmatrix} -3 & -3 & -4 \\ 0 & \frac{1}{4} & \frac{1}{2} \\ 2 & 2 & 3 \end{bmatrix}$$

For each pair of matrices, determine if either matrix is the inverse of the other.

$$31. \begin{bmatrix} -5 & -2 \\ -7 & 4 \end{bmatrix}, \begin{bmatrix} 10 & 4 \\ 14 & -8 \end{bmatrix} \quad 32. \begin{bmatrix} 9 & -18 \\ 3 & 12 \end{bmatrix}, \begin{bmatrix} -3 & -6 \\ -1 & -4 \end{bmatrix}$$

$$33. \begin{bmatrix} -6 & -1 & 1 \\ 4 & -1 & -2 \\ 1 & -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -2 & 3 \\ 2 & 5 & -8 \\ -3 & -7 & 10 \end{bmatrix} \quad 34. \begin{bmatrix} -1 & 4 & 5 \\ 3 & -11 & -17 \\ 4 & -17 & -19 \end{bmatrix}, \begin{bmatrix} -80 & -9 & -13 \\ -11 & -1 & -2 \\ -7 & -1 & -1 \end{bmatrix}$$

$$35. \begin{bmatrix} 2 & 0 & -1 \\ 3 & 4 & 2 \\ 1 & 1 & -3 \end{bmatrix}, \begin{bmatrix} 4 & 0 & -2 \\ 6 & 8 & 4 \\ 2 & 2 & -6 \end{bmatrix} \quad 36. \begin{bmatrix} -7 & 0 & -2 \\ -10 & -1 & -2 \\ -7 & -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 2 & -2 \\ 4 & -7 & 6 \\ 3 & -7 & 7 \end{bmatrix}$$

Solve the following systems by the inverse matrix method, if possible. If the inverse matrix method doesn't apply, use any other method to determine if the system is inconsistent or dependent. See Example 4.

$$37. \begin{cases} -2x - 2y = 9 \\ -x + 2y = -3 \end{cases} \quad 38. \begin{cases} 3x + 4y = -2 \\ -4x - 5y = 9 \end{cases} \quad 39. \begin{cases} -2x + 3y = 1 \\ 4x - 6y = -2 \end{cases}$$

$$40. \begin{cases} -2x + 4y = 5 \\ x - 4y = -3 \end{cases} \quad 41. \begin{cases} -5x = 10 \\ 2x + 2y = -4 \end{cases} \quad 42. \begin{cases} -3x + y = 2 \\ 9x - 3y = 5 \end{cases}$$

$$43. \begin{cases} 8x + 2y = 26 \\ -16x - 2y = -90 \end{cases} \quad 44. \begin{cases} 3x - 7y = -2 \\ -6x + 14y = 4 \end{cases} \quad 45. \begin{cases} 3y = 15 \\ 8x + 4y = 20 \end{cases}$$

$$46. \begin{cases} 4y + 3z = -254 \\ 2x - 2y - z = 100 \\ -x + y - 2z = 155 \end{cases} \quad 47. \begin{cases} 2x - y - 3z = -10 \\ 2y - z = 11 \\ -x + 4z = 0 \end{cases} \quad 48. \begin{cases} 3y - 4z = 15 \\ x + 2y - 3z = 9 \\ -x - y + 2z = -5 \end{cases}$$

 **WRITING & THINKING**

Solve the following sets of systems by the inverse matrix method. **Hint:** The coefficient matrix is the same for the three systems within a set.

$$49. \begin{cases} x+2y-z=2 \\ 3x+3y-z=-5 \\ 4x+4y-z=1 \end{cases} \quad \begin{cases} x+2y-z=1 \\ 3x+3y-z=1 \\ 4x+4y-z=1 \end{cases} \quad \begin{cases} x+2y-z=0 \\ 3x+3y-z=1 \\ 4x+4y-z=1 \end{cases}$$

$$50. \begin{cases} -x-y-2z=4 \\ x+3y+3z=0 \\ -3y-2z=9 \end{cases} \quad \begin{cases} -x-y-2z=1 \\ x+3y+3z=0 \\ -3y-2z=0 \end{cases} \quad \begin{cases} -x-y-2z=-2 \\ x+3y+3z=-3 \\ -3y-2z=1 \end{cases}$$

$$51. \begin{cases} -x+z=6 \\ -x+3y+2z=-11 \\ 2x-4y-3z=13 \end{cases} \quad \begin{cases} -x+z=-2 \\ -x+3y+2z=2 \\ 2x-4y-3z=-1 \end{cases} \quad \begin{cases} -x+z=-4 \\ -x+3y+2z=2 \\ 2x-4y-3z=0 \end{cases}$$

 **TECHNOLOGY**

Using a graphing utility, find the inverse of each of the following matrices, if possible. Round your answers to three decimal places if necessary.

52.  $\begin{bmatrix} -7 & 3 \\ -1 & 2 \end{bmatrix}$

53.  $\begin{bmatrix} -6 & 2 \\ -5 & 5 \end{bmatrix}$

54.  $\begin{bmatrix} -2 & 0 & 2 \\ 2 & -3 & 1 \\ 1 & -2 & 3 \end{bmatrix}$

55.  $\begin{bmatrix} 2.3 & 7.8 \\ -3.4 & 1.6 \end{bmatrix}$

56.  $\begin{bmatrix} 4.5 & -9.4 & 6.9 \\ 8.6 & -2.8 & 1.2 \\ 3.1 & 0.3 & -7.0 \end{bmatrix}$

57.  $\begin{bmatrix} 38 & -44 & 72 \\ -93 & 16 & 29 \\ 65 & 23 & -19 \end{bmatrix}$